A Distance-based Method for the Choice of Direct or Indirect Seasonal Adjustment

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Abstract

In this paper we deal with the problem of the choice of direct or indirect seasonal adjustment procedure. This is a problem for which the literature has not proposed many solutions; on the other hand, a solution of this problem is a crucial task, particularly for the National Statistical Institutes. In a model-based framework, the direct seasonal adjusted series is preferable, but if the discrepancy with respect the indirect seasonal adjusted series is large it can cause confusion in the users. We propose a new approach, based on the idea that the two data generating processes of the alternative series (the direct and the indirect sesonal adjusted series) can be compared in terms of a dissimilarity measure between ARMA models. A small dissimilarity implies that the difference between direct and indirect series is negligible and that the direct approach can be used. Our approach differs from the others because it is based on general statistical properties of the series and not on particular aspects, as the revisions, the smoothness, etc. An example referred to the Italian industrial production series is explained, in which the new procedure is applied. The procedure is performed in terms of hypothesis test, so that its application is standard and very simple.

1. Introduction

A time series can be the result of adding up two or more sub-series (eventually weighted); generally, the seasonal adjusted aggregate series is not identical to the sum of seasonal adjusted sub-series.

From a point of view of the seasonal adjusted policy, two natural alternatives arise to seasonal adjust the aggregate series:

1) the aggregated series is seasonal adjusted on its own (direct method);

2) the aggregate seasonal adjusted series is obtained as the sum of the seasonal adjusted sub-series (indirect method).

It is clear that summing up two or more series and seasonally adjusting the total does not necessarily give the same result as seasonally adjusting and then summing up the sub-series. In other terms, the two approaches can give different results and a problem of choice of the method arises.

The direct method can be preferable since the aggregate adjusted series is clearly of higher quality. Furthermore, following a model-based approach, the direct method is the natural choice, the seasonal adjusted series being derived from the ARIMA model of the rough series. On the other side the indirect method allows the consistency in aggregation.

Generally, quality and consistency are considered equally important, so that various criteria was proposed in literature in order to choose the direct or indirect method. Dagum (1979) uses two measures of lack of smoothness of the seasonal adjusted series to decide between the direct or the indirect method; Lothian and Morry (1977) indicate small revision errors as an important aspect in order to choose the method for seasonal adjustment, whereas Ghysels (1997) suggests the final estimation error. A criterion based on the stability of the seasonal adjusted series is used in the X-12-RegARIMA program, based on sliding spans and month-to month changes (Findley et al., 1998). Another kind of approach was proposed by den Butter and Fase (1991), allocating the discrepancy between direct and indirect methods among the seasonal adjusted sub-series, with weights proportional to the variance of the sub-series. Practically, they create a new seasonal adjusted series, different from those obtained from direct or indirect methods.

The most recent developments refer directly to the model-based approach. Planas and Campolongo (2000) base their analysis both on final estimation errors and total revisions in concurrent estimates, applying this procedure for the industrial production series of European Monetary Union countries. Gómez (2000) has proposed a criterion based on empirical revisions, measured with three alternative statistics.

The limit of these approaches is that the choice is based on a single aspect of the seasonal adjustment (smoothness, errors, revisions, stability,...), that can change with the kind of series or subjectively, according to the point of view of the researcher.

A typical common criterion (recommended by Eurostat too) is the use of direct seasonal adjustment if the discrepancy between methods is acceptable, and the use of indirect seasonal adjustment if this discrepancy is relevant. But when the discrepancy is relevant?

The simple evaluation of the size of the discrepancies is not sufficient. The same discrepancy in a series in level has a different interpretation than in a series of indices; in addition two series can be similar for the absolute discrepancies but they can have different behaviors in terms of period-to-period variations.[1]

If the seasonal adjustment method is a model based one, we retain that a correct approach should consider the stochastic properties of the series. In other terms, we retain that a discrepancy is relevant when the data generating process (DGP) of the indirect seasonal adjusted series is different from the DGP of the direct seasonal adjusted one. This approach is different from the others because we do not consider a particular aspect of the seasonal adjustment, but its global statistical properties, concerning the DGP. We propose the use of Piccolo's (1990) distance as a measure of difference between the two DGP's. In the next section we formalize the problem of direct and indirect method; in section 3 the new procedure is illustrated and in the section 4 an application of the approach is described. Concluding remarks follow. In Appendix A are reported the tables and in Appendix B the figures.

[1] The period-to-period variations are relevant with seasonal adjusted data.

2. The Consistency Problem

The purpose of this section is to provide a formal framework to analyze the so-called consistency problem. Let us consider a seasonal observed time series Y_t composed of two series X_t and Z_t, via the relationship

$$Y_t = X_t + Z_t; \qquad t = 1; ::: ; T:$$

Furthermore, we assume that the observable time series $Y_t; X_t; Z_t$ can be expressed as

$$\begin{aligned} Y_t &= Y_t^{ns} + Y_t^{s} \\ X_t &= X_t^{ns} + X_t^{s} \\ Z_t &= Z_t^{ns} + Z_t^{s} \end{aligned}$$

where Y_t^{ns}, $X_t^{ns}; Z_t^{ns}$ are the nonseasonal components containing the trend, the cycles and the irregular components, and Y_t^{s}, $X_t^{s}; Z_t^{s}$ are the seasonal components.

A desired property of seasonal adjustment procedures is that

$$Y_t^{ns} = X_t^{ns} + Z_t^{ns}: \tag{1}$$

In general the consistency requirement (1) is not satisfied: the seasonal adjusted series directly obtained from composed series Y_t is not equal to the sum of the seasonal adjusted components. In this case, a natural question arises: is it “better” to seasonally adjust the aggregate series (*direct* method) or to aggregate the seasonally adjusted sub-series (*indirect* method)?

If the discrepancy between direct and indirect seasonal adjustment

$$D^{ns} = Y^{ns} \; ¡ \; (X_t^{ns} + Z_t^{ns})$$

is “negligible”, the direct seasonal adjustment is preferable, because the seasonally adjusted composite series is clearly of a higher quality, especially when a model-based approach is used. Furthermore, the correlation structure between X_t and Z_t can not be captured with the indirect method, whereas modelling directly the aggregate series this problem does not exist. Finally the seasonal adjusted series obtained by using the indirect method may still exhibit spurious seasonality.

On the other hand, a strong difference between the direct seasonal adjusted series and the sum of the seasonal adjusted sub-series (a large D^{ns}) can cause confusion in the data users.

In this paper we utilize a formal statistical test to evaluate if the discrepancy is negligible or not. In particular, we consider the model-based approach, so that every series follows an ARIMA model; it implies that non-seasonal and seasonal components follow ARIMA models too. If the ARIMA model for the observed series is known, the models for the components can be derived through the canonical decomposition (Hillmer and Tiao, 1982).[2]

[2] This approach is followed in the seasonal adjusted routine named TRAMO-SEATS, developed by Gómez and Maravall (1997).

3. A Distance-based Approach

From the previous section it should be clear that the principal problem in choosing between direct and indirect methods is the evaluation of the size of the discrepancy. A natural solution can be obtained measuring the distance between the DGP of Y_t^{ns} and the DGP of $(X_t^{ns} + Z_t^{ns})$. Dealing with model-based methods for seasonal adjustment, the DGP's can be represented by an ARIMA model, so this purpose is achieved using a dissimilarity measure between ARIMA models. In order to obtain this measure, a useful tool is the AR metric introduced by Piccolo (1989) and (1990).

Let W_t be a zero-mean stochastic process and $\mathcal{F}$ the class of ARIMA invertible processes. It is well known that if $W_t \in \mathcal{F}$ then there exists a sequence of constant $\{\pi_i\}$ such that

$$\sum_{i=1}^{\infty} |\pi_i| < \infty$$

and

$$W_t = \sum_{i=1}^{\infty} \pi_i W_{t-i} + \varepsilon_t \qquad (2)$$

where $\varepsilon_t \sim WN(0; \sigma^2)$.

Following Piccolo (1989, 1990) we define the *distance* between $W_t^{(1)}; W_t^{(2)} \in \mathcal{F}$ as

$$d(W_t^{(1)}; W_t^{(2)}) = \left[\sum_{i=1}^{\infty} (\pi_{1i} - \pi_{2i})^2 \right]^{1/2}.$$

This distance measure can be used to compare the DGP of the direct seasonal adjusted series and the DGP of the indirect seasonal adjusted series. Using a model based approach, the canonical decomposition produces automatically a model for each component of the series, so that the direct seasonal adjusted series is immediately obtained. For example, a typical seasonal model, as the ARIMA(0,1,1)(0,1,1), produces a seasonal adjusted series following an IMA(2,2) model.

The DGP of the indirect seasonal adjusted series can be obtained summing up the ARIMA models relative to the seasonal adjusted sub-series. The state-space representation and the Kalman filters can be utilized to obtain the implicit model relative to the aggregate series. For example, let us consider two MA processes of order q_1 and q_2 respectively:

$$\begin{aligned} X_t &= \zeta_t + \vartheta_1 \zeta_{t-1} + \cdots + \vartheta_{q_1} \zeta_{t-q_1}; \\ Z_t &= \eta_t + \theta_1 \eta_{t-1} + \cdots + \theta_{q_2} \eta_{t-q_2}; \end{aligned}$$

where ζ_t and η_t are white noises with:

$$\begin{aligned} Cov(\zeta_t; \eta_t) &= \gamma \quad \text{for } t = 1; \ldots; T; \\ Cov(\zeta_t; \eta_s) &= 0 \quad \text{for } t \neq s. \end{aligned}$$

The process $V_t = X_t + Z_t$ follows a MA(max{$q_1; q_2$}) model. In order to obtain the coefficients of this MA model we express the process V_t in the following state-space form:

Observation equation: $V_t = a\xi_t$
State equation: $\xi_{t+1} = B\xi_t + e_t$

where:

$$a = \begin{bmatrix} 1 & \vartheta_1 & \cdots & \vartheta_{q_1} & 1 & \theta_1 & \cdots & \theta_{q_2} \end{bmatrix},$$

$$B = \begin{bmatrix} \underset{(1xq_1)}{0} & 0 & \underset{(1xq_2)}{0} & 0 \\ I_{q_1} & \underset{(q_1x1)}{0} & \underset{(q_1xq_2)}{0} & \underset{(q_1x1)}{0} \\ \underset{(1xq_1)}{0} & 0 & \underset{(1xq_2)}{0} & 0 \\ \underset{(q_2xq_1)}{0} & \underset{(q_2x1)}{0} & I_{q_2} & \underset{(q_2x1)}{0} \end{bmatrix},$$

$$\xi_t^{\prime} = \begin{bmatrix} \zeta_t & \zeta_{t-1} & \cdots & \zeta_{t-q_1} & \eta_t & \eta_{t-1} & \cdots & \eta_{t-q_2} \end{bmatrix},$$

$$e_t^{\prime} = \begin{bmatrix} \zeta_t & 0 & \cdots & 0 & \eta_t & 0 & \cdots & 0 \end{bmatrix}$$

and $\underset{(hxk)}{0}$ is a hxk matrix with all the elements equal to zero, whereas I_k is the identity kxk matrix. Denoting with K the steady-state Kalman gain, defined as:

$$K = BPa^{\prime}(aPa^{\prime})^{-1}; \tag{3}$$

where P is the steady-state MSE matrix of the state vector ξ_t, it can be demonstrated that the coefficients of the MA process of V_t are (see Hamilton, 1994, chapter 13):

$$\delta_j = aB^{j-1}K; \quad j = 1; \ldots; \max(q_1; q_2) \tag{4}$$

The (3) does not imply burdensome calculations; in fact the steady-state MSE matrix is expressed as:

$$P = \lim_{t \to \infty} P_{t|t-1};$$

where $\left\{P_{t|t-1}\right\}$ is the sequence of variance matrix calculated in each step of the Kalman filter. In particular, this sequence can be calculated by:[3]

$$P_{t+1|t} = B\left[P_{t|t-1} - P_{t|t-1}a^{\prime}\left(aP_{t|t-1}B\right)^{-1}aP_{t|t-1}\right]B^{\prime} + Q; \tag{5}$$

where Q is the variance matrix of e_t (invariant with t). If B is a kxk matrix whose eigenvalues are all inside the unit circle and $P_{1|0}$ is the initializing matrix of the sequence, satisfying:

$$vec(P_{1|0}) = [I_{k^2} - (B \otimes B)]^{-1} vec(Q);$$

it can be demonstrated that $\left\{P_{t|t-1}\right\}$ is a monotonically non increasing sequence that converge to:

$$P = B\left[P - Pa^{\prime}(aPB)^{-1}aP\right]B^{\prime} + Q:$$

[3] The formulas are referred to the particular state-space model used in this paper. For a general formulation, see Hamilton (1994), chapter 13.

In addition, if Q is strictly positive definite, the convergence is unique for any positive semidefinite symmetric matrix $P_{1|0}$. In other terms, iterating (5) we can obtain P, that provides the calculation of (3) and (4).

So, we are able to explicit the model representing the DGP of the indirect seasonal adjusted series, if the sub-series follow MA processes.

Now let us suppose that X_t and Z_t are two AR processes of order p_1 and p_2 respectively:

$$
\begin{aligned}
X_t &= \phi_1 X_{t-1} + \dots + \phi_{p_1} X_{t-p_1} + \epsilon_t; \\
Z_t &= \psi_1 Z_{t-1} + \dots + \psi_{p_2} Z_{t-p_2} + \eta_t;
\end{aligned}
$$

with:

$$
\begin{aligned}
Cov(\epsilon_t; \eta_t) &= \gamma \quad \text{for } t = 1; \dots; T; \\
Cov(\epsilon_t; \eta_s) &= 0 \quad \text{for } t \neq s.
\end{aligned}
$$

In this case, the process $V_t = X_t + Z_t$ follows an ARMA($p_1 + p_2$; max$\{p_1; p_2\}$) model:

$$\gamma(L)V_t = \delta(L)\varepsilon_t$$

where $\gamma(L) = \phi(L)\psi(L)$ and $\delta(L)\varepsilon_t = \psi(L)\epsilon_t + \phi(L)\eta_t$. The coefficients of $\delta(L)$ are obtained by (4).

Obtaining the estimates of the direct and indirect seasonal adjusted models, it is possible to calculate the Piccolo's distance between them. If this distance is not significantly different from zero, the discrepancy between direct and indirect methods can be considered negligible and we can use the former; if the distance is significantly different from zero, it is convenient to use the indirect method.

Now, the problem is to establish when the distance is not significantly different from zero. Piccolo (1989) showed that the asymptotic distribution of d^2 for an AR(p) process is a linear combination of independent Chi-Square variables and Corduas (1996) approximated it with a single Chi-Square distribution. Anyway, in a seasonal adjustment framework, we are often interested to general ARMA(p,q) models. For this reason we prefer to simulate the empirical distribution of d in the practical cases.

The procedure that we propose follows these steps:

1) seasonal adjust the aggregate series and each sub-series with the model-based procedure; the former is the direct seasonal adjusted series;

2) sum the ARIMA models relative to the seasonal adjusted sub-series, obtaining the parameters via the (4); they are the coefficients of indirect seasonal adjusted model;

3) express the direct and indirect seasonal adjusted series in the AR form (2);

4) calculate the distance d between the two AR models; if this is significantly different from zero use the direct method, otherwise use the indirect method.

4. An Example: the Italian Industrial Production Index

Let us consider the monthly series of the general Italian industrial production index (IPI$^{(0)}$) from January 1985 to December 1999, plotted in Figure 1. This series, which presents a clear seasonal behavior, is obtained by weighted aggregation of the indices relative to consumption (IPI$^{(1)}$), investments (IPI$^{(2)}$) and intermediate goods (IPI$^{(3)}$). The weights change with the different bases used to construct the indices and are reported in Table 1.

In order to apply our procedure, we have estimated the following model for each series:

$$IPI_t^{(i)} = \beta_{TD}^{(i)} TD_t + \beta_{LY}^{(i)} LY_t + \beta_{EE}^{(i)} EE_t + \beta_{H}^{(i)} H_t + \varepsilon_t^{(i)} \qquad i = 0; 1; 2; 3 \tag{6}$$

where TD_t is a regressor that represents the trading days effect, LY_t is the leap year effect at time t, EE_t is the Easter effect and H_t is the holidays effect;[4] $\varepsilon_t^{(i)}$ is a disturbance that follows an ARIMA(0,1,1)(0,1,1) model:

$$\varepsilon_t^{i} = \frac{\left(1 + \theta_1^{(i)} B\right)\left(1 + \theta_{12}^{(i)} B^{12}\right) w_t^{(i)}}{(1 - B)(1 - B^{12})} \qquad w_t^{(i)} \sim IIN(0; \sigma^2_{(i)})$$

Using the routine TRAMO-SEATS (Gómez and Maravall, 1997), we obtain the canonical decomposition of the model in a trend-cycle, a seasonal component and an irregular part.

The presence of deterministic effects does not affect our analysis. Calling *linearized series* the series obtained subtracting from $IPI_t^{(i)}$ (i =0, 1, 2, 3) the estimated deterministic effects, we can note in Figure 2 that the discrepancy between the linearized $IPI^{(0)}$ series and the aggregate linearized series is negligible.

The estimated ARIMA parameters for the three series are reported in Table 2. They produce the following models for the seasonal adjusted series:

$$(1 - B)^2 SA^{(0)} = \left(1 - 1.4550B + 0.4724B^2\right) v^{(0)} \tag{7}$$

$$\begin{aligned}(1 - B)^2 SA^{(1)} &= \left(1 - 1.5575B + 0.5770B^2\right) v^{(1)} \\ (1 - B)^2 SA^{(2)} &= \left(1 - 1.5119B + 0.5264B^2\right) v^{(2)} \\ (1 - B)^2 SA^{(3)} &= \left(1 - 1.3597B + 0.3801B^2\right) v^{(3)}\end{aligned} \tag{8}$$

where $SA^{(i)}$ (i =0, 1, 2, 3) is the seasonal adjusted series for $IPI^{(i)}$ and $v^{(i)}$ is a white noise. Summing the stationary parts of the processes relative to $SA^{(1)}$, $SA^{(2)}$, $SA^{(3)}$, we obtain the estimated DGP of the indirect seasonal adjusted series:

$$(1 - B)^2 SA_i^{(0)} = \left(1 - 1.4259B + 0.4449B^2\right) v_i^{(0)}.$$

The distance between the direct and indirect seasonal adjusted series is 0.5360. To verify the significance of this value we have calculated the empirical distribution of the statistic d under the null hypothesis that the distance is 0 (supposing the direct series as true). The

[4] The regressors are obtained as:

TD_t = # of (Mon, Tus, Wed, Thu, Fri)-#(Sat, Sun)$\frac{5}{2}$ in the month t;

$$LY_t = \begin{cases} 0.75 & \text{if } t \text{ is referred to a February in a leap year} \\ -0.25 & \text{if } t \text{ is referred to a February in a non leap year} \\ 0 & \text{otherwise} \end{cases}$$

$EE_t = \frac{j}{6}$ where j is the number of days of the month t that lies in the temporal interval:

$$[(\text{date of Easter}) - (6\text{ days}); \text{date of Easter}];$$

H_t is the number of national holidays, not coincident with Saturday and Sunday, that lies in the month t.

Monte Carlo experiments was performed with 1000 couples of series generated under the (7) with a variance for the white noise equal to $3{:}2083$, that is the estimated variance of (7).

The Figure 3 plots the empirical distribution; the critical value corresponding to a 0.05 size is $16{:}32$, so that the null hypothesis of distance zero is largely accepted. The statistical result is confirmed from the graphic comparison of Figure 4, in which the direct and indirect seasonal adjusted series are plotted. We are not able to distinguish the two lines because the two series are quasi-coincident. In Figure 5 the absolute discrepancies are reported and in Figure 6 the month-to-month percentage variations; we can note that the maximum absolute discrepancy is less than 0.22 and that the variations are very similar for the two series. These last considerations confirm the result of the test.

5. Final remarks

In this paper, a new approach to establish the use of direct or indirect seasonal adjustment method was proposed. The solution is valid only in the case of additive model, because in the case of moltiplicative model (or additive in logarithms) the approach is no longer valid because of the impossibility to obtain analitically the model for the indirect methods. Actually, we are looking for some approximation in this important and frequent case.

To apply the procedure we have used the Piccolo's distance, but other measures of dissimilarity could be used; for example, an interesting alternative is the distance between filters proposed by Depoutot and Planas (1998), that was used by Bruno and Otranto (2000) choosing the time interval to perform the seasonal adjustment. This tool does not imply the transformation of MA parameters in AR parameters, as in (2), with obvious advantages from a computational point of view. Another approach could use the test proposed by Maharaj and Inder (1996), who verify if two AR processes are generated from the same DGP. They perform a set of simulations only for the AR case, with a good power and respect of the nominal size.

The principal advantage of our method is that the choice between direct and indirect procedure is based on difference between DGP's and not on particular aspects of the estimation procedures, as the revisions, the stability, etc.

It will be interesting evaluate the empirical size and the power of the test proposed with Monte Carlo simulations.

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Appendix A. Tables

Table 1: Weights of the sub-series to obtain $IPI^{(0)}$

Years	$IPI^{(1)}$	$IPI^{(2)}$	$IPI^{(3)}$
1985-1989	0:267	0:177	0:556
1990-1994	0:257	0:158	0:585
1995-1999	0:232	0:165	0:603

Table 2: Estimates of MA parameters (t-values in parentheses):

	MA(1)	MA(12)
IPI^0	¡ 0:484	¡ 0:660
	(¡ 6:89)	(¡ 9:12)
IPI^1	¡ 0:598	¡ 0:543
	(¡ 9:43)	(¡ 7:25)
IPI^2	¡ 0:539	¡ 0:679
	(¡ 7:83)	(¡ 9:92)
IPI^3	¡ 0:387	¡ 0:664
	(¡ 5:29)	(¡ 8:87)

Appendix B. Figures

Figure 1: General Industrial production Index

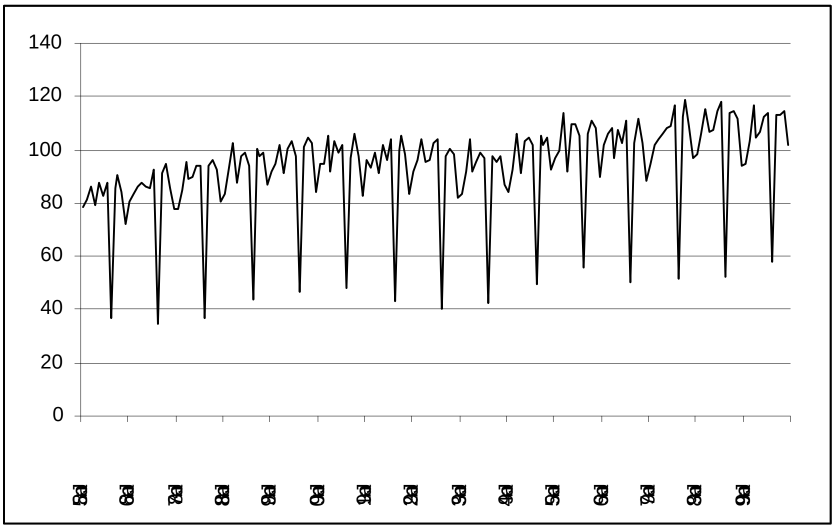

Figure 2: Discrepancy between direct and indirect linearized series

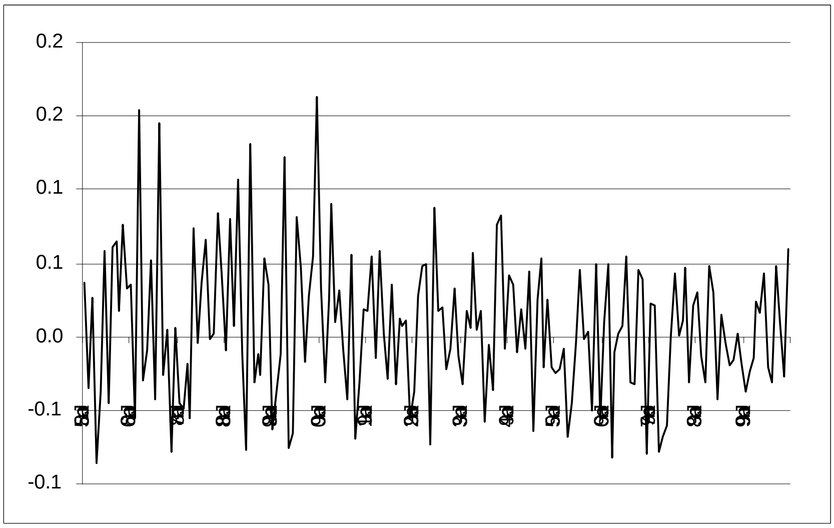

Figure 3: Empirical distribution of the distance under the null hypothesis of direct=indirect

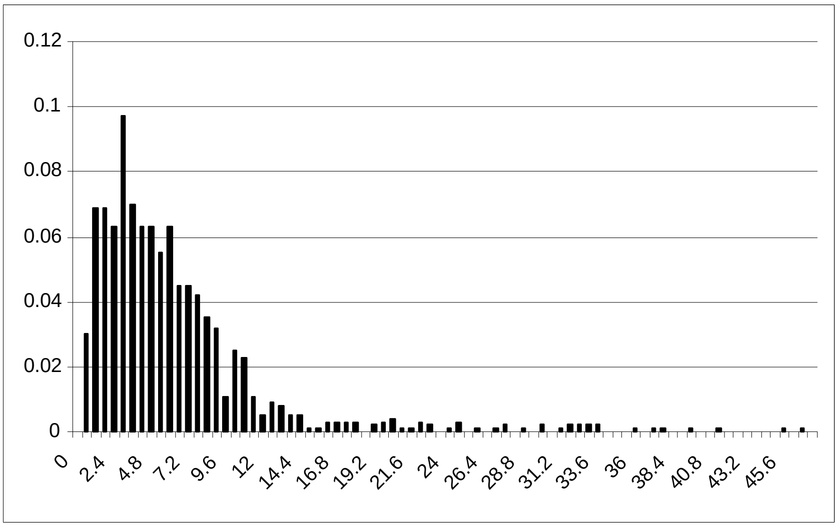

Figure 4: Direct (continuous line) and indirect (dot line) seasonal adjusted series.

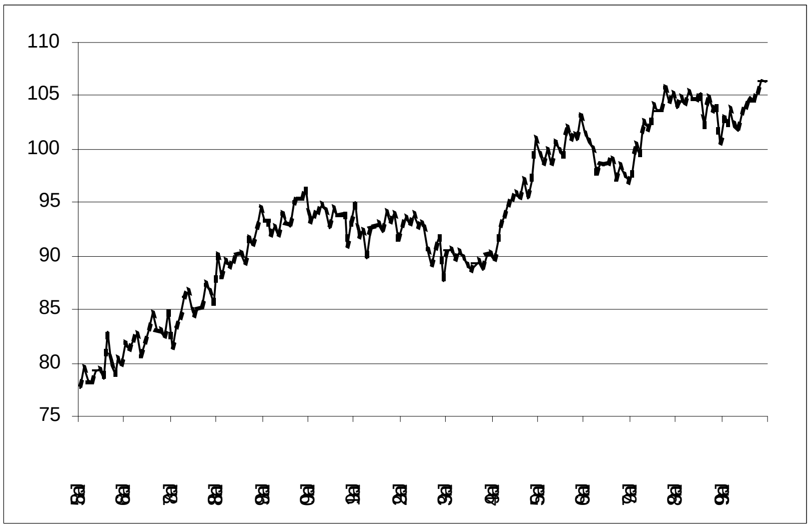

Figure 5: Discrepancy between the direct and indirect seasonal adjusted series.

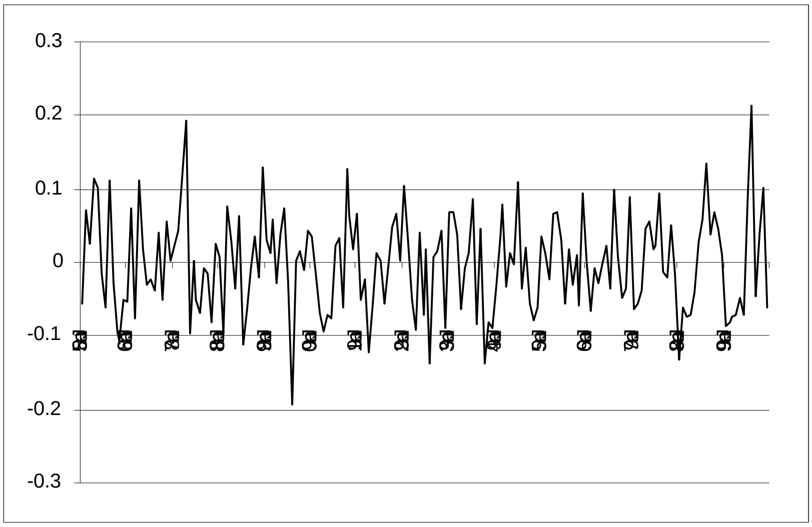

Figure 6: Month-to-month variation with direct method (continuous line) and indirect method (dot line)