A Debiased Machine Learning Single-Imputation Framework for Item Nonresponse in Surveys

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4th Workshop on Methodologies for Official Statistics
Rome, Italy

December 1, 2025

The setup

- $U = \{1, 2, ..., N\}$: finite population of size N.
- Y: Survey variable
- Goal: estimate the finite population mean

$$\mu = \frac{1}{N} \sum_{k \in U} y_k.$$

- S: sample of size n selected according to a probability sampling design with first-order inclusion probabilities $\pi_k = \mathbb{P}(I_k = 1) > 0$, where $I_k = 1$ if $k \in S$ and $I_k = 0$, otherwise.
- $\mathbf{I} = (I_1, \cdots, I_k, \cdots, I_N)^\top$: vector of sample selection indicators
- Full sample (Narain–Horvitz–Thompson) estimator of μ:

$$\widehat{\mu}_{\mathrm{HT}} = \frac{1}{N} \sum_{k \in S} w_k y_k \quad \text{with } w_k = 1/\pi_k$$

- Design-unbiased: $\mathbb{E}_p(\widehat{\mu}_{\mathrm{HT}}) = \mu$ for any survey variable Y.

Item nonresponse: Imputation

- The survey variable Y is prone to missing values.
- Let r_k be the response indicator such that

$$r_k = \begin{cases} 1 & \text{if } y_k \text{ is observed,} \\ 0 & \text{if } y_k \text{ is missing.} \end{cases}$$

- $\mathbf{r} = (r_1, \cdots, r_k, \cdots, r_N)^\top$: vector of response indicators.
- Set of respondents: $S_r = \{k \in S; r_k = 1\}$.
- Set of nonrespondents: $S_m = \{k \in S; r_k = 0\}$.

Assumptions

- $\mathbf{x}_k$: vector of fully observed variables associated with $k \in S$.
- Data are assumed to be MAR (Rubin, 1976):

$$\mathbb{P}\left[r_k = 1 | y_k, \mathbf{x}_k\right] = \mathbb{P}\left[r_k = 1 | \mathbf{x}_k\right] \equiv p(\mathbf{x}_k).$$

- Positivity assumption: $\mathbb{P}\left[r_k = 1 | \mathbf{x}_k\right] > 0$.
- Imputation model:

$$y_k = m(\mathbf{x}_k) + \epsilon_k, \qquad k \in U$$

  - $m(\mathbf{x})$: unknown and assumed to be continuous.
  - $\{\epsilon_k\}_{k \in U}$: independent such that $\mathbb{E}\left[\epsilon_k | \mathbf{x}_k\right] = 0$ and $\mathbb{V}\left[\epsilon_k | \mathbf{x}_k\right] = \sigma_k^2$.

Imputed estimator

- Imputed values:

$$y_k^* := \widehat{m}(\mathbf{x}_k), \qquad k \in S_m,$$

with $\widehat{m}$ an estimator of m fitted on

$$D_r := \{(\mathbf{x}_k, y_k)\,; k \in S_r\}\,.$$

- Imputed estimator of μ:

$$\widehat{\mu}_{imp} := \frac{1}{N}\left\{\sum_{k\in S} w_k r_k y_k + \sum_{k\in S} w_k (1-r_k)\widehat{m}(\mathbf{x}_k)\right\}.$$

- $\widehat{m}(\mathbf{x}_k)$: nuisance function, i.e., not of primary interest
- What are the properties of $\widehat{\mu}_{imp}$ and how to draw statistical inferences?

Parametric imputation

$$\widehat{\mu}_{imp} := \frac{1}{N}\left\{\sum_{k\in S} w_k r_k y_k + \sum_{k\in S} w_k (1-r_k)\widehat{m}(\mathbf{x}_k)\right\}$$

- For instance, linear regression imputation:

$$\widehat{m}(\mathbf{x}_k) = \mathbf{x}_k^\top \widehat{\boldsymbol{\beta}}$$

- First moment of the imputation model is correctly specified:
  - Correct functional form;
  - Correct specification of the $\mathbf{x}$-vector (interactions, terms accounting for curvature, etc.);
  - $\widehat{\mu}_{\text{imp}}$ is $\sqrt{n}$–consistent and asymptotically normal;
  - Several variance estimation methods have been developed (e.g., Haziza and Vallée, 2020).
- Misspecification of the first moment of the imputation model :
  - Bias does not vanish, even asymptotically $\longrightarrow$ Invalid inferences!

ML-Based Imputation

$$\widehat{\mu}_{imp} := \frac{1}{N}\left\{\sum_{k\in S} w_k r_k y_k + \sum_{k\in S} w_k(1-r_k)\widehat{m}(\mathbf{x}_k)\right\}$$

Nice features:

- Offers robustness against model misspecification such as incorrect functional form, omitted interaction terms, and unmodeled nonlinearities

Inference is challenging when using ML to obtain $\widehat{m}(\mathbf{x}_k)$:

- ML estimators typically converge slower than $n^{-1/2}$ $\longrightarrow$ $\widehat{\mu}_{imp}$ is consistent ;
- A CLT usually fails to hold $\longrightarrow$ difficult to justify confidence intervals;
- Lack of Neyman orthogonality $\longrightarrow$ $\widehat{\mu}_{\text{imp}}$ is affected by first-order errors in $\widehat{m}(\mathbf{x}_k)$
  $\longrightarrow$ Estimating the variance of $\widehat{\mu}_{\text{imp}}$ is very challenging for arbitrary ML imputation methods

Double-debiased imputation based on machine learning

- Double debiased ML framework: Chernozhukov et al. (2018)
- Relies on two key ingredients:
  - Neyman orthogonality
    - Small errors in nuisance parts have only second-order (negligible) impact on the target parameter (here, the population mean)
    - It provides "local immunity".
  - Crossfitting
- Doubly robust estimators: satisfy the Neyman orthogonality property.
- Originally, DR estimators were proposed to provide some robustness against the misspecification of either the imputation model or the nonresponse model: e.g., Robins et al. (1994), Kott (1994), Haziza and Rao (2006), Cao et al. (2009), and Kim and Haziza (2014).

Oracle doubly robust estimator

- We start by considering an oracle doubly robust (DR) estimator of μ:

$$\begin{aligned}\widehat{\mu}_{dr}(m,p) &= \frac{1}{N}\left\{\sum_{k\in S} w_k m(\mathbf{x}_k) + \sum_{k\in S} w_k \frac{r_k}{p(\mathbf{x}_k)}\{y_k - m(\mathbf{x}_k)\}\right\}, \\ &= \frac{1}{N}\sum_{k\in S} w_k \eta_k,\end{aligned}$$

where η_k is a pseudo-value associated with unit k:

$$\eta_k = m(\mathbf{x}_k) + \frac{r_k}{p(\mathbf{x}_k)}\{y_k - m(\mathbf{x}_k)\} \quad \text{for } k \in S.$$

Oracle doubly robust estimator: Its properties

- Unbiasedness:

$$\mathbb{E}\left\{\widehat{\mu}_{dr}(m,p)-\mu\right\}=0$$

$\mathbb{E}(.)$: expectation with respect to the joint distribution induced by the outcome regression model, the sampling design, and the propensity score model.

- Root-n consistency:

$$\widehat{\mu}_{dr}(m,p)-\mu=O_p\left(n^{-1/2}\right)$$

Oracle doubly robust estimator: Its properties

- Variance based on the reverse approach (Shao and Steel, 1999):

$$\mathbb{V}_{tot}\left(\widehat{\mu}\right) = \mathbb{E}\mathbb{V}\left(\widehat{\mu} \mid \mathbf{X}, \mathbf{r}, \mathbf{y}\right) + \mathbb{E}\mathbb{V}\left\{\mathbb{E}(\widehat{\mu} - \mu \mid \mathbf{X}, \mathbf{r}, \mathbf{y}) \mid \mathbf{X}, \mathbf{r}\right\},$$

  where $\mathbf{y} = (y_1, \ldots, y_N)^\top$ and $\mathbf{X}$ is the matrix whose kth row is $\mathbf{x}_k^\top$.

- Total variance of $\widehat{\mu}_{dr}(m, p)$ (assuming n/N negligible):

$$\mathbb{V}_{tot}\left\{\widehat{\mu}_{dr}(m, p)\right\} = \frac{1}{N^2} \sum_{k \in U} \sum_{\ell \in U} (\pi_{k\ell} - \pi_k \pi_\ell) \frac{\eta_k}{\pi_k} \frac{\eta_\ell}{\pi_\ell}$$

- Special case: SRSWOR

$$\mathbb{V}_{tot}\left\{\widehat{\mu}_{dr}(m, p)\right\} = \left(1 - \frac{n}{N}\right) \frac{S_\eta^2}{n}$$

Oracle doubly robust estimator: Its properties

- Unbiased estimator of $\mathbb{V}_{tot}\{\widehat{\mu}_{dr}(m,p)\}$:

$$\widehat{\mathbb{V}}_{tot}\{\widehat{\mu}_{dr}(m,p)\} := \frac{1}{N^2}\sum_{k\in S}\sum_{\ell\in S}\frac{(\pi_{k\ell}-\pi_k\pi_\ell)}{\pi_{k\ell}}\frac{\eta_k}{\pi_k}\frac{\eta_\ell}{\pi_\ell},$$

where

$$\eta_k = m(\mathbf{x}_k) + \frac{r_k}{p(\mathbf{x}_k)}\{y_k - m(\mathbf{x}_k)\} \quad \text{for } k \in S.$$

- Special case: SRSWOR

$$\widehat{\mathbb{V}}_{tot}\{\widehat{\mu}_{dr}(m,p)\} = \left(1-\frac{n}{N}\right)\frac{s_\eta^2}{n}$$

- Under some regularity conditions:

$$\frac{\widehat{\mu}_{dr}(m,p)-\mu}{\sqrt{\widehat{\mathbb{V}}_{tot}\{\widehat{\mu}_{dr}(m,p)\}}} \to \mathcal{N}(0,1).$$

DR estimation based on estimated nuisance functions

- Consider a DR estimator:

$$\begin{aligned}\widehat{\mu}_{dr}(\widehat{m},\widehat{p}) &= \frac{1}{N}\left\{\sum_{k\in S} w_k \widehat{m}(\mathbf{x}_k) + \sum_{k\in S} w_k \frac{r_k}{\widehat{p}(\mathbf{x}_k)}\{y_k - \widehat{m}(\mathbf{x}_k)\}\right\}, \\ &= \frac{1}{N}\sum_{k\in S} w_k \widehat{\eta}_k,\end{aligned}$$

where

$$\widehat{\eta}_k = \widehat{m}(\mathbf{x}_k) + \frac{r_k}{\widehat{p}(\mathbf{x}_k)}\{y_k - \widehat{m}(\mathbf{x}_k)\} \quad \text{for } k \in S.$$

- $\widehat{m}(\mathbf{x}_k)$: predicted value for the missing $y_k, k \in S$, based on some ML method
- $\widehat{p}(\mathbf{x}_k)$: estimated response probability for $k \in S_r$ based on some ML method.
- Neither $\widehat{m}(\mathbf{x})$ nor $\widehat{p}(\mathbf{x})$ is of primary interest $\longrightarrow$ can be viewed as nuisance functions

DR estimation based on estimated nuisance functions

- Assuming n/N is negligible, an estimator of the variance of $\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})$ is

$$\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})\right\} := \frac{1}{N^2} \sum_{k \in S} \sum_{\ell \in S} \frac{(\pi_{k\ell} - \pi_k \pi_\ell)}{\pi_{k\ell}} \frac{\widehat{\eta}_k}{\pi_k} \frac{\widehat{\eta}_\ell}{\pi_\ell},$$

$$\widehat{\eta}_k := \widehat{m}(\mathbf{x}_k) + \frac{r_k}{\widehat{p}(\mathbf{x}_k)} \left\{y_k - \widehat{m}(\mathbf{x}_k)\right\}, \quad k \in S.$$

- Special case: SRSWOR

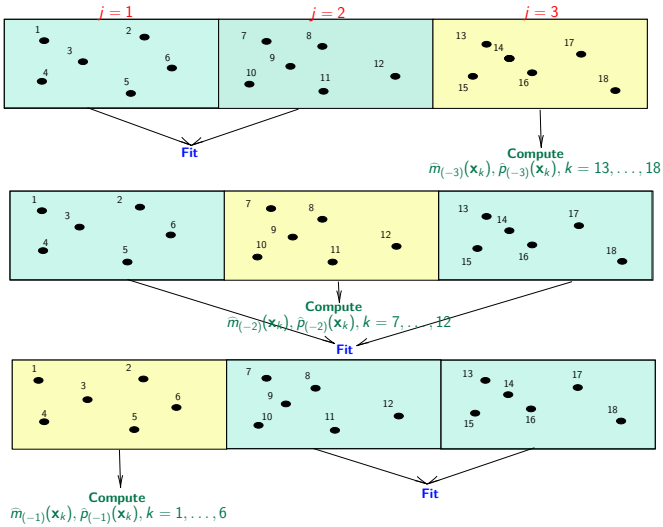
$$\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr}(m, p)\right\} = \left(1 - \frac{n}{N}\right) \frac{s^2_{\widehat{\eta}}}{n}$$

- Issue: Complex ML procedures can overfit noise and end up with artificially small residuals

$\longrightarrow$ Both $\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})$ and $\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})\right\}$ may exhibit bias in finite samples.

- Solution: Crossfitting!

Crossfitting with $J = 3$ folds



DR estimation based on crossfitting

- A cross-fitted version of $\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})$ is given by

$$\begin{aligned}\widehat{\mu}_{dr,cf}(\widehat{m}, \widehat{p}) &= \frac{1}{N}\left\{\sum_{k\in S} w_k \widehat{m}_{(-j)}(\mathbf{x}_k) + \sum_{k\in S} w_k \frac{r_k}{\widehat{p}_{(-j)}(\mathbf{x}_k)}\{y_k - \widehat{m}_{(-j)}(\mathbf{x}_k)\}\right\} \\ &= \frac{1}{N}\sum_{k\in S} w_k \widehat{\eta}_{k,cf},\end{aligned}$$

where

$$\widehat{\eta}_{k,cf} = \widehat{m}_{(-j)}(\mathbf{x}_k) + \frac{r_k}{\widehat{p}_{(-j)}(\mathbf{x}_k)}\Big\{y_k - \widehat{m}_{(-j)}(\mathbf{x}_k)\Big\}, \quad \text{for } k \in S.$$

- The residuals based on crossfitting, $y_k - \widehat{m}_{(-j)}(\mathbf{x}_k)$, are called "honest residuals".

DR estimation based on crossfitting

Theorem

We make the following assumption: The sequence of estimators $\{\widehat{m}_v\}_{v\in\mathbb{N}}$ *and* $\{\widehat{p}_v\}_{v\in\mathbb{N}}$ *satisfy the following rate condition:*

$$\lim_{n\to\infty}\sqrt{n}\times\mathbb{E}\left[\frac{1}{|\mathcal{F}_j|}\sum_{k\in\mathcal{F}_j}\left(\widehat{m}_{(-j)}\left(\mathbf{x}_k\right)-m\left(\mathbf{x}_k\right)\right)^2\right]^{1/2}\mathbb{E}\left[\frac{1}{|\mathcal{F}_j|}\sum_{k\in\mathcal{F}_j}\left(\widehat{p}_{(-j)}\left(\mathbf{x}_k\right)-p\left(\mathbf{x}_k\right)\right)^2\right]^{1/2}=0.$$

Then,

$$\widehat{\mu}_{dr,cf}\left(\widehat{m},\widehat{p}\right)-\widehat{\mu}_{dr}\left(m,p\right)=o_p\left(1/\sqrt{n}\right).$$

Both estimators are asymptotically equivalent! $\longrightarrow$ The estimator $\widehat{\mu}_{dr,cf}\left(\widehat{m},\widehat{p}\right)$ inherits all the properties of the oracle estimator $\widehat{\mu}_{dr}\left(m,p\right)$!

DR estimation based on crossfitting

- Root-n consistency:

$$\widehat{\mu}_{dr,cf}\left(\widehat{m},\widehat{p}\right)-\mu=O_p\left(\frac{1}{\sqrt{n}}\right)$$

- If n/N is negligible, an estimator of $\mathbb{V}_{tot}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}))$, is

$$\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})\right\}=\frac{1}{N^2}\sum_{k\in S}\sum_{\ell\in S}\frac{(\pi_{k\ell}-\pi_k\pi_\ell)}{\pi_{k\ell}}\frac{\widehat{\eta}_{k,cf}}{\pi_k}\frac{\widehat{\eta}_{k,cf}}{\pi_\ell},$$

where

$$\widehat{\eta}_{k,cf}=\widehat{m}_{(-j)}(\mathbf{x}_k)+\frac{r_k}{\widehat{p}_{(-j)}(\mathbf{x}_k)}\left\{y_k-\widehat{m}_{(-j)}(\mathbf{x}_k)\right\},\quad \text{for } k\in S.$$

- Special case: SRSWOR

$$\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr}(m,p)\right\}=\left(1-\frac{n}{N}\right)\frac{s^2_{\widehat{\eta},cf}}{n}$$

DR estimation based on crossfitting

- Assume the previous assumptions + higher moment conditions on the sampling design. Then,

$$\frac{\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})\right\}}{\mathbb{V}_{tot}\left\{\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})\right\}} \xrightarrow[v\to+\infty]{p} 1.$$

- Asymptotic normality.

$$\frac{\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})-\mu}{\sqrt{\widehat{\mathbb{V}}_{tot}\left\{\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})\right\}}} \to \mathcal{N}(0,1).$$

Implementation

- The estimator $\widehat{\mu}_{dr,cf}(\widehat{m}, \widehat{p})$ is not written in a convenient form $\longrightarrow$ we seek an estimator of the form

$$\widehat{\mu}_{imp} = \frac{1}{N}\left\{\sum_{k\in S_r} w_k y_k + \sum_{k\in S_m} w_k \widehat{m}(\mathbf{x}_k)\right\}.$$

- Data file after imputation:

Unit	x_1	$\cdots$	x_p	r	w	$ry + (1-r)\widehat{m}$
1	x_{11}	$\cdots$	x_{1p}	1	w_1	y_1
2	x_{21}	$\cdots$	x_{2p}	0	w_2	$\widehat{m}(\mathbf{x}_2)$
3	x_{31}	$\cdots$	x_{3p}	1	w_3	y_3
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	x_{n1}	$\cdots$	x_{np}	0	w_n	$\widehat{m}(\mathbf{x}_n)$

Implementation

- Obtain the modified imputed values for $k \in S_m$:

$$\widehat{m}^*(\mathbf{x}_k) = \widehat{m}_{(-j)}(\mathbf{x}_k) + \frac{1}{\sum_{\ell \in S_m} w_\ell} \sum_{\ell \in S_r} w_\ell \left\{ \frac{1 - \widehat{p}_{(-j)}(\mathbf{x}_\ell)}{\widehat{p}_{(-j)}(\mathbf{x}_\ell)} \right\} \left\{ y_\ell - \widehat{m}_{(-j)}(\mathbf{x}_\ell) \right\}$$

- Data file after imputation:

Unit	x_1	$\cdots$	x_p	r	w	$ry + (1-r)\widehat{m}^*$
1	x_{11}	$\cdots$	x_{1p}	1	w_1	y_1
2	x_{21}	$\cdots$	x_{2p}	0	w_2	$\widehat{m}^*(\mathbf{x}_2)$
3	x_{31}	$\cdots$	x_{3p}	1	w_3	y_3
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	x_{n1}	$\cdots$	x_{np}	0	w_n	$\widehat{m}^*(\mathbf{x}_n)$

- As a result, $\frac{1}{N}\left\{\sum_{k \in S_r} w_k y_k + \sum_{k \in S_m} w_k \widehat{m}^*(\mathbf{x}_k)\right\} = \widehat{\mu}_{dr,cf}(\widehat{m}, \widehat{p})$.

A recap...

- **Step 1**: Select a ML procedure for $m(\mathbf{x}_k)$ and a ML procedure for $p(\mathbf{x}_k)$. Optimize the hyperparameters for each ML procedure (e.g., cross-validation).
- **Step 2**: Compute

$$\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}) = \frac{1}{N}\left\{\sum_{k\in S} w_k \widehat{m}_{(-j)}(\mathbf{x}_k) + \sum_{k\in S} w_k \frac{r_k}{\widehat{p}_{(-j)}(\mathbf{x}_k)}\{y_k - \widehat{m}_{(-j)}(\mathbf{x}_k)\}\right\}$$

- **Step 3**: Modify the imputed values $\longrightarrow \widehat{m}^*(\mathbf{x}_k)$.
- **Step 4**: Estimate the variance of $\widehat{\mu}_{dr,cf}$ by

$$\widehat{\mathbb{V}}_{tot}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})) = \frac{1}{N^2}\sum_{k\in S}\sum_{\ell\in S}\frac{(\pi_{k\ell}-\pi_k\pi_\ell)}{\pi_{k\ell}}\frac{\widehat{\eta}_{k,cf}}{\pi_k}\frac{\widehat{\eta}_{\ell,cf}}{\pi_\ell}$$

- **Step 5**: Compute a 95% confidence interval for μ:

$$\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}) \pm 1.96\sqrt{\widehat{\mathbb{V}}_{tot}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}))}$$

Simulation study: Generating the population

- We generated 5 mutually independent predictors $x_1, \ldots, x_5$, according to a standard normal distribution.
- We then repeated $R = 10,000$ iterations of the following process:

  (1) Given the predictors $x_1, \ldots, x_5$, we generated four finite populations with 2 different values of N: 4,000; 16,000.

  One survey variables Y:

  $$Y = 20 \times \mathbb{1}_{x_1 > 0} + 10 \sin(x_2) + 5x_3^2 + \mathcal{N}(0, 1);$$

  (2) A sample S was selected from each population according to simple random sampling without replacement with $n/N = 10\%$
  $\longrightarrow$ n: 400; 1, 600.

Simulation study: Generating nonresponse

(3) In each sample:

The missing values to Y were generated independently from a Bernoulli distribution with probability

$$p(\mathbf{x}_k) = \frac{1}{6}\left\{0.5 + \sin(x_1) + \sin(x_2) + \sin(x_3)\right\}.$$

In each sample, we obtained approximately 50% of missing values.

Simulation study: Estimation through ML

(4) In each sample, we computed $\widehat{\mu}_{dr}(\widehat{m}, \widehat{p})$ and $\widehat{\mu}_{dr,cf}(\widehat{m}, \widehat{p})$ based on $J = 10$ folds:

ML procedure for $m(\mathbf{x}_k)$:

(i) Random forests (RF) with 15 observations in each terminal node and $B = 250$ trees;

(ii) Cubist;

(iii) XGBoost with a learning rate set to 1, a depth set to 5, and $B = 50$ trees.

ML procedure for $p(\mathbf{x}_k)$:

(i) Regression trees with 40 observations in each terminal node

(ii) Random forests (RF) with 20 observations per terminal node and $B = 250$ trees

Simulation study: Monte Carlo Measures

1. Monte Carlo percent relative bias of a point estimator:

$$\mathrm{RB}(\widehat{\mu}) = 100 \times \frac{\mathbb{E}_{MC}(\widehat{\mu} - \mu)}{\mu}.$$

2. Relative efficiency of $\widehat{\mu}$:

$$\mathrm{RE}(\widehat{\mu}) = 100 \times \frac{\mathrm{MSE}_{MC}(\widehat{\mu})}{\mathrm{MSE}_{MC}(\widehat{\mu}_{\mathrm{HT}})}.$$

3. Monte Carlo percent relative bias of variance estimator:

$$\mathrm{RB}\left\{\widehat{\mathbb{V}}(\widehat{\mu})\right\} = 100 \times \frac{\mathbb{E}_{MC}\left\{\widehat{\mathbb{V}}(\widehat{\mu})\right\} - \mathbb{V}_{MC}(\widehat{\mu})}{\mathbb{V}_{MC}(\widehat{\mu})}.$$

4. Monte Carlo coverage rate of 95% normal-based confidence intervals.

Simulation study: Results for the variable Y

	$\widehat{\mu}_{dr}(\widehat{m},\widehat{p})$	$\widehat{\mathbb{V}}(\widehat{\mu}_{dr}(\widehat{m},\widehat{p}))$	$\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})$	$\widehat{\mathbb{V}}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}))$
$[m(\mathbf{x}_k); p(\mathbf{x}_k)]$ = [Cubist; Trees]				
$n = 400$	0.8 (130)	-14.2 (92.6)	0.5 (130)	-2.1 (94.7)
$n = 1,600$	-0.1 (112)	-8.8 (93.8)	-0.1 (113)	-0.8 (94.6)
$[m(\mathbf{x}_k); p(\mathbf{x}_k)]$ = [Cubist; Trees]				
$n = 400$	-0.6 (116)	-14.6 (92.4)	0.2 (129)	-3.5 (94.2)
$n = 1,600$	-0.1 (116)	-11.3 (93.7)	0.0 (120)	0.0 (95.0)

Table 1: Monte Carlo statistics associated with point and variance estimator for the survey variable Y

Simulation study: Results for the variable Y

	$\widehat{\mu}_{dr}(\widehat{m},\widehat{p})$	$\widehat{\mathbb{V}}(\widehat{\mu}_{dr}(\widehat{m},\widehat{p}))$	$\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})$	$\widehat{\mathbb{V}}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}))$
$[m(\mathbf{x}_k); p(\mathbf{x}_k)] = [\text{RF; RF}]$				
$n = 400$	0.1 (158)	-20.2 (92.6)	0.1 (160)	-0.5 (95.3)
$n = 1,600$	0.1 (152)	-23.0 (91.8)	0.1 (152)	-6.4 (94.9)
$[m(\mathbf{x}_k); p(\mathbf{x}_k)] = [\text{XGBoost; Trees}]$				
$n = 400$	-0.2 (201)	-52.0 (82.4)	0.2 (228)	-2.1 (94.7)
$n = 1,600$	-0.1 (195)	-50.4 (83.9)	0.0 (218)	0.4 (95.0)

Table 2: Monte Carlo statistics associated with point and variance estimator for the survey variable Y

What happens in the case of a poor architecture?

	$\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p})$	$\widehat{\mathbb{V}}(\widehat{\mu}_{dr,cf}(\widehat{m},\widehat{p}))$	MSE($\widehat{m}$)	MSE($\widehat{p}$)
	$[m(\mathbf{x}_k); p(\mathbf{x}_k)] = [\text{Cubist}; \text{XGBoost}]$			
$n = 200$	-1.2 (583)	50.8 (98.3)	0.29	0.108
$n = 400$	-0.5 (514)	52.5 (98.4)	0.18	0.108
$n = 800$	-0.3 (484)	51.7 (98.3)	0.13	0.103
$n = 1,600$	-0.2 (446)	48.3 (98.3)	0.09	0.093

Table 3: Results for the survey variable Y

Final Remarks

- Instead of a single ML procedure for estimating $p(\mathbf{x}_k)$ and $m(\mathbf{x}_k)$, we can use an aggregation procedure for either or both of them.
- Extension to random imputation for categorical variables: Use of a rejective imputation procedure
- Extension to finite population parameters that can be expressed as a solution of an estimating equation (e.g., distribution function, quantiles): Under investigation
- Unified theory for model-assisted estimation based on ML procedures: Under investigation.
- Small area estimation with ML: Under investigation