House Price Indexes: A Comparison of Repeat Sales and Other Multilateral Methods

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Abstract: This paper compares multilateral methods for measuring house price change: geometric and arithmetic repeat sales, time product dummy, GEKS-Jevons, and Geary-Khamis. These methods do not use any characteristics information other than a property identifier such as address. Another common feature is that they only make use of data on houses that have been sold at least twice during the sample period; in this sense they are all repeat sales or "matched pairs" methods. Empirical results are presented for the Netherlands using transactions data from the land registry. We also provide evidence on the magnitude of index revisions.

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1. Introduction

One of the best known approaches to constructing a house price index is the repeat sales method (Bailey, Muth and Nourse, 1963). In the United States, the house price indexes published by S&P Global (2020), the Federal Housing Finance Agency (Calhoun, 1996) and Freddie Mac (Stephens et al., 1995) are all based on repeat sales methods. Repeat sales is a multilateral method. Multilateral price indexes are estimated simultaneously for multiple periods from pooled cross section data; they are transitive and independent of the choice of base period. A drawback of multilateral price indexes is that they will be revised when data is added to the sample.

The standard (geometric) repeat sales method uses linear regression to derive the price indexes. Another regression-based multilateral approach to estimating house price indexes is the time dummy hedonic method, which makes use of the price-determining characteristics of properties to adjust for quality changes. An advantage of the hedonic method is that it can use the data for all properties sold whereas the repeat sales method, as the name suggests, depends on matched pairs and ignores data on properties that have been sold only once during the whole sample period.

Some statistical agencies have access to transactions data from the land registry. Typically, the land registry's transactions data set covers all property sales in a country and contains selling price, address (or exact location in terms of longitude and latitude), and a few other property characteristics, such as type of house. Limited information on characteristics precludes the estimation of accurate hedonic house price indexes. Since the land registry's data set is a complete enumeration, the use of repeat sales is possible to measure price change, provided that the sample period is long enough.

There are several alternative multilateral methods that can be considered for the construction of house price indexes from land registry data. The first is what we call the time product dummy method.[1] This method includes property fixed effects rather than property characteristics as independent variables in the regression model. Fixed effects

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[1] These alternative multilateral index methods were originally developed for spatial price comparisons. For example, the time product dummy method adapts the country product dummy method proposed by Summers (1973) to the time domain. It is currently used in New Zealand (Bentley, 2018) and Belgium to compile price indexes for rents, which are components of the Consumer Price Index (CPI). Following the work of Ivancic, Diewert and Fox (2011), some statistical agencies have implemented multilateral index methods to handle scanner data in the CPI. For an overview of multilateral methods, see e.g., van Kints, de Haan and Webster (2019) and Diewert (2020).

modelling is normally applied to panel data, but here we face a highly unbalanced panel since houses are sold infrequently. As far as we know, Gao and Wang (2007) were the first to use the time product dummy method – which they called "multiple transactions model" – in the context of housing; Grimes and Young (2010) and Grimes, Sorensen and Young (2021) applied it to New Zealand house prices, Osland (2013) to Norwegian house prices.

The second alternative multilateral method is GEKS.[2] GEKS is a straightforward approach to imposing transitivity on a set of bilateral price indexes. In its original form, GEKS has bilateral Fisher indexes as elements. In the housing context, where quantities sold are equal to 1 because each property can be deemed unique, unweighted bilateral price indexes are appropriate. We use bilateral Jevons price indexes and thus construct GEKS-Jevons indexes. Melser (2013) was the first to examine GEKS-Jevons for house prices.

Geary-Khamis is our third alternative multilateral method. This method can be viewed as the arithmetic counterpart to the time product dummy method although it is not regression-based. To the best of our knowledge, Geary-Khamis has not been applied before to residential property.

In this paper we compare repeat sales, time product dummy, GEKS-Jevons and Geary-Khamis methods, and present empirical evidence using data from the Dutch land registry. Section 2 outlines the underlying model for the regression-based methods and briefly discusses the time dummy hedonic method. Section 3 discusses the related time product dummy method and shows that, similar to the repeat sales method, properties that are sold only once during the sample period are ignored in the estimation. Section 4 addresses geometric repeat sales. It is shown that the time product dummy method is a constrained version of repeat sales which estimates a unique base period price for each property. Section 5 discusses GEKS-Jevons and argues that (geometric) repeat sales is a form of weighted GEKS-Jevons. Section 6 explains Geary-Khamis and its relationship with arithmetic repeat sales. Section 7 addresses index revisions and a few other issues. Section 8 describes our monthly data set, which consists of nearly all sales of existing residential property in the Netherlands for a period of more than 26 years, and presents empirical results. Section 9 concludes.

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[2] GEKS is an acronym based on the surnames of the "inventors": Gini (1931), Eltetö and Köves (1941), and Szulc (1964).

2. The basic model and hedonic regression

Our sample period, or estimation window, consists of time periods $0,1,...,T$. In practice the periods will be months or quarters. A simple model for house price is

$$p_i^t = P^t a_i , \tag{1}$$

where p_i^t denotes the price of house *i* in period *t*, P^t is the general price level in period *t*, and a_i is a time-invariant house-specific term.[3] The house price index going (directly) from the base period 0 to the comparison period t $(t = 1,...,T)$ is given by the ratio of price levels P^t / P^0. In equation (1), price p_i^t is strictly proportional to the price level P^t. It implies $p_i^t / p_i^0 = P^t / P^0$, the same price change for all houses. This is not useful; the model should allow for "disturbances" that lead to price change differences. Taking natural logarithms of both sides of (1), setting $\ln P^t = \delta^t$ and $\ln a_i = \gamma_i$, and then adding random errors with mean 0 leads to our basic stochastic log-linear model:

$$\ln p_i^t = \delta^t + \gamma_i + \varepsilon_i^t . \tag{2}$$

Equation (2) is the starting point for at least three regression-based multilateral methods that have been proposed in the literature for the construction of price indexes: time dummy hedonic, time product dummy, and repeat sales. In this section, we briefly review the first method, Time Dummy Hedonic (TDH). This is arguably the best-known method for measuring quality-adjusted price change, for residential property as well as for other goods.

A good can be viewed as a bundle of characteristics or attributes that consumers value. For housing, location is obviously important. The hedonic hypothesis postulates that price is determined by these characteristics. That is, γ_i in (2) is a function of price-determining characteristics x_{ki} $(k = 1,..., K)$. If, as usual, a linear combination is chosen, $\gamma_i = \sum_{k=1}^{K} \beta_k x_{ik}$, the estimating equation for the TDH approach reads

$$\ln p_i^t = \alpha + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{k=1}^{K} \beta_k x_{ik} + \varepsilon_i^t , \tag{3}$$

where D_i^t is a dummy variable that has the value 1 if house *i* is sold in period *t* and 0 otherwise. Since an intercept term α is included, the time dummy for the base period 0 is excluded, i.e. δ^0 is set equal to 0, to identify the model. We assume that equation (3)

[3] Diewert (2021) relates this model to linear preferences of the (representative) consumer.

is estimated using Ordinary Least Squares (OLS) regression on the pooled cross section data of the entire sample period, producing parameter estimates $\hat{\alpha}$, $\hat{\delta}^t$ $(t=1,...,T)$, and $\hat{\beta}_k$ $(k=1,...,K)$. The estimated TDH index equals $P_{TDH}^{0t}=\exp(\hat{\delta}^t)$, with $P_{TDH}^{00}=1$.

The OLS TDH index can be expressed in different ways. We will denote the sets of houses sold in periods 0 and t $(t=1,...,T)$ by S^0 and S^t with size N^0 and N^t. The first expression for the TDH index is

$$P_{TDH}^{0t}=\frac{\prod_{i\in S^t}(p_i^t)^{1/N^t}}{\prod_{i\in S^0}(p_i^0)^{1/N^0}}\exp\left[\sum_{k=1}^{K}\hat{\beta}_k\left(\overline{x}_k^0-\overline{x}_k^t\right)\right], \qquad (4)$$

where $\overline{x}_k^0=\sum_{i\in S^0}x_{ik}/N^0$ and $\overline{x}_k^t=\sum_{i\in S^t}x_{ik}/N^t$ denote the average characteristics in the periods compared. Equation (4) shows that the ratio of geometric average prices is adjusted for changes in the average characteristics. The geometric form stems from the combination of a log-linear model and least squares estimation. In the (unlikely) event that the sets S^0 and S^t coincide, so that $\overline{x}_k^t=\overline{x}_k^0$ in (4), the TDH index simplifies to the Jevons index $\prod_{i\in S^t}(p_i^t)^{1/N^t}\Big/\prod_{i\in S^t}(p_i^0)^{N^t}=\prod_{i\in S^t}(p_i^t/p_i^0)^{1/N^t}$. This result is intuitively understandable: because there is now no quality mix change to adjust for, the choice of characteristics does not impact the aggregate index – it is only the price relatives p_i^t/p_i^0 of the (matched) houses that matter.

Another expression for the OLS TDH index is (de Haan, 2015)

$$P_{TDH}^{0t}=\prod_{i\in S^t}\left(\frac{p_i^t}{\hat{p}_i^0}\right)^{1/N^t}, \qquad (5)$$

where $\hat{p}_i^0=\exp(\hat{\alpha})\exp[\sum_{k=1}^{K}\hat{\beta}_k x_{ik}]=\exp(\hat{\alpha})\exp(\hat{\gamma}_i)$ is the predicted base period price from the TDH regression. The right-hand side of (5) is an imputation Jevons price index defined on the set of houses sold in period *t*. The "missing" base period prices of houses that were not sold in period 0 are imputed by $\hat{p}_i^0$. Also, the prices p_i^0 of the houses that were sold in both period *t* and period 0 are replaced by their predicted values $\hat{p}_i^0$.

Two more features of the method are worth mentioning. First, the TDH index is transitive: measured price change between any two periods is independent of the choice of base period (period 0 in the estimating equation (3)). Second, we had to assume time fixity of the property characteristics to make TDH compatible with the basic model (2). But this is a strong assumption: quality changes of the individual houses resulting from depreciation and renovations are ignored.

3. Time product dummy

In this section, we discuss the multilateral Time Product Dummy (TPD) method, which is closely related to the TDH method. If (sufficient) information on price-determining characteristics is lacking, we could try and estimate the property-specific parameters γ_i in the basic model (2) using property dummy variables. Suppose that *N* different houses (identified by exact location) are traded across the whole sample period. The estimating equation for the pooled data becomes

$$\ln p_i^t = \alpha + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{n=1}^{N-1} \gamma_i D_i + \varepsilon_i^t , \tag{6}$$

where D_i is a dummy variable which has the value 1 if the price observation pertains to house *i* and 0 otherwise; a dummy variable for house *N* is excluded in order to identify the model. The time dummy for the base period 0 is also excluded, as in (3). The OLS parameter estimates for the TPD equation (6) are denoted by $\tilde{\alpha}$, $\tilde{\delta}^t$ $(t = 1,...,T)$ and $\tilde{\gamma}_i$ $(i = 1,..., N-1)$, with $\tilde{\gamma}_N = 0$. The TPD index going from period 0 to period *t* is given by $P_{TPD}^{0t} = \exp(\tilde{\delta}^t)$. Because of the similarity between TPD and TDH, we should be able to write the TPD index in the form of equation (5), i.e. as

$$P_{TPD}^{0t} = \prod_{i \in S^t} \left(\frac{p_i^t}{\tilde{p}_i^0} \right)^{1/N^t} , \tag{7}$$

where $\tilde{p}_i^0$ are again imputed/predicted base period prices.

Importantly, houses which are sold once during the sample period cannot affect the price index; they are "zeroed out". For housing, this was first mentioned by Gao and Wang (2007), but Diewert (2004) had already noticed it in the context of cross-country comparisons. Thus, the same price indexes would be obtained if the regression was run on a (repeat sales) data set that only includes dwellings sold more than once. Still, the TPD index can be written in the form of (7). To show this, we will derive an alternative expression for the predicted base period prices $\tilde{p}_i^0 = \exp(\tilde{\alpha})\exp(\tilde{\gamma}_i)$.

The dummy variable (fixed effects) specification of the model ensures that the average regression residuals for each property *i* are equal to zero across the set of periods when the property is sold, which we denote by S_i with size $N(S_i)$. That is, we have $\sum_{t \in S_i} \ln(p_i^t / \tilde{p}_i^t) / N(S_i) = 0$, and taking antilogs yields $\prod_{t \in S_i} (p_i^t / \tilde{p}_i^t)^{1/N(S_i)} = 1$. Using $\tilde{p}_i^t = \tilde{p}_i^0 P_{TPD}^{0t}$ we find

$$\tilde{p}_i^0 = \prod_{t \in S_i} \left(\frac{p_i^t}{P_{TPD}^{0t}} \right)^{1/N(S_i)} \qquad (i = 1, ..., N). \tag{8}$$

Thus, the predicted base period price equals the geometric average of deflated observed prices, where the TPD index acts as the deflator. Notice that (7) and (8) form a system of simultaneous equations. The OLS estimators provide the exact solution, but it is also possible to solve the system iteratively.

Now suppose house 1 is sold only once, say in period t^* $(0 \le t^* \le T)$. Since this house is zeroed out from the regression, the TPD index in this period estimated on the repeat sales data set will be equal to

$$P_{TPD}^{0t^*} = \prod_{i=2}^{N^t} \left(\frac{p_i^{t^*}}{\tilde{p}_i^0} \right)^{1/(N^{t^*}-1)}. \tag{9}$$

Next, multiply both sides of equation (9) by $(p_1^{t^*} / \tilde{p}_1^0)^{1/(N^{t^*}-1)}$, where $\tilde{p}_1^0$ is the predicted base period price for house 1, to obtain

$$P_{TPD}^{0t^*} \left(\frac{p_1^{t^*}}{\tilde{p}_1^0} \right)^{1/(N^{t^*}-1)} = \left(\frac{p_1^{t^*}}{\tilde{p}_1^0} \right)^{1/(N^{t^*}-1)} \prod_{i=2}^{N^{t^*}} \left(\frac{p_i^{t^*}}{\tilde{p}_i^0} \right)^{1/(N^{t^*}-1)} = \prod_{i=1}^{N^{t^*}} \left(\frac{p_i^{t^*}}{\tilde{p}_i^0} \right)^{1/(N^{t^*}-1)}. \tag{10}$$

It is easy to check that (10) becomes

$$P_{TPD}^{0t^*} = \prod_{i \in S^t} \left(\frac{p_i^{t^*}}{\tilde{p}_i^0} \right)^{1/N^{t^*}}, \tag{11}$$

which is equal to (7) for $t = t^*$, if we set $\tilde{p}_1^0 = p_1^{t^*} / P_{TPD}^{0t^*}$. This shows that if the predicted base period prices for properties sold only once during the sample period are defined in accordance with (8), then the TPD index can be written as (7). Note that $\tilde{p}_1^0 = p_1^{t^*} / P_{TPD}^{0t^*}$ holds because $\tilde{p}_1^{t^*} = p_1^{t^*}$ (zeroing-out).[4]

Just like the TDH index, the TPD index *i*) is transitive, and *ii*) cannot adjust for quality changes of the individual houses. But while the TDH method uses all the data, the TPD method *iii*) only uses data on houses traded more than once during the sample period. The last point emphasizes that TPD is a matched pairs approach. In Section 4, the relationship with the most famous matched pairs method for real estate, repeat sales, will be explained.

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[4] Gao and Wang (2007) wrongly mention that the fixed effect for the single-observation house cannot be estimated.

4. Repeat sales

The dependent variable in a Repeat Sales (RS) model is price relative rather than price. For a house that has been sold twice during the sample period, a single price relative is available, while for a house that has been sold three (or more) times, two (or multiple) consecutive price relatives are available. However, the RS method merely distinguishes between "first" sale and "second" sale; if a house is sold for the third time, a new pair of repeat sales is created. Thus, RS treats consecutive price relatives for a particular house as if they related to different houses. The total number of repeat sales in the data set will be denoted by N_{RS}.

We denote the periods of first and second sales of property *i* by $f(i)$ and $s(i)$ $(0 \le f(i) < s(i) \le T)$. According to basic model (2), we have $\ln p_i^{f(i)} = \delta^{f(i)} + \gamma_i + \varepsilon_i^{f(i)}$ and $\ln p_i^{s(i)} = \delta^{s(i)} + \gamma_i + \varepsilon_i^{s(i)}$, so that the log of price relative can be written as

$$\ln\left(\frac{p_i^{s(i)}}{p_i^{f(i)}}\right) = \ln(p_i^{s(i)}) - \ln(p_i^{f(i)}) = \left(\delta^{s(i)} - \delta^{f(i)}\right) + \varepsilon_i^{f(i)s(i)}, \tag{12}$$

with $\varepsilon_i^{f(i)s(i)} = \varepsilon_i^{s(i)} - \varepsilon_i^{f(i)}$. The (holding) period between subsequent sales differs among houses, but given that, apart from random errors, all house prices are assumed to change at the same rate, the repeat sales data can be pooled. The estimating equation for model (12) becomes

$$\ln\left(\frac{p_i^{s(i)}}{p_i^{f(i)}}\right) = \sum_{t=0}^{T} \delta^t D_{i(RS)}^t + \varepsilon_i^{f(i)s(i)}, \tag{13}$$

where $D_{i(RS)}^t$ is a (dummy) variable that takes on the value 1 in the period that the resale of house *i* occurred, i.e. if $t = s(i)$, -1 in the period that the previous sale of *i* took place, i.e. if $t = f(i)$, and 0 otherwise. Using OLS regression to estimate equation (13) yields coefficients $\breve{\delta}^t$ $(t = 1,...,T)$; the RS index going from period 0 to period *t* is given by $P_{RS}^{0t} = \exp(\breve{\delta}^t)$.

Wang and Zorn (1997) derived an analytical expression for the OLS repeat sales index. Let us denote the set of houses sold for the first time in period *t* by S_f^t and the set of houses sold for the second time in period *t* by S_s^t; $S_{RS}^t = S_f^t \cup S_s^t$, with size N_{RS}^t, is the total set of houses sold in period *t* in the repeat sales data set. It turns out that the RS index is defined by the following system of equations (see Kirby-McGregor and Martin, 2019, and Martin, 2019):

$$P_{RS}^{0t} = \prod_{i \in S_f^t} \left(\frac{p_i^t}{\breve{p}_i^0} \right)^{1/N_{RS}^t} \prod_{i \in S_s^t} \left(\frac{p_i^t}{\breve{p}_i^0} \right)^{1/N_{RS}^t} ; \tag{14}$$

$$\breve{p}_i^0 = \frac{p_i^{s(i)}}{P_{RS}^{0s(i)}} \text{ for } i \in S_f^t ; \tag{15a}$$

$$\breve{p}_i^0 = \frac{p_i^{f(i)}}{P_{RS}^{0f(i)}} \text{ for } i \in S_s^t . \tag{15b}$$

Thus, the RS index can be written as an imputation Jevons price index where the base period prices are imputed by the deflated second selling price for houses that are sold for the first time in period t ($i \in S_f^t$) and by the deflated first selling price for houses that are sold for the second time in period t ($i \in S_s^t$).

Note that $\breve{p}_i^0$ for $i \in S_f^t$ will usually differ from $\breve{p}_i^0$ for $i \in S_s^t$, because in general $p_i^{s(i)} / p_i^{f(i)} \neq P_{RS}^{0s(i)} / P_{RS}^{0f(i)}$. So, if a house is sold twice during the sample period, it has two different implicitly estimated base period prices. An alternate system with a unique base period price estimate is obtained by taking the geometric mean of the two deflated prices:

$$P^{0t} = \prod_{i \in S_{RS}^t} \left(\frac{p_i^t}{\bar{p}_i^0} \right)^{1/N_{RS}^t} ; \tag{16}$$

$$\bar{p}_i^0 = \left(\frac{p_i^{f(i)}}{P^{0f(i)}} \right)^{1/2} \left(\frac{p_i^{s(i)}}{P^{0s(i)}} \right)^{1/2} . \tag{17}$$

If each house is sold twice during the sample period, then the system given by equations (16) and (17) is identical to the TPD system given by (7) and (8). More generally, when properties may be sold multiple times, TPD can be seen as a constrained version of RS: it is a matched pairs method that, through averaging all available deflated selling prices, produces a unique base period price estimate for each property rather than two or more (unconstrained) estimates as RS does.

Like the TPD index, the RS index *i*) is transitive, *ii*) does not adjust for quality changes of the individual houses, and *iii*) is restricted to properties traded twice or more in the sample period. An advantage of RS compared with TPD is that, without property fixed effects, fewer model parameters have to be estimated. On the other hand, when the average holding period is relatively short and a fairly large proportion of houses is sold multiple times in the sample period, we might prefer TPD because it identifies multiple sales for the "same" house and estimates a unique base period price.

5. GEKS-Jevons

An estimation window of $T+1$ time periods $t=0,...,T$ has $T+1$ possible base periods b for bilateral price indexes. The generic bilateral index going from b to t is denoted by P^{bt}; b can be greater than t. If P^{bt} satisfies the time reversal test, i.e. $P^{bt}=1/P^{tb}$, then price change between 0, the starting period of the time series, and t $(t=1,...,T)$ can be measured by $P^{0t(b)}=P^{bt}/P^{b0}=P^{0b}\times P^{bt}$ for each b. And if all base periods are deemed equally valid, taking the geometric mean of $P^{0t(b)}$ across all possible b is a reasonable strategy. This GEKS procedure to attain transitivity leads to

$$P_{GEKS}^{0t}=\prod_{b=0}^{T}\left(P^{0t(b)}\right)^{1/(T+1)}=\prod_{b=0}^{T}\left[P^{bt}/P^{b0}\right]^{1/(T+1)}=\prod_{b=0}^{T}\left[P^{0b}\times P^{bt}\right]^{1/(T+1)}. \tag{18}$$

We will use matched-model bilateral Jevons price indexes (when available) as inputs in equation (18): $P_J^{0b}=\prod_{i\in S^{0b}}(p_i^b/p_i^0)^{1/N^{0b}}$ and $P_J^{bt}=\prod_{i\in S^{bt}}(p_i^t/p_i^b)^{1/N^{bt}}$, where $S^{0b}=S^0\cap S^b$ and $S^{bt}=S^b\cap S^t$ denote the sets of matched houses (sold in the periods compared); N^{0b} and N^{bt} are the corresponding sizes, i.e. the numbers of repeat sales. So, the GEKS-Jevons price index is defined as

$$P_{GEKS-J}^{0t}=\prod_{b=0}^{T}\left[P_J^{0b}\times P_J^{bt}\right]^{1/(T+1)}. \tag{19}$$

Due to transitivity, the index between two periods r and t $(r,t=0,...,T)$ equals

$$P_{GEKS-J}^{rt}=\prod_{b=0}^{T}\left[P_J^{rb}\times P_J^{bt}\right]^{1/(T+1)}, \tag{20}$$

where P_J^{rb} and P_J^{bt} are matched-model Jevons house price indexes going from r to b and from b to t, respectively.

Rao (2001) and others have shown that the GEKS price index can be conceived as the solution to a least squares minimization problem. Consider the following model (with $\delta^0=0$ for identification):

$$\ln P_J^{rt}=\delta^t-\delta^r+\varepsilon^{rt}. \tag{21}$$

Minimizing the sum of squared errors $\sum_{r=0}^{T}\sum_{t=0}^{T}[\ln P_J^{rt}-(\delta^t-\delta^r)]^2$ is equivalent to running an OLS regression of model (21) using all of the available bilateral Jevons price indexes. The OLS estimates are $\hat{\delta}^r$ and $\hat{\delta}^t$, and we have

$$\exp(\hat{\delta}^t-\hat{\delta}^r)=P_{GEKS-J}^{rt}. \tag{22}$$

Let us take another look at the RS method. For periods *r* and *t* (instead of *f*(*i*) and *s*(*i*)), the RS model given by (12) reads

$$\ln\left(\frac{p_i^t}{p_i^r}\right) = \ln(p_i^t) - \ln(p_i^r) = \delta^t - \delta^r + \varepsilon_i^{rt} . \tag{23}$$

Summing over all properties *i* that belong to the set S^{rt} of matched properties, or repeat sales, between *r* and *t*. and dividing by the corresponding number N^{rt} yields a model that is similar to (21):

$$\ln P_J^{rt} = \delta^t - \delta^r + \bar{\varepsilon}^{rt} , \tag{24}$$

where $\bar{\varepsilon}^{rt} = \sum_{i \in S^{rt}} \varepsilon_i^{rt} / N^{rt}$. Now if the errors ε_i^{rt} have constant variance σ^2, then the average errors $\bar{\varepsilon}^{rt}$ have variance $\sigma^2 / (N^{rt})^2$. The use of WLS regression with weights N^{rt} when estimating (24) will adjust for this type of heteroskedasticity. This is standard procedure in econometrics when a regression model with homoscedastic errors at the micro level is estimated using sample means. In Appendix 1 we provide an alternative interpretation of the use of these weights.

To reiterate: model (21) applies to both GEKS and RS. Estimating this model on all the available matched-model bilateral Jevons price indexes between periods *r* and *t* $(r = 0,...,T; t = 0,...,T)$ using OLS regression produces the GEKS-Jevons index while estimating it using WLS regression with the corresponding number of repeat sales, N^{rt}, as weights (or with normalized values N^{rt} / N_{RS}) produces the RS index. This confirms Melser's (2013) observation that RS is essentially weighted GEKS.[5] If the accuracy of the price changes depends inversely on the number of observations, which seems quite likely, we expect the RS index to be more stable than the GEKS-Jevons index because price changes with low accuracy will be downweighted.

6. Geary-Khamis and arithmetic repeat sales

All the methods discussed so far were based on geometric aggregation, either explicitly (GEKS) or implicitly (through log-linear regressions). It is possible though to construct arithmetic property price indexes. Shiller (1991) and others have argued that arithmetic

[5] Melser (2013) also accounted for potential non-constant variance of the error terms in (23). In this case weighted RS regressions might be useful; see also Section 7 below.

aggregation is appropriate for a house price index that aims to measure price change of the entire housing stock. In this section we discuss two arithmetic multilateral methods, Geary-Khamis and arithmetic repeat sales.

As far as we know, the Geary-Khamis (GK) method has not been applied before to residential property.[6] As outlined in Appendix 2, the GK index can be written as the following set of equations:

$$P_{GK}^{0t} = \frac{\sum_{i \in S^t} p_i^t / N^t}{\sum_{i \in S^t} \breve{p}_i^0 / N^t} = \frac{\sum_{i \in S^t} \breve{p}_i^0 \left(\frac{p_i^t}{\breve{p}_i^0} \right)}{\sum_{i \in S^t} \breve{p}_i^0} \qquad (t = 1, \ldots, T); \tag{25}$$

$$\breve{p}_i^0 = \lambda_i \left[\frac{\sum_{i \in S^0} p_i^0}{\sum_{i \in S^0} \lambda_i} \right] \qquad (i = 1, \ldots, N); \tag{26a}$$

$$\lambda_i = \frac{1}{N(S_i)} \sum_{t \in S_i} \left(\frac{p_i^t}{P_{GK}^{0t}} \right). \tag{26b}$$

The first expression of (25) shows that the GK index is an imputation Dutot price index; the second expression tells us why Dutot is sometimes referred to as a "value weighted" index. The imputed base period prices, $\breve{p}_i^0$, are measured as unweighted averages of the deflated observed prices, as shown by (26b), and normalized, as in (26a), such that the price index is equal to 1 in period 0. Like the TPD system, the GK system can be solved iteratively. And here, too, properties sold only once during the entire sample period do not affect the results. So, the sums in (25) can be taken over those properties that belong to S_{RS}^t, the sub-set of properties in the repeat sales data set traded in period *t* (with size N_{RS}^t).

Equations (25), (26a) and (26b) might be viewed as the arithmetic counterpart to equations (7) and (8) that define TPD. In Section 4 we argued that TPD is a constrained version of (geometric) RS. Likewise, GK is a constrained version of the value weighted variant of Arithmetic Repeat Sales (ARS), proposed by Shiller (1991).[7] ARS is defined by the following system of equations:

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[6] This method is used by Statistics Netherlands to compile price indexes from scanner data; see Chessa (2016).

[7] The S&P Global house price index in the U.S. mentioned in the introduction as well as the house price index compiled by Teranet Inc. and the National Bank in Canada are based on arithmetic, not geometric, repeat sales.

$$P_{ARS}^{0t} = \frac{\sum_{i \in S_f^t} p_i^t + \sum_{i \in S_s^t} p_i^t}{\sum_{i \in S_f^t} \hat{p}_i^0 + \sum_{i \in S_s^t} \hat{p}_i^0} = \frac{\sum_{i \in S_{RS}^t} p_i^t / N_{RS}^t}{\sum_{i \in S_{RS}^t} \hat{p}_i^0 / N_{RS}^t} = \frac{\sum_{i \in S_{RS}^t} \hat{p}_i^0 \left(\frac{p_i^t}{\hat{p}_i^0} \right)}{\sum_{i \in S_{RS}^t} \hat{p}_i^0}; \tag{27}$$

$$\hat{p}_i^0 = \frac{p_i^{s(i)}}{P_{ARS}^{0s(i)}} \text{ for } i \in S_f^t; \tag{28a}$$

$$\hat{p}_i^0 = \frac{p_i^{f(i)}}{P_{ARS}^{0f(i)}} \text{ for } i \in S_s^t. \tag{28b}$$

Thus, the arithmetic RS index can also be seen as an imputation Dutot price index, but where the base period prices are now imputed by the deflated second selling price for properties sold for the first time in period t ($i \in S_f^t$) and by the deflated first selling price for properties sold for the second time in period t ($i \in S_s^t$).

Like geometric RS, arithmetic RS provides two unconstrained base period price estimates if each property is sold twice during the sample period. In the general case, properties which are sold multiple times have multiple base period price estimates. GK, on the other hand, yields a unique base period price estimate for each property through averaging all the available deflated selling prices, as can be seen from (26). In that sense GK can be called a constrained version of arithmetic RS.

When there is variation in the data, an arithmetic average is always greater than the corresponding geometric average. However, it is not necessarily true that $P_{ARS}^{0t} \geq P_{RS}^{0t}$. This is due to "value weighting" of the price relatives in the arithmetic index, and also because the imputed prices differ between the arithmetic and geometric variants.

7. Revisions and other issues

Multilateral price indexes are estimated simultaneously for multiple time periods from pooled cross section data, and previously estimated index numbers will change, though perhaps just slightly, when time passes and data for a new period is added to the sample. That is, the indexes are subject to revisions. Revisions can be a nuisance to users but are not necessarily a "bad" feature; the use of more data can improve the efficiency of the estimators. For instance, when the sample period is extended and the TDH model is re-estimated, the new coefficients $\hat{\beta}_k$ in (4) are likely to have smaller standard errors than the original coefficients as they are based on a bigger data set.

If the hedonic model holds true, we do not expect to find systematic revisions in the TDH. This could be different for multilateral methods estimated from data which are restricted to houses sold twice or more during the sample period. Clapp and Giaccotto (1999) showed that revisions in RS indexes can indeed be systematic and substantial. Systematic revisions in RS indexes, or in other multilateral indexes that do not use any characteristics information (TPD and GEKS-Jevons), suggests bias due to inappropriate treatment of unmatched houses. In other words, these multilateral indexes are not only less efficient because they do not include unmatched observations, they can also suffer from sample selection bias (Gatzlaff and Haurin, 1997) when the (unobservable) price change of the excluded unmatched houses differs in a systematic fashion from the price change of included houses.

Even if continuous revisions of the indexes were random rather than systematic, statistical agencies would not accept them. There are ways to construct a non-revisable time series, in particular splicing, possibly combined with a rolling window approach.[8] A practical problem could be that, due to the long holding period of residential property, the window must be exceptionally long, and many years of data must be available before the agency can start publishing a non-revisable time series. Our data set covers a period of over 25 years, which is long enough to examine revisions. However, we will not address splicing methods in the empirical Section 7; this is beyond the scope of our paper.

Case and Shiller (1989) argued that property price changes include components whose variance increases with the length of holding period. To adjust for this type of heteroskedasticity, they proposed a WLS version of RS. Melser (2013) showed how the weights in GEKS-Jevons must be adapted to make the resulting index equal to the WLS RS index. The findings in the literature on heteroskedasticity are somewhat ambiguous. Leishman and Watkins (2002) and Jansen et al. (2008), who compared OLS repeat sales indexes with several weighted versions, concluded that OLS performed just as well as WLS. In Section 7, we will not estimate a weighted RS index or an adapted version of GEKS-Jevons.

[8] Shimizu et al. (2010) used a rolling window approach combined with a so-called movement splice for a TDH house price index to deal with structural changes in the parameters while at the same time avoiding revisions. Martin (2019) argued that a rolling window approach may not be sensible for RS "as this would discard properties that sold for the first time early in the series".

Our data set (see Section 8 below) does not have information on the age of the dwelling. As we are basically tracking addresses, with no information on age we cannot be sure that the same building is compared across time. The original building may have been demolished and replaced by a new one. In any case, it is unlikely that renovations exactly offset depreciation; the quality of a dwelling typically changes over time. But if we had information on age, the following could be worth pursuing.[9] Diewert, Huang and Burnett-Isaacs (2017) proposed a TDH model where information on age of the structure was required to measure the effect of net depreciation (the other housing characteristics were land area and structure area). Differencing their model for periods when a property is traded produces a generalized RS model which adds holding period as an explanatory variable (instead of using it to weight the RS regression). If the corresponding estimated parameter is strongly negative, we need to be a bit wary of RS – the standard RS index is then likely to have downward bias.

A disadvantage of TPD as compared with RS is the potentially huge number of parameters to be estimated due to use of property fixed effects in the regression model. Not only are the fixed effects likely to have large standard errors because properties are sold infrequently, but we also found that performing the regression sometimes ran into computational difficulties. This problem was in fact one of the reasons for Bailey, Muth and Nourse (1963) to propose the RS approach. We therefore decided to estimate TPD indexes by solving the system of equations (7) and (8) iteratively rather than by running regressions. Given the similarity between TPD and RS, the RS system of equations can also be solved iteratively, but we do not follow this route.

One way to reduce the number of parameters to be estimated in TPD regressions is to stratify the sample according to region and/or type of house and construct indexes at the stratum level. Stratification may also mitigate sample selection bias in multilateral house price indexes when selectivity is related to the stratification variables. A question raised by stratification is how to combine the strata indexes. This is not just a practical matter; it concerns the preferred target index, i.e., the choice of index number formula. Should the weights pertain to just sales or to the entire housing stock, and should we use arithmetic or geometric weighting? While we do estimate indexes for several sub-sets, we leave the question of how best to combine these indexes for further research.

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[9] We thank Erwin Diewert for this suggestion.

8. Data and empirical findings

We use monthly transactions data as collected by the Dutch land registry. Our data set covers January 1995 – July 2021, i.e., a period of more than 26 years (319 months). We cleaned the data, mainly to be able to track properties over time but we did not follow Clapp and Giacotto's (1999) suggestion to eliminate very short holds. The total number of residential property sales, excluding newly built dwellings, during the sample period was 4,884,394 (on average 15,312 per month) according to Statistics Netherlands. After cleaning the data, we were left with 4,462,155 (13,988 per month) sales.

Figure 1: Index of the number of residential property sales

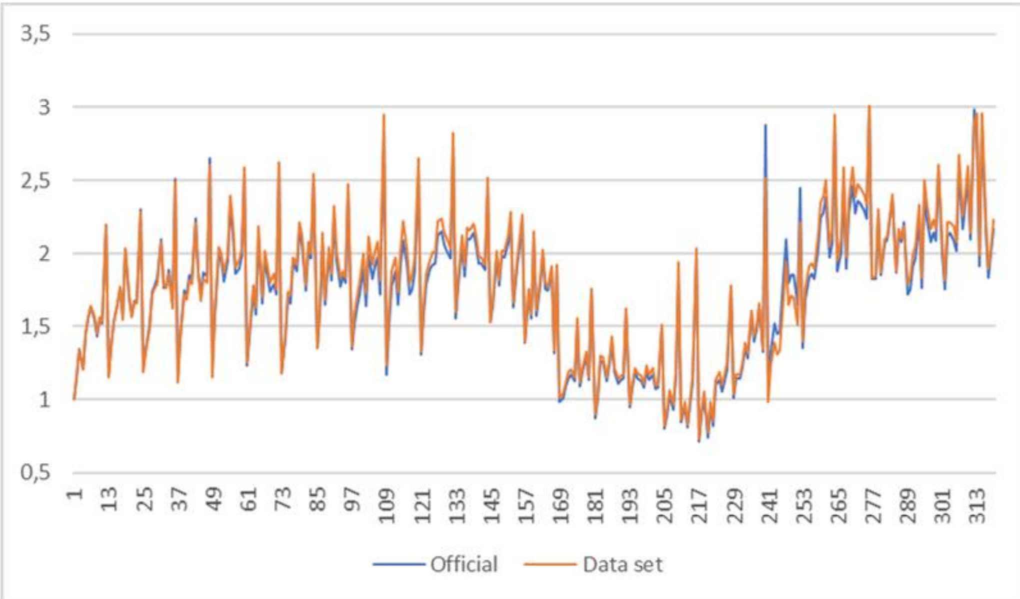
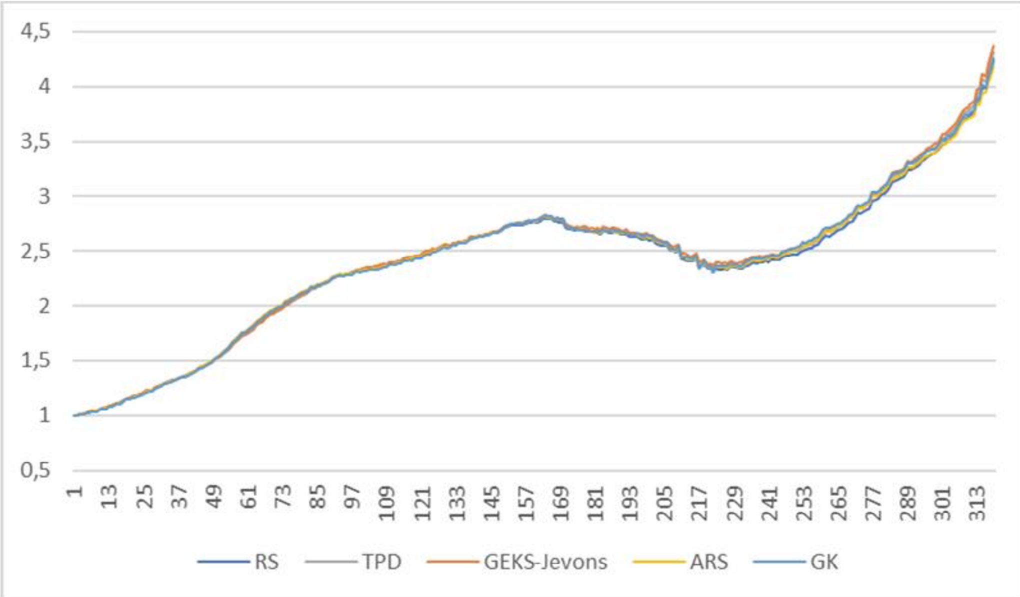


Figure 1 shows the index of the total number of sales, with January 1995 as the base. The change in the number of sales in our data set closely tracks the change in the official number. There is a distinct seasonal pattern in the number of sales with peaks in December, and to a lesser extent July, and troughs in January.

Figure 2 depicts the multilateral price indexes discussed in the previous sections – Repeat Sales (RS), Time Product Dummy (TPD), GEKS-Jevons, Arithmetic Repeat Sales (ARS), and Geary-Khamis (GK) – for the whole country. The differences between the five indexes are rather small, and they also identify the same turning points. Thus, at the nation-wide level, the choice of (non-hedonic) multilateral method does not seem to be an important issue. Note again that all these indexes are explicitly or implicitly based on the repeat sales data set that excludes dwellings sold once during the sample period. We ended up with 2,927,840 (matched) observations, i.e., we deleted 40% of the total number of officially registered sales.

Figure 2: Five multilateral price indexes for the Netherlands



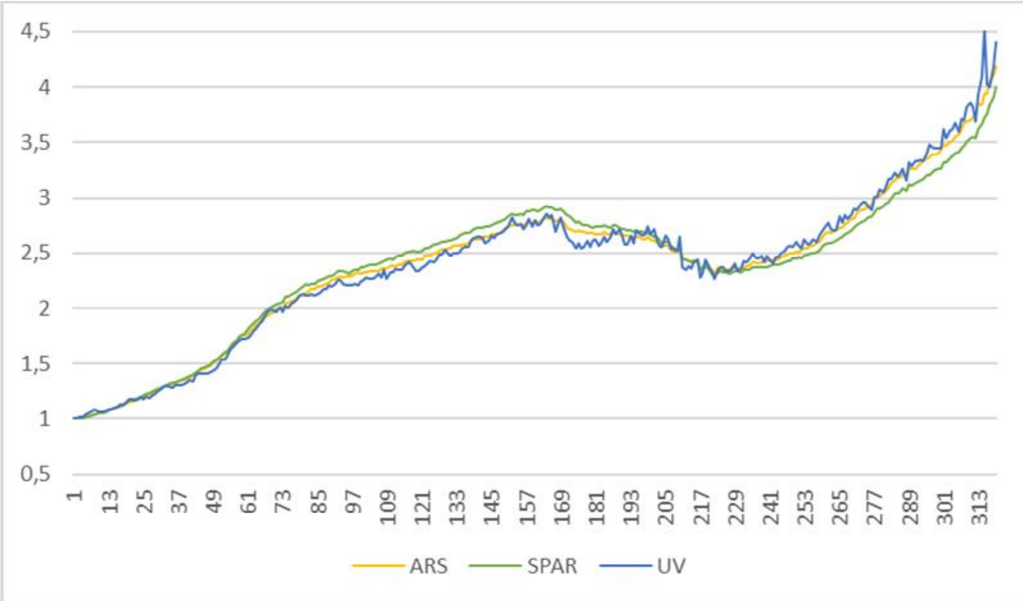
For the construction of the official house price index, Statistics Netherlands also exploits the land registry data. However, the official index is not based on one of the five multilateral methods. Instead, Statistics Netherlands relies on the SPAR (Sale Price Appraisal Ratio) method proposed by Bourassa, Hoesli and Sun (2006). This method compares the selling price of a property in the current period to its appraised value in some earlier base period. For details on the Dutch SPAR approach, see de Vries et al. (2009).[10] The SPAR index has an arithmetic structure, and Figure 3 therefore compares the official index with the ARS index rather than the standard RS index.[11] The unit value index, which measures the change in the (arithmetic) average selling price is shown too.

[10] Official appraisals are used in the Netherlands for tax purposes. They are available for every property and updated in January of each year. The house price index compiled by Statistics Netherlands is actually a stratified version of SPAR where the appraisals also provide stock weights. As the weights are updated, the index is chained annually, with January as link period. In other words, the official house price index is a *stock-based* index that aims to measure the price change of the entire housing stock.

[11] De Haan, van der Wal and de Vries (2009) discussed a geometric variant of SPAR. This variant would better compare to the (geometric) RS, TPD and GEKS-Jevons indexes.

Interestingly, the trend of the SPAR index differs slightly from that of the ARS index. This could be related to the fact that the SPAR index is based on data of all properties traded across the entire window, not just matched properties. The unit value index is not restricted to matched pairs either.[12] Because this index does not adjust for compositional change, it is quite volatile. The trend of the unit value index is very similar to that of the ARS index though.

Figure 3: ARS, SPAR and UV indexes for the Netherlands



The Netherlands is a small country with less than 18 million inhabitants (as of September 2021). Nonetheless, the number of properties sold in our data set appears to be large enough to estimate nation-wide multilateral house price indexes with sufficient precision, i.e., without "too much" volatility or noise. Obviously, for sub-sets of the data we would expect volatility to increase. Our focus is on methods, and we are particularly interested in whether volatility differs across the various indexes and whether the trends differ. From a housing market perspective, it is important to know if price change varies across regions/cities and across types of dwellings. Below, we provide evidence on the latter, but we do this just to explore the effect of choice of method.

[12] It should be mentioned that the data cleaning procedure for the official SPAR index is more restrictive than our procedure. So, the monthly data sets used to construct the unit value index and the SPAR index were not exactly the same.

Figure 4 compares the RS index for four parts of the country: North, East, South, and West. During the upswing that began in mid-2013, house prices have gone up faster in West than elsewhere. As Figure 5 shows, all the multilateral methods produce very similar trends. The western part of the Netherlands is where most people live, and a lot of data was available to estimate the indexes.

Figure 4: RS index for four different parts of the Netherlands

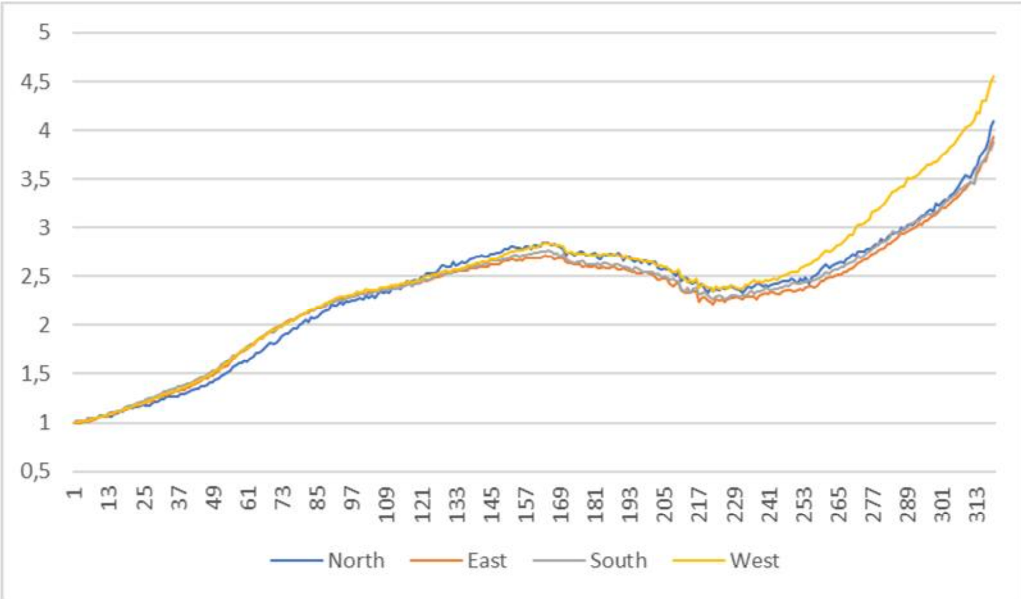
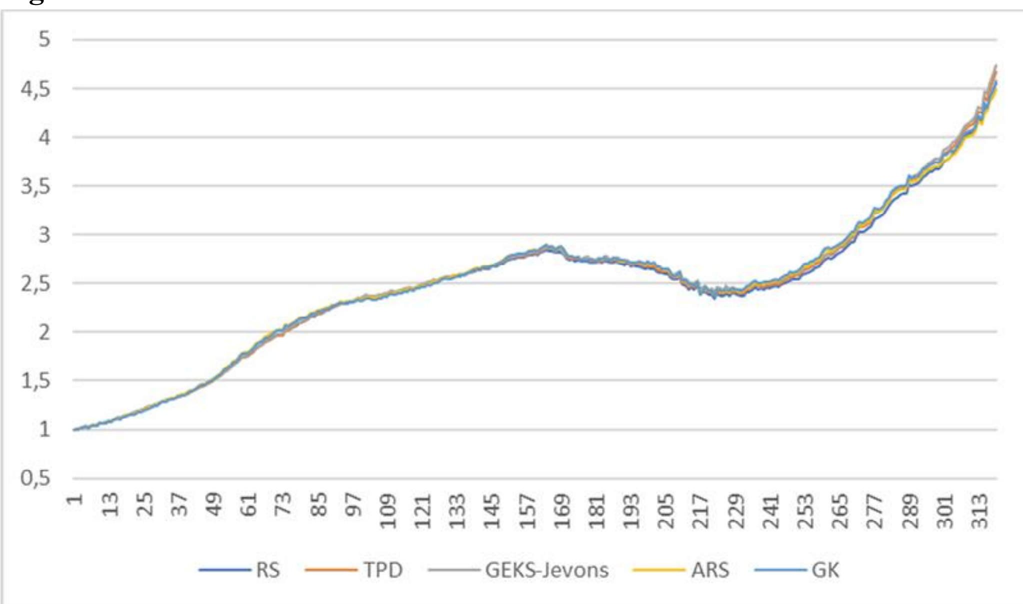


Figure 5: Multilateral indexes for the West of the Netherlands



The four largest cities in the Netherlands, Amsterdam, Rotterdam, The Hague, and Utrecht, are all in the western part. Figure 6 shows that, according to the RS index, house prices increased most in the capital Amsterdam. The volatility of the indexes is becoming an issue now. Looking at the different multilateral indexes for Amsterdam in Figure 7, it looks like the volatility of the GEKS-Jevons index is greater than that of the other indexes.

Figure 6: RS index for the four largest cities in the Netherlands

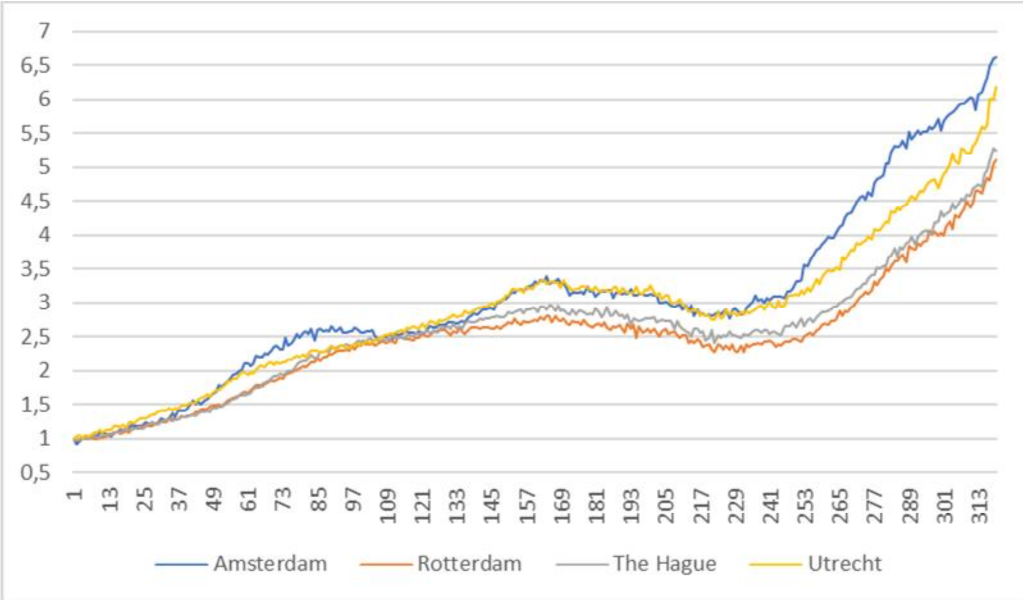
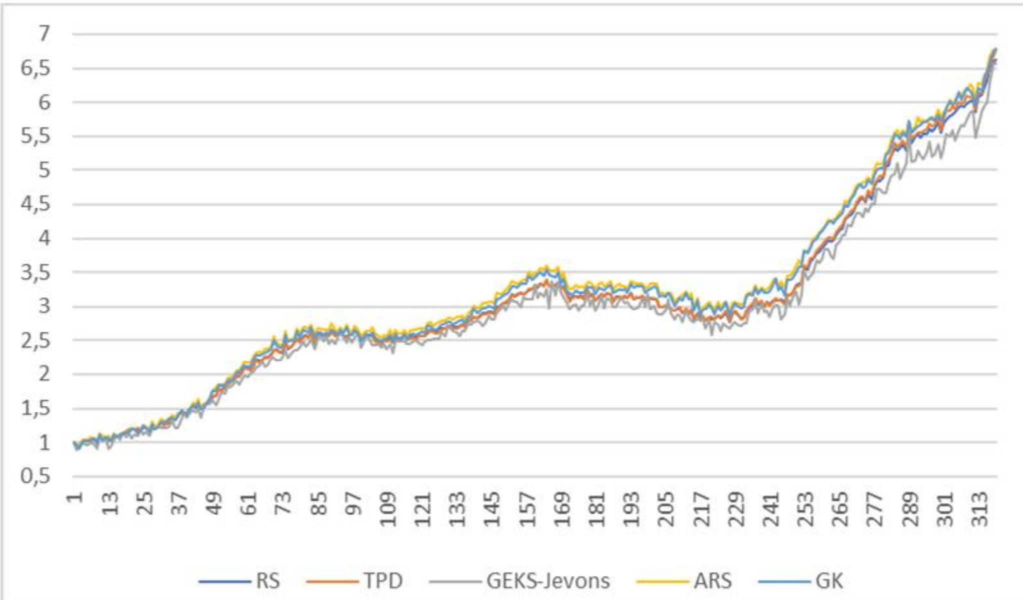


Figure 7: Multilateral indexes for Amsterdam



Figures 8 and 9 delve a bit deeper into the bigger volatility of the GEKS-Jevons index; see also Appendix 3. Figure 8 plots the GEKS-Jevons for the northern part of the Netherlands (which is very similar to the RS index in Figure 4) and for two cities in the northern part, i.e. Groningen – the capital of the province with the same name – and the much smaller city of Assen. Index volatility clearly increases with a decreasing size of the population. For Assen in particular, the volatility is huge.

Figure 8: GEKS-Jevons index for the northern part of the Netherlands and for two cities

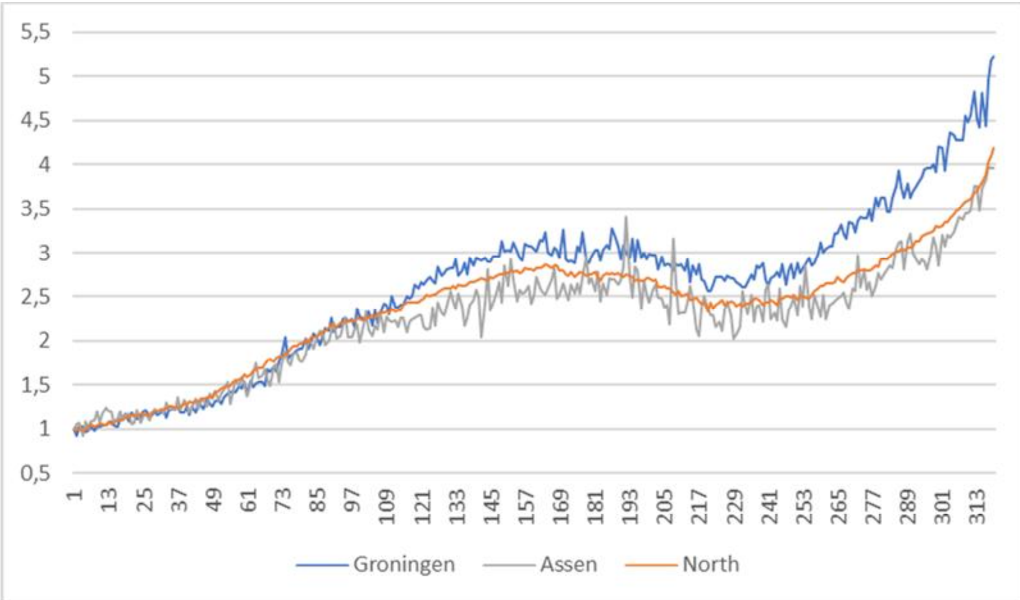


Figure 9: RS and GEKS-Jevons indexes for Assen

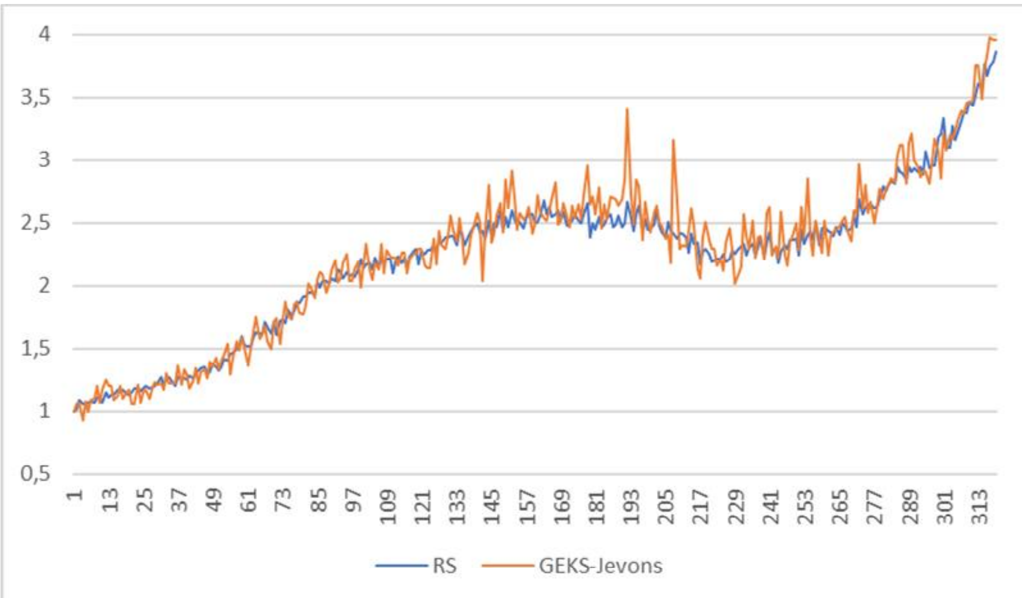


Figure 9 copies the GEKS-index for Assen from Figure 8 and compares it with the RS index. The volatility of the RS index is a lot smaller. This is consistent with the analysis in Section 5 where we argued that RS is essentially a form of weighted GEKS-Jevons that downweights price comparisons with relatively few observations. Since the latter comparisons are generally noisier, the RS index is expected to be less volatile than the GEKS-Jevons index.

In Section 4 we argued that the TPD is a constrained version of RS that might be preferred since it identifies multiple sales for the "same" house and estimates a unique base period price. Yet, using our data, the differences between the TPD and RS indexes are negligible, even for small cities with sparse data. This is illustrated in Figure 10 for Assen; it is almost impossible to distinguish the two lines.

Figure 10: RS and TPD indexes for Assen

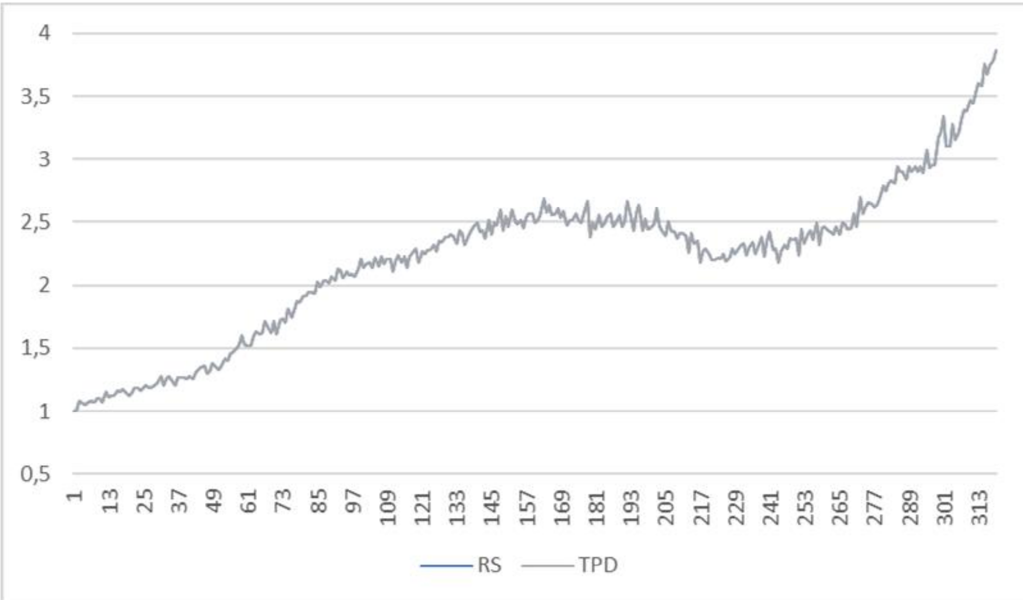
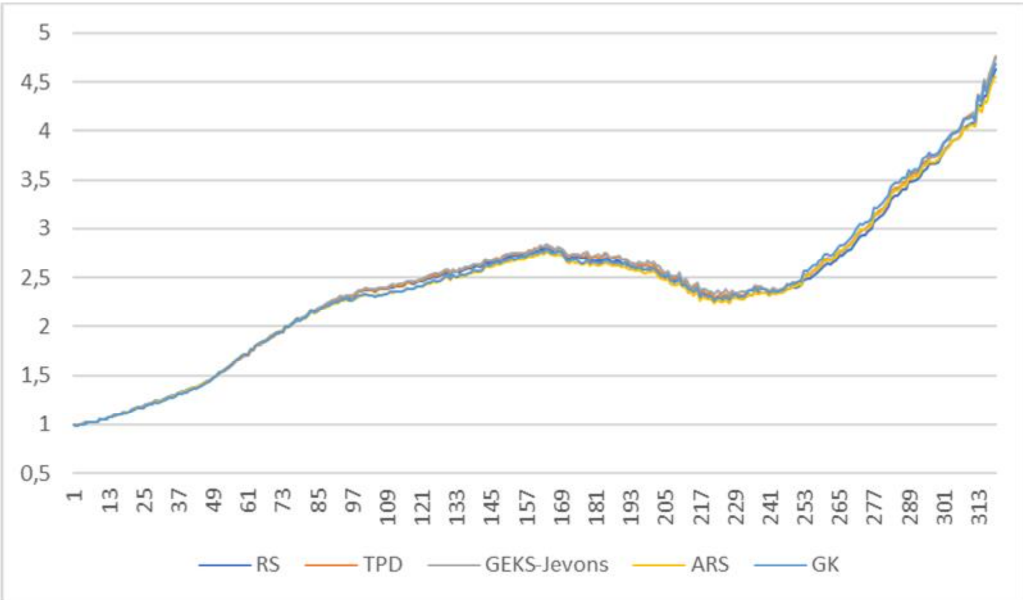


Figure 11 depicts the RS index for five types of dwelling that are distinguished in our data set. During the last six or seven years, prices of apartments increased faster than prices of other dwelling types. To some extent, this could reflect a locational effect because most apartments are obviously located in cities, especially in the largest cities in the western part of the Netherlands. As shown in Figure 12 for apartments, the trends of the various multilateral indexes are again very similar. This is also true for the other dwelling types.

Figure 11: RS index for different types of dwelling in the Netherlands

Figure 12: Multilateral indexes for apartments in the Netherlands



The multilateral methods shown here are all matched pairs methods in that they only include properties that were (re-)sold twice or multiple times during our data set. Figure 13 gives an idea of how long-term price change differs between properties with different holding periods; it plots RS indexes according to the number of times that the properties in our data set (for the whole country) were sold, i.e., two, three, four, five, or six or more times. Shorter holds seem to have appreciated a bit faster during upswings. This finding suggests that if we had started with a relatively short sample period, say of five years, and then gradually extended it, index revisions would have been downward, at least until the start of the downturn in 2008, as more and more longer holds would be included.

Figure 13: RS index according to number of times sold in the Netherlands

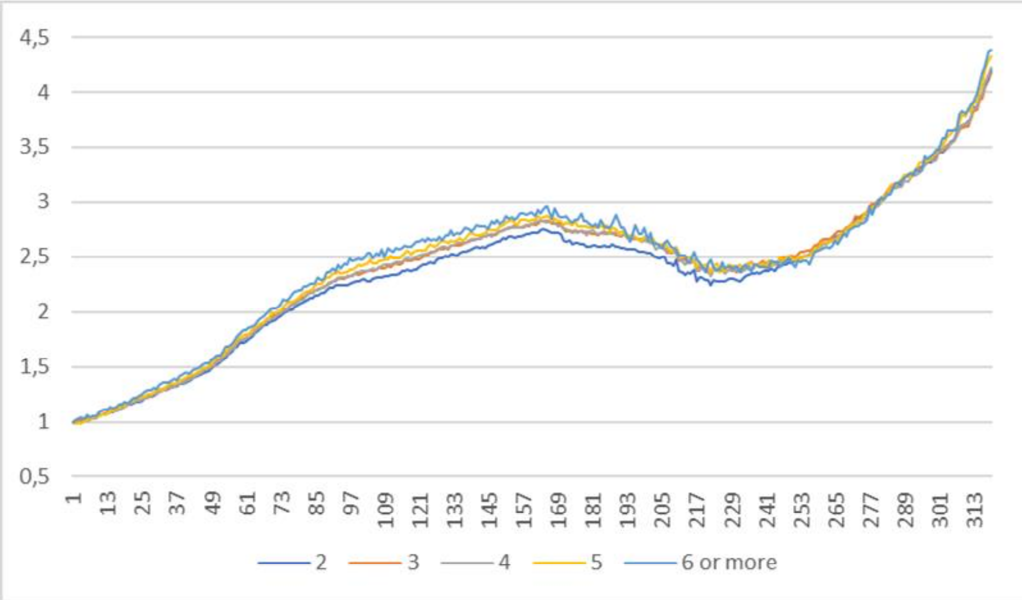


Figure 14: Full-window TPD index and two chained TPD indexes for Amsterdam

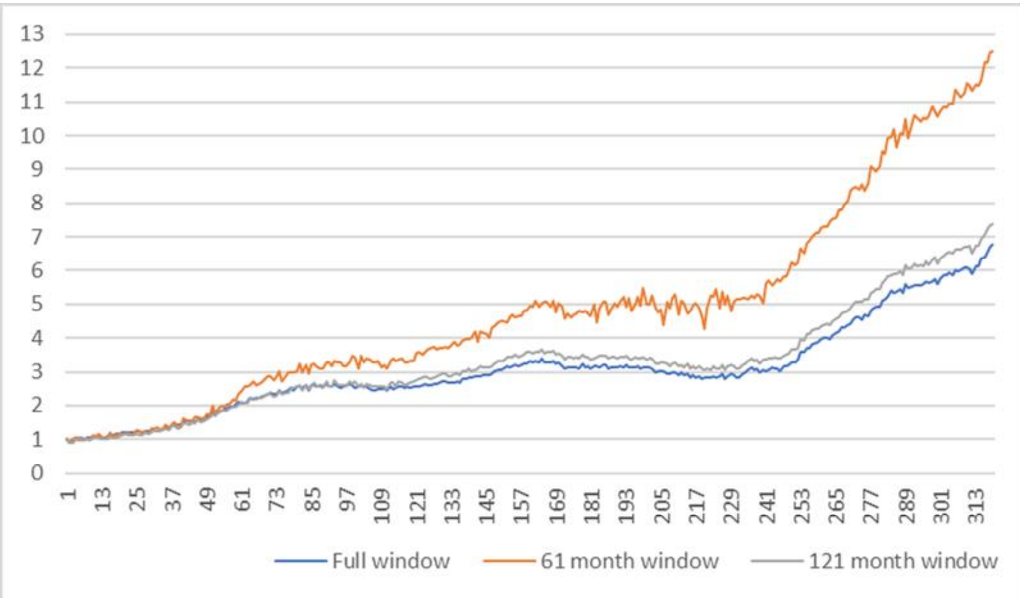


Figure 14 provides evidence on the magnitude of the revisions in the TPD index for Amsterdam. The full-window index, copied from Figure 7, is shown along with two chained versions. The orange line is a chain of five sub-period indexes. Each of the first four sub-periods has a window length of 61 months; the first sub-period starts in period 1 (January 1995) and ends in period 61 (January 2000), the second sub-period starts in period 61 (January 2000) and ends in period 121 (January 2005). The fifth sub-period is a little longer. The grey line is made up of two sub-period indexes covering 121 months plus a third sub-period index of about five years. Figure 14 confirms our expectations: the index revisions are indeed downward. The other multilateral indexes show similar downward revisions.

9. Main findings and conclusions

Statistical agencies have become familiar with multilateral methods, in particular TPD, GK and GEKS, in the context of the CPI; see IMF et al. (2020). These methods can also be used to construct a house price index, but the international Handbook on Residential Property Price Indices (de Haan and Diewert, 2013) did not discuss them. The purpose of this paper was to explain the similarities and differences between TPD, GK, GEKS and conventional repeat sales methods, which are multilateral too.

The various multilateral methods are all matched pairs (or repeat sales) methods that ignore houses sold once during the sample period. In contrast to hedonic regression methods, these methods do not adjust for quality changes of the structures. Furthermore, multilateral indexes will be revised when the sample period is extended.

We derived the following theoretical results.

- TPD and GK can be viewed as constrained versions of geometric and arithmetic RS, respectively. They yield a single base period price estimate for each house in the (repeat sales) data set.
- Geometric RS is essentially a form of weighted GEKS-Jevons that downweights price movements with relatively few observations, and so GEKS-Jevons indexes are likely to be more volatile.

Our empirical findings can be summarized as follows.

- For our nation-wide data set, the choice between geometric/arithmetic RS, TPD, GK and GEKS-Jevons was not very important.

- For subsections of the data set, the differences were often bigger – in particular, GEKS-Jevons indexes were much more volatile than other multilateral indexes, as expected.
- Index revisions were downward,[13] and a very long window would be required in practice.

When sufficient characteristics information is lacking so that hedonic regression is not an option for the compilation of an official house price index (and appraisal data is unavailable so that SPAR cannot be used), we suggest using TPD or geometric RS.[14] These two methods are based on the same underlying model and produced very similar results, even for small data sets. If TPD is used, it may be necessary to solve the system iteratively, like we did, due to the potentially huge number of property dummy variables in a TPD regression.

Appendix 1: RS versus GEKS-Jevons

This Appendix provides an alternative interpretation of the difference between RS and GEKS-Jevons. Following Melser (2013), let us define the indicator function δ_i^{rt}, which is equal to 1 when a price for property i is observed in both period r and period t, and 0 otherwise. The RS index going from r to t is found by minimizing the sum of squared errors from model (23)

$$\sum_{r=0}^{T}\sum_{t=0}^{T}\sum_{i=1}^{N_{RS}}\delta_i^{rt}\left[\ln\left(\frac{p_i^t}{p_i^r}\right)-\left(\delta^t-\delta^r\right)\right]^2=\sum_{r=0}^{T}\sum_{t=0}^{T}\sum_{i\in S^{rt}}\left[\ln\left(\frac{p_i^t}{p_i^r}\right)-\left(\delta^t-\delta^r\right)\right]^2, \qquad \text{(A.1)}$$

where, as in the main text, N_{RS} is the total number of matched properties or repeat sales in the data set and N^{rt} is the number of repeat sales between periods r and t, i.e. the size of S^{rt}. This minimization problem involves running an OLS regression using all of the

[13] Clapp and Giacotto (1992), Case, Pollakowski and Wachter (1997), and Gatzlaff and Haurin (1997) found that smaller properties, which are often “starter homes”, trade more frequently than properties in general. Clapham et al. (2004) thus argued that revisions could be related to differences in price change between starter homes and other dwellings, but it is unclear whether this is what drove our revisions.

[14] Hill et al. (2018) note that no country in Europe is using the RS method to construct an official house price index. As far as we know, Statistics Canada is the only statistical agency in the world that uses RS. They have a license to publish the (arithmetic) RS index compiled by Teranet Inc. and National Bank of Canada; for details, see Martin (2019).

available price relatives between r and t $(r = 0,...,T; t = 0,...,T)$; the OLS estimates are $\tilde{\delta}^r$ and $\tilde{\delta}^t$, and $\exp(\tilde{\delta}^t - \tilde{\delta}^r) = P_{RS}^{rt}$.

Using $\sum_{i \in S^{rt}} \ln(p_i^t / p_i^r) / N^{rt} = \ln P_J^{rt}$, it is easy to check that the summation over $i \in S^{rt}$ in (A.1) can be written as

$$N^{rt} \left[\frac{1}{N^{rt}} \sum_{i \in S^{rt}} \left[\ln\left(\frac{p_i^t}{p_i^r} \right) \right]^2 - \left[\frac{1}{N^{rt}} \sum_{i \in S^{rt}} \ln\left(\frac{p_i^t}{p_i^r} \right) \right]^2 + \left[\ln P_J^{rt} - \left(\delta^t - \delta^r \right) \right]^2 \right]. \qquad \text{(A.2)}$$

Now let $(\sigma^{rt})^2 = \sum_{i \in S^{rt}} [\ln(p_i^t / p_i^r)]^2 / N^{rt} - [\sum_{i \in S^{rt}} \ln(p_i^t / p_i^r) / N^{rt}]^2$ denote the variance of $\ln(p_i^t / p_i^r)$ within the set S^{rt}. It follows from (A.1) and (A.2) that RS is equivalent to minimizing

$$\sum_{r=0}^{T} \sum_{t=0}^{T} N^{rt} \left[(\sigma^{rt})^2 + \left[\ln P_J^{rt} - \left(\delta^t - \delta^r \right) \right]^2 \right]. \qquad \text{(A.3)}$$

with respect to the parameters δ^r and δ^t. We can write (A.3) as

$$\sum_{r=0}^{T} \sum_{t=0}^{T} N^{rt} (\sigma^{rt})^2 + \sum_{r=0}^{T} \sum_{t=0}^{T} N^{rt} \left[\ln P_J^{rt} - \left(\delta^t - \delta^r \right) \right]^2 . \qquad \text{(A.4)}$$

The first term in (A.4) is "given". Thus, minimizing (A.3), hence (A.1), is equivalent to minimizing the second term in (A.4). In turn, this is equivalent to estimating model (21) by WLS regression with N^{rt} as weights using all available matched-model Jevons price indexes. This result is in accordance with Section 5 and confirms Melser's (2013) point: RS is a form of weighted GEKS.

Appendix 2: The GK index for residential property

Is this Appendix we derive equations (25), (26a) and (26b) that define the GK index for residential property. We start with a generic definition for any good. The quantity sold (the number of sales) of good *i* in period *t* $(t = 0,...,T)$ is denoted by q_i^t; as in the main text, S_i denotes the set of time periods when *i* is actually sold $(q_i^t > 0)$. The GK index is defined by the following system of simultaneous equations:

$$P_{GK}^{0t} = \frac{\sum_{i \in S^t} p_i^t q_i^t \Big/ \sum_{i \in S^t} \lambda_i q_i^t}{\sum_{i \in S^0} p_i^0 q_i^0 \Big/ \sum_{i \in S^0} \lambda_i q_i^0} \qquad (t = 1,...,T); \qquad \text{(A.5)}$$

$$\lambda_i = \frac{\sum_{t \in S_i} q_i^t \left(\frac{p_i^t}{P_{GK}^{0t}} \right)}{\sum_{t \in S_i} q_i^t} \quad (i = 1, \ldots, N). \tag{A.6}$$

If we interpret λ_i as a "quality parameter", $\sum_{i \in S^t} \lambda_i q_i^t$ is the sum of quality-adjusted quantities across all goods. The ratio of total expenditure to the sum of quality-adjusted quantities, $\sum_{i \in S^t} p_i^t q_i^t / \sum_{i \in S^t} \lambda_i q_i^t$, can be viewed as a "quality-adjusted unit value", and the GK index given by (A.5) has been called a quality-adjusted unit value index. Prices, or relative prices really, inform us about consumer valuations of quality differences. GK uses the deflated prices in the periods a good is sold to measure the quality parameter; (A.6) shows that a weighted arithmetic mean of the deflated prices is taken with relative quantities as weights.

Another interpretation of the GK index is as follows. Being a mean of deflated prices, λ_i can be viewed as an estimate of the base period price. So, we can interpret the numerator of (A.5) as an *imputation Paasche price index* (based on S^t, the product set in period *t*) where the base period prices, whether observable or "missing", are imputed according to (A.6). Since this index is likely to differ from 1 in the base period, it must be normalized, i.e., divided by the value in period 0, which is the denominator of (A.5). We can of course adjust λ_i such that the GK index is equal to 1 in the base period. So, using

$$\breve{p}_i^0 = \lambda_i \frac{\sum_{i \in S^0} p_i^0 q_i^0}{\sum_{i \in S^0} \lambda_i q_i^0}, \tag{A.7}$$

we can write the GK index as

$$P_{GK}^{0t} = \frac{\sum_{i \in S^t} p_i^t q_i^t}{\sum_{i \in S^t} \breve{p}_i^0 q_i^t}. \tag{A.8}$$

The system of equations (A.6), (A.7) and (A.8) is another way to define the GK system.

For a house price based on the GK method we can simply set $q_i^t = 1$ for all *i* and all *t* because each property is considered unique (and assuming a property will not be sold twice in the same period). It is easy to verify that this leads to equations (25), (26a) and (26b) in the main text.

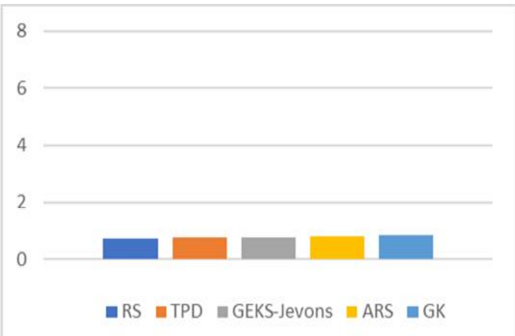
Appendix 3: Index volatility

A simple measure for the volatility of the multilateral indexes presented in Section 8 is the standard deviation of the monthly percentage changes. Panel A.1 shows the standard deviations for the whole country and for the cities of Amsterdam, Groningen and Assen. Notice that the volatility of the arithmetic ARS and GK indexes is consistently larger than that of the geometric RS and TPD indexes. The difference in volatility between RS and TPD is small (and the trends of the two indexes were also similar, as we have seen, so that the choice between RS and TPD does not seem to be important from a numerical perspective).

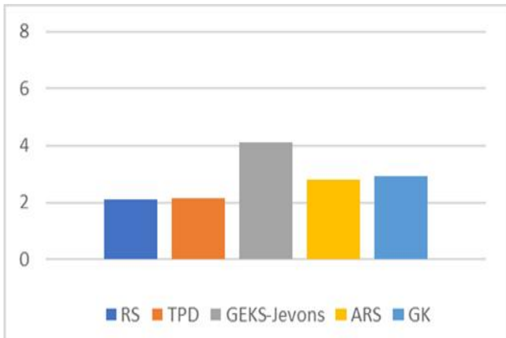
The panel nicely illustrates our finding that the RS index is expected to be less volatile than the GEKS-Jevons index; see also Appendix 1. In particular for the small city of Assen, the standard deviation (7.9%) is huge compared with the average monthly percentage change in the GEKS-Jevons index (0.7%).

Panel A.1: Standard deviation of monthly percentage index changes

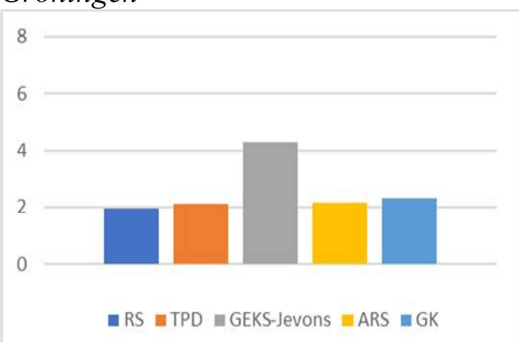
The Netherlands



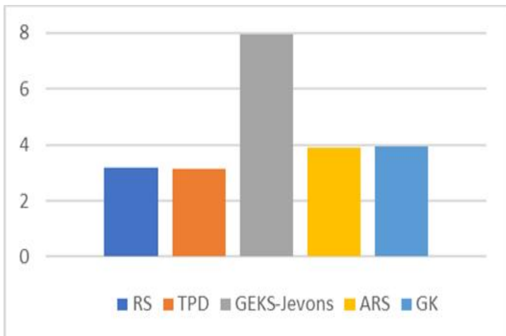
Amsterdam



Groningen



Assen



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