# Price and Quantity Indicators and Index Numbers

**Jan de Haan**[a]

21 February 2022

**Abstract:** Indicators split the absolute change in some aggregate value additively into price and quantity effects. This paper examines the relationship between the well-known Bennet indicators and Laspeyres and Paasche price and quantity indexes. The paper also explores several alternative decompositions, including a decomposition based on Fisher price and quantity indexes. The analysis is illustrated using data on energy expenses by Dutch households.

**Keywords:** decomposition analysis, index numbers, indicators.

**JEL Classification:** C43, E31.

---

[a] Statistics Netherlands and Delft University of Technology; email: jandehaan1@gmail.com. I would like to thank Arlen Hoebergen for providing the data and Jacco Daalmans for comments on an earlier version of this paper.

The views expressed in this paper are those of the author and do not necessarily reflect the views of Statistics Netherlands.

# 1. Introduction

Using appropriate index number formulae, an aggregate value ratio for two periods can be written as the product of a price index and quantity index. Well-known combinations are the Laspeyres price index and Paasche quantity index, the Paasche price index and Laspeyres quantity index, and the Fisher price and quantity indexes. In some cases, we want to decompose the value difference rather than the ratio into (additive) price and quantity components. These components are referred to as price and quantity indicators. The best-known indicators are due to Bennet (1920).

Section 2 of this paper shows how the Bennet indicators can be written in terms of Laspeyres and Paasche price and quantity indexes. Section 3 focuses on decomposing the percentage value change and examines the decomposition that follows directly from the Bennet indicators and a few alternative decompositions. Loosely following Diewert (2005), Section 4 then derives a decomposition of the percentage value change that is based on Fisher price and quantity indexes and discusses the corresponding indicators., Just like the Bennet indicators, the Fisher indicators satisfy the important time reversal test. Section 5 provides an empirical example using data on energy expenses by Dutch households. Section 6 concludes.

# 2. Bennet indicators

The price of product $i$ in periods 0 and 1 is denoted by $p_i^0$ and $p_i^1$; the corresponding quantities purchased are $q_i^0$ and $q_i^1$. Thus, $\sum_i p_i^0 q_i^0 = V^0$ and $\sum_i p_i^1 q_i^1 = V^1$ denote total expenditures on all products. I assume that every product is purchased in both periods; there are no new or disappearing products. The aim is to decompose the absolute change in the total value, i.e. the difference $V^1 - V^0$, additively into a price effect and a quantity effect.

To measure the effect of price changes, we want to hold quantities constant, and to measure the effect of quantity changes, we want to hold prices constant. There are two obvious choices to derive price effects: holding constant the quantities pertaining to either period 0 or period 1. Similarly, for deriving quantity effects, we can hold constant the prices of either period 0 or period 1. Hence, we find the following two alternative decompositions:

$$V^1 - V^0 = \left[\sum_i p_i^1 q_i^0 - \sum_i p_i^0 q_i^0\right] + \left[\sum_i p_i^1 q_i^1 - \sum_i p_i^1 q_i^0\right]$$
$$= \left[\sum_i q_i^0 \left(p_i^1 - p_i^0\right)\right] + \left[\sum_i p_i^1 \left(q_i^1 - q_i^0\right)\right]; \quad (1)$$

$$V^1 - V^0 = \left[\sum_i p_i^1 q_i^1 - \sum_i p_i^0 q_i^1\right] + \left[\sum_i p_i^0 q_i^1 - \sum_i p_i^0 q_i^0\right]$$
$$= \left[\sum_i q_i^1 \left(p_i^1 - p_i^0\right)\right] + \left[\sum_i p_i^0 \left(q_i^1 - q_i^0\right)\right], \quad (2)$$

where the first terms between square brackets in (1) en (2) are the price effects and the second terms are the quantity effects.

The two alternatives are equally "good" in that there is no reason to prefer one over the other. This suggests taking their (arithmetic) average, which was first proposed by Bennet (1920).[1] This yields

$$V^1 - V^0 = \left[\sum_i \left(\frac{q_i^0 + q_i^1}{2}\right)\left(p_i^1 - p_i^0\right)\right] + \left[\sum_i \left(\frac{p_i^0 + p_i^1}{2}\right)\left(q_i^1 - q_i^0\right)\right]. \quad (3)$$

The first term between square brackets in (3) is the Bennet price indicator, $IP_B^{01}$, and the second term between square brackets is the Bennet quantity indicator, $IQ_B^{01}$. It is easy to verify that $IP_B^{01}$ and $IQ_B^{01}$ satisfy the (indicator counterpart to) the time reversal test: $IP_B^{10} = -IP_B^{01}$ and $IQ_B^{10} = -IQ_B^{01}$. The price and quantity effects in (1) and (2) violate this important test.[2] For a single product *i*, the price and quantity contributions in equation (3) are equal to $(q_i^0 + q_i^1)(p_i^1 - p_i^0)/2$ and $(p_i^0 + p_i^1)(q_i^1 - q_i^0)/2$. Across all products, the contributions add up to $IP_B^{01}$ and $IQ_B^{01}$.

If we are not interested in the contributions of individual products, the indicators can be expressed in terms of price and quantity indexes. In particular, we can write (1) and (2) as

$$V^1 - V^0 = V^0\left[\left(P_L^{01} - 1\right)\right] + V^0\left[P_L^{01}\left(Q_P^{01} - 1\right)\right]; \quad (4)$$
$$V^1 - V^0 = V^0\left[Q_L^{01}\left(P_P^{01} - 1\right)\right] + V^0\left[\left(Q_L^{01} - 1\right)\right], \quad (5)$$

---

[1] For more information, see Balk, Färe and Grosskopf (2004), Diewert (2005), Balk (2008a), Diewert and Mizobuchi (2009), and De Boer and Rodrigues (2020).

[2] Decomposition (1) can be regarded as a first-order Taylor series approximation of Montgomery's (1929, 1937) decomposition. The latter uses the logarithmic mean and satisfies the time reversal test.

where $P_L^{01} = \sum_i p_i^1 q_i^0 / \sum_i p_i^0 q_i^0$ is the Laspeyres price index, $P_P^{01} = \sum_i p_i^1 q_i^1 / \sum_i p_i^0 q_i^1$ is the Paasche price index, $Q_L^{01} = \sum_i p_i^0 q_i^1 / \sum_i p_i^0 q_i^0$ is the Laspeyres quantity index, and $Q_P^{01} = \sum_i p_i^1 q_i^1 / \sum_i p_i^1 q_i^0$ is the Paasche quantity index.[3] Averaging (4) and (5) gives

$$V^1 - V^0 = V^0 \left[ \frac{\left(P_L^{01} - 1\right) + Q_L^{01}\left(P_P^{01} - 1\right)}{2} \right] + V^0 \left[ \frac{\left(Q_L^{01} - 1\right) + P_L^{01}\left(Q_P^{01} - 1\right)}{2} \right]. \tag{6}$$

To reiterate, the two components on the right-hand side of (6) are the Bennet price and quantity indicators, now expressed in terms of Laspeyres and Paasche price and quantity indexes (and the period 0 aggregate value).

## 3. Decomposing percentage value change

Dividing both sides of (6) by $V^0$ yields a decomposition of the percentage value change into price and quantity effects:

$$\frac{V^1}{V^0} - 1 = \left[ \frac{\left(P_L^{01} - 1\right) + Q_L^{01}\left(P_P^{01} - 1\right)}{2} \right] + \left[ \frac{\left(Q_L^{01} - 1\right) + P_L^{01}\left(Q_P^{01} - 1\right)}{2} \right]. \tag{7}$$

This result follows directly from the use of Bennet indicators. Alternate decompositions of the percentage value change are possible.

Since $P_L^{01} Q_P^{01} = V^1 / V^0 = P_P^{01} Q_L^{01}$, it is obvious that the following decompositions hold true:

$$\frac{V^1}{V^0} - 1 = P_L^{01} Q_P^{01} - 1 = \left(Q_P^{01} - 1\right) P_L^{01} + \left(P_L^{01} - 1\right); \tag{8}$$

$$\frac{V^1}{V^0} - 1 = P_P^{01} Q_L^{01} - 1 = \left(Q_L^{01} - 1\right) P_P^{01} + \left(P_P^{01} - 1\right). \tag{9}$$

Taking the average of (8) and (9) gives

$$\frac{V^1}{V^0} - 1 = \left[ \frac{\left(P_L^{01} - 1\right) + \left(P_P^{01} - 1\right)}{2} \right] + \left[ \frac{P_P^{01}\left(Q_L^{01} - 1\right) + P_L^{01}\left(Q_P^{01} - 1\right)}{2} \right]. \tag{10}$$

An alternative set of decompositions would be

[3] The advantage of decomposition (4) is perhaps that it points to the relationship with the Consumer Price Index (CPI), assuming the CPI is based on the Laspeyres formula.

$$\frac{V^1}{V^0}-1=P_L^{01}Q_P^{01}-1=\left(P_L^{01}-1\right)Q_P^{01}+\left(Q_P^{01}-1\right); \tag{11}$$

$$\frac{V^1}{V^0}-1=P_P^{01}Q_L^{01}-1=\left(P_P^{01}-1\right)Q_L^{01}+\left(Q_L^{01}-1\right). \tag{12}$$

Again, taking the average yields

$$\frac{V^1}{V^0}-1=\left[\frac{Q_P^{01}\left(P_L^{01}-1\right)+Q_L^{01}\left(P_P^{01}-1\right)}{2}\right]+\left[\frac{\left(Q_L^{01}-1\right)+\left(Q_P^{01}-1\right)}{2}\right]. \tag{13}$$

Unless there is no aggregate price change or aggregate quantity change, (10) and (13) will in general yield different results. We could take the average of (10) and (13), but I would still prefer (7) due to its close relationship with the Bennet indicators. Note, however, that decompositions (7), (10) and (13) are likely to produce similar results if the percentage changes are relatively modest. Taking natural logarithms of $P_L^{01}Q_P^{01}$ and $P_P^{01}Q_L^{01}$ and then applying the first-order Taylor series approximations $\ln(P_L^{01})\simeq P_L^{01}-1$, $\ln(Q_P^{01})\simeq Q_P^{01}-1$, $\ln(P_P^{01})\simeq P_P^{01}-1$, and $\ln(Q_L^{01})\simeq Q_L^{01}-1$, the following approximation can be derived:

$$\frac{V^1}{V^0}-1\simeq\left[\frac{\left(P_L^{01}-1\right)+\left(P_P^{01}-1\right)}{2}\right]+\left[\frac{\left(Q_L^{01}-1\right)+\left(Q_P^{01}-1\right)}{2}\right]. \tag{14}$$

## 4. An alternative based on Fisher indexes

Decomposition (7) for the aggregate percentage value change, which follows from the Bennet indicators, is based on Laspeyres and Paasche price and quantity indexes. I will now derive a similar decomposition, and the corresponding indicators, based on Fisher (ideal) price and quantity indexes $P_F^{01}=(P_L^{01}P_P^{01})^{1/2}$ and $Q_F^{01}=(Q_L^{01}Q_P^{01})^{1/2}$. For a detailed discussion, see section 4 in Diewert (2005).

Fisher indexes satisfy the product test, i.e. $P_F^{01}Q_F^{01}=V^1/V^0$. This suggests the following decompositions of the percentage change in the aggregate value:

$$\frac{V^1}{V^0}-1=P_F^{01}Q_F^{01}-1=\left(P_F^{01}-1\right)Q_F^{01}+\left(Q_F^{01}-1\right); \tag{15}$$

$$\frac{V^1}{V^0}-1=P_F^{01}Q_F^{01}-1=\left(Q_F^{01}-1\right)P_F^{01}+\left(P_F^{01}-1\right). \tag{16}$$

Taking the average yields

$$\frac{V^1}{V^0}-1=\left[\frac{\left(P_F^{01}-1\right)+Q_F^{01}\left(P_F^{01}-1\right)}{2}\right]+\left[\frac{\left(Q_F^{01}-1\right)+P_F^{01}\left(Q_F^{01}-1\right)}{2}\right]$$

$$=\left[\left(\frac{Q_F^{01}+1}{2}\right)\left(P_F^{01}-1\right)\right]+\left[\left(\frac{P_F^{01}+1}{2}\right)\left(Q_F^{01}-1\right)\right]. \tag{17}$$

The first component on the right-hand side of (17) measures the price effect, the second component the quantity effect. Note the similarity between decompositions (7) and (17); replacing the Laspeyres and Paasche price and quantity indexes in (7) by Fisher indexes leads to the first expression of (17).

Multiplying (17) by $V^0$ gives

$$V^1-V^0=V^0\left[\left(\frac{Q_F^{01}+1}{2}\right)\left(P_F^{01}-1\right)\right]+V^0\left[\left(\frac{P_F^{01}+1}{2}\right)\left(Q_F^{01}-1\right)\right]. \tag{18}$$

The components on the right-hand side of (18) are alternatives for the Bennet price and quantity indicators in (6). It can be shown that these Fisher(-based) indicators, denoted by $IP_F^{01}$ and $IQ_F^{01}$, also satisfy the time reversal test, i.e. $IP_F^{10}=-IP_F^{01}$ and $IQ_F^{10}=-IQ_F^{01}$. Because they rely on superlative indexes, I would prefer them to the Bennet indicators if the only aim is to construct aggregate measures. The Fisher indicators cannot be exactly decomposed into additive contributions of the various products, however, and statistical agencies might therefore prefer the Bennet indicators.[4]

Using (18) and (6), the difference between the Fisher and Bennet price indicators can be written as

$$IP_F^{01}-IP_B^{01}=\frac{V^0}{2}\left[\left(P_F^{01}-P_L^{01}\right)-\left(Q_F^{01}-Q_L^{01}\right)\right]. \tag{19}$$

Not surprisingly, this difference depends on the Paasche-Laspeyres spread. If $P_P^{01}=P_L^{01}$, and hence $P_F^{01}=P_L^{01}$ and $Q_F^{01}=Q_L^{01}$, then $IP_F^{01}=IP_B^{01}$. Often, though not in the empirical example of Section 5, we find $P_F^{01}<P_L^{01}$ and $Q_F^{01}<Q_L^{01}$, in which case the term between square brackets in (19) becomes relatively small. The difference between the Fisher and Bennet quantity indicators is of course the opposite of (19).

[4] Diewert and Mizobuchi (2009) called the Bennet price and quantity indicators "(strongly) superlative" and recommended "their use in practical applications of cost benefit analysis when ex post variations must be calculated".

## 5. An empirical illustration

Recently, Statistics Netherlands decided to use Bennet indicators in order to decompose and publish the absolute change in (expected) average household expenditure on energy into a price effect and a quantity effect. The quantity data pertain to expected rather than actual energy consumption. The aim is to calculate how much more, or less, the average household is expected to spend in euros on natural gas and electricity in the current year compared with the previous year, given the prevailing prices/tariffs, and to analyse what drives this change. The example below uses observable energy prices for January 2020 and January 2021.

Table 1 lists the data for the products distinguished, including the annual values.[5] The difference of the aggregate value is -€67.84 (=€2071.56 – €2139.39). The Bennet price and quantity indicators in Table 2, calculated according to (3), are equal to -€29.11 and -€38.73, respectively. The contributions of the various products are also shown. For "natural gas, variable tariff", for example, the values in 2020 and 2021 are €392.73 and €317.07 (Table 1), leading to a difference of -€75.66. The price and quantity effects are equal to -€68.30 and -€7.36 (Table 2).

**Table 1: Price and quantity data**

| | 2020, prices January | | | 2021, prices January | | |
|---|---|---|---|---|---|---|
| | p20 | q20 | value20 | p21 | q21 | value21 |
| *Natural gas* | | | | | | |
| Transport tariff | 185.36 | 1.00 | 185.36 | 187.85 | 1.00 | 187.85 |
| Fixed tariff | 66.52 | 1.00 | 66.52 | 70.19 | 1.00 | 70.19 |
| Variable tariff | 0.32 | 1217.00 | 392.73 | 0.27 | 1192.00 | 317.07 |
| Surcharge renew. energy | 0.09 | 1217.00 | 114.13 | 0.10 | 1192.00 | 122.74 |
| Energy tax | 0.40 | 1217.00 | 490.46 | 0.42 | 1192.00 | 502.74 |
| *Electricity* | | | | | | |
| Transport tariff | 241.85 | 1.00 | 241.85 | 257.36 | 1.00 | 257.36 |
| Fixed tariff | 67.50 | 1.00 | 67.50 | 71.33 | 1.00 | 71.33 |
| Variable tariff | 0.08 | 2547.00 | 195.61 | 0.07 | 2464.00 | 171.74 |
| Surcharge renew. energy | 0.03 | 2547.00 | 84.13 | 0.04 | 2464.00 | 89.44 |
| Energy tax | 0.12 | 2547.00 | 301.11 | 0.11 | 2464.00 | 281.09 |
| **Total** | | | 2139.39 | | | 2071.56 |

[5] The original data set included an item called reduction in energy tax that had a negative "price". Indexes cannot deal with negative prices, and I therefore excluded this item. The item has been included though in the official publication; see https://www.cbs.nl/nl-nl/nieuws/2021/07/prijs-van-energie-3-8-procent-lager (Dutch only).

**Table 2: Bennet price and quantity indicators**

| | Price effect | | | Quantity effect | | |
|---|---|---|---|---|---|---|
| | p21-p20 | (q21+q20)/2 | product | (p21+p20)/2 | q21-q20 | product |
| *Natural gas* | | | | | | |
| Transport tariff | 2.49 | 1 | 2.49 | 186.61 | 0 | 0.00 |
| Fixed tariff | 3.67 | 1 | 3.67 | 68.36 | 0 | 0.00 |
| Variable tariff | -0.06 | 1204.5 | -68.30 | 0.29 | -25 | -7.36 |
| Surcharge renew. energy | 0.01 | 1204.5 | 11.07 | 0.10 | -25 | -2.46 |
| Energy tax | 0.02 | 1204.5 | 22.58 | 0.41 | -25 | -10.31 |
| *Electricity* | | | | | | |
| Transport tariff | 15.51 | 1 | 15.51 | 249.61 | 0 | 0.00 |
| Fixed tariff | 3.83 | 1 | 3.83 | 69.42 | 0 | 0.00 |
| Variable tariff | -0.01 | 2505.5 | -17.79 | 0.07 | -83 | -6.08 |
| Surcharge renew. energy | 0.00 | 2505.5 | 8.19 | 0.03 | -83 | -2.88 |
| Energy tax | 0.00 | 2505.5 | -10.37 | 0.12 | -83 | -9.64 |
| **Total** | | | -29.11 | | | -38.73 |

**Table 3: Index numbers**

| | |
|---|---|
| Laspeyres price index | 0.98607 |
| Paasche price index | 0.98647 |
| Fisher price index | 0.98627 |
| | |
| Laspeyres quantity index | 0.98158 |
| Paasche quantity index | 0.98197 |
| Fisher quantity index | 0.98177 |

**Table 4: Decompositions percentage change**

| | Price effect | Quantity effect |
|---|---|---|
| Equation (9) [Bennet] | -1.36 | -1.81 |
| Equation (12) | -1.37 | -1.80 |
| Equation (15) | -1.35 | -1.82 |
| Equation (19) [Fisher] | -1.36 | -1.81 |
| | | |
| Approximation (16) | -1.37 | -1.82 |

Table 3 lists the Laspeyres, Paasche and Fisher price and quantity index numbers that follow from the data in Table 1. It is easy to check, using these index numbers and the 2020 aggregate value (€2139.39), that formula (8) yields the same Bennet indicators as formula (3). The index numbers can also be used to decompose the percentage value change (-3.17%) according to (9), (12), (15), and (19), and to calculate approximation (16). Table 4 shows the results. Since the Laspeyres and Paasche indexes hardly differ, the Bennet- and Fisher-based decompositions are numerically almost identical. While it slightly overstates the percentage value change, the approximation works rather well, as expected.

## 6. Conclusions

There is a straightforward relationship between the Bennet price and quantity indicators and Laspeyres and Paasche price and quantity indexes. This paper discussed a number of alternative indicators, and the corresponding decompositions of the percentage value change, including indicators based on Fisher price and quantity indexes. Not only do the Fisher indicators satisfy the important time reversal test, just like the Bennet indicators, they are superlative (Diewert, 2005).[6]

A practical disadvantage of the Fisher indicators is that they cannot be exactly decomposed into contributions of the various products so that statistical agencies might prefer using the Bennet indicators. If the spread between the Laspeyres and Paasche price and quantity indexes is small, then the difference between the Fisher and Bennet indicators will also be small.

## References

Balk, B.M. (2008a), *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference*. New York: Cambridge University Press.

Balk, B.M. (2008b), "Searching for the Holy Grail of Index Number Theory", *Journal of Economic and Social Measurement* 33, 19-25.

Balk, B.M, R. Färe en S. Grosskopf (2004), "The Theory of Economic Price and Quantity Indicators", *Economic Theory* 23, 149-164.

Bennet, T.L. (1920), "The Theory of Measurement of Changes in Cost of Living", *Journal of the Royal Statistical Society* 83, 455-462.

Casler, S.D. (2006), "Discrete Growth, Real Output, and Inflation: An Additive Perspective on the Index Number Problem", *Journal of Economic and Social Measurement* 31, 69-88.

[6] Other indicators have been proposed in the literature, e.g. by Casler (2006). Balk (2008b) showed that the Casler indicators do not satisfy the time reversal test, which is problematic.

De Boer, P. and J.F.D. Rodrigues (2020), "Decomposition Analysis: When to Use Which Method?", *Economic Systems Research* 32, 1-28.

Diewert, W.E. (2005), "Index Number Theory Using Differences Rather Than Ratios", *The American Journal of Economics and Sociology* 64, 311-360.

Diewert, W.E. and H. Mizobuchi (2009), "Exact and Superlative Price and Quantity Indicators", Macroeconomic Dynamics 13, Supplement S2: Measurement with Theory, 335-380.

Montgomery, J.K. (1929), "Is There a Theoretically Correct Price Index of a Group of Commodities?", International Institute of Agriculture, Rome.

Montgomery, J.K. (1937), *The Mathematical Problem of the Price Index*. London: P.S. King & Son.