

The Treatment of Durable Goods and Housing

April 28, 2022

Erwin Diewert,¹ University of British Columbia and University of New South Wales, Sydney;
Chihiro Shimizu, Hitotsubashi University, Japan.

Discussion Paper 22-02,
Vancouver School of Economics,
The University of British Columbia,
Vancouver, Canada V6T 1L4.

Abstract: The paper considers alternative approaches to the treatment of household purchases of durable goods (including housing) in a Consumer Price Index. The four main approaches are: (i) the acquisitions approach; (ii) the rental equivalence approach; (iii) the user cost approach and (iv) the payments approach. A fifth approach is the opportunity cost approach which takes the maximum of the rental price and the user cost price of the durable. Alternative models of depreciation are also discussed. The decomposition of house and condominium purchase prices into land and structure components is also discussed. The construction of rent indexes when there are new and disappearing units is also discussed. Various hedonic regression approaches to modelling the demand for housing are considered..

Key Words: Consumer Price Index, Treatment of durable goods, housing, user costs, opportunity costs, depreciation models, rent indexes, hedonic regressions, residential property price decompositions.

Journal of Economic Literature Classification Numbers: C43, D31, D32, E31

¹ Email address: erwin.diewert@ubc.ca. This chapter draws on Chapter 23 of the *Consumer Price Index Manual*; see ILO IMF OECD UNECE Eurostat World Bank (2004; 419-441) and on Chapter 6 of Diewert, Nishimura, Shimizu and Watanabe (2020; 223-298). The authors thank Paul Armknecht, John Astin, Corinne Becker-Vermeulen, David Fenwick, Dennis Fixler, Elspeth Hazell, Michael Henderson, Brian Graf, Ronald Johnson, Shaima Kamleh, Jill Leyland, Jens Mehrhoff, Paul Schreyer, Nigel Stapledon, Valentina Stoevska, Randall Verbrugge and Alice Xu for helpful comments on earlier drafts. This paper is a draft of Chapter 10 of volume *Consumer Price Index Theory*. For other draft Chapters, see <https://www.imf.org/en/Data/Statistics/cpi-manual>.

Table of Contents

| | |
|--|----------------|
| 1. Introduction | Page 3 |
| 2. The Acquisitions Approach | Page 5 |
| 3. The Rental Equivalence Approach | Page 7 |
| 4. The User Cost Approach for Pricing the Services of a Durable Good | Page 10 |
| 5. The Opportunity Cost Approach | Page 15 |
| 6. A General Model of Depreciation for Consumer Durables | Page 16 |
| 7. Geometric or Declining Balance Depreciation | Page 18 |
| 8. Alternative Depreciation Models | Page 20 |
| 9. The Relationship Between User Costs and Acquisition Costs | Page 24 |
| 10. User Costs for Storable Goods | Page 26 |
| 11. Decomposing Residential Property Prices into Land and Structure Components | Page 33 |
| 12. Decomposing Condominium Sales Prices into Land and Structure Components | Page 42 |
| 13. Demand Side Property Price Hedonic Regressions | Page 49 |
| 14. Price Indexes for Rental Housing: The Modified Repeat Rents Approach | Page 54 |
| 15. Price Indexes for Rental Housing: Hedonic Regression Approaches | Page 61 |
| 16. Owner Occupied Housing: The User Cost Perspective | Page 65 |
| 17. Valuing the Services of OOH: User Cost versus Rental Equivalence | Page 67 |
| 18. The Payments Approach and the Household Costs Index | Page 72 |
| 19. The Treatment of Household Monetary Balances in a CPI | Page 81 |
| 20. Summary and Conclusion | Page 84 |
| Appendix: Adjusting Household Rental Price Indexes for New and Disappearing Units | Page 88 |
| References | Page 92 |

1. Introduction

When a durable good (other than housing) is purchased by a consumer, national Consumer Price Indexes typically attribute *all* of that expenditure to the period of purchase even though the use of the good extends beyond the period of purchase.² By definition, a durable good delivers services longer than the accounting period under consideration.³ The *System of National Accounts 1993* defines a *durable good* as follows:

“In the case of goods, the distinction between acquisition and use is analytically important. It underlies the distinction between durable and non-durable goods extensively used in economic analysis. In fact, the distinction between durable and non-durable goods is not based on physical durability as such. Instead, the distinction is based on whether the goods can be used once only for purposes of production or consumption or whether they can be used repeatedly, or continuously. For example, coal is a highly durable good in a physical sense, but it can be burnt only once. A durable good is therefore defined as one which may be used repeatedly or continuously over a period of more than a year, assuming a normal or average rate of physical usage. A consumer durable is a good that may be used for purposes of consumption repeatedly or continuously over a period of a year or more.” *System of National Accounts 1993*, (1993; 208).

This Chapter will be concerned with the problems involved in pricing the services provided by durable goods according to the above definition. Thus durability is more than the fact that a good can physically persist for more than a year (this is true of most goods): a *durable good* is distinguished from a *nondurable good* due to its property that it can deliver useful services to a consumer through repeated use over an extended period of time. Examples of durable goods are automobiles and washing machines. A *storable good* is a good that can be stored over at least two periods of time but can only be used in a single period; e.g., a can of beans. A *perishable good* is a good that can be stored for only a limited period of time; e.g., a carton of milk. Thus perishable goods are like services: depending on the length of the period, they must be consumed in their period of purchase. Most of this chapter will be concerned with the treatment of durable goods but section 10 will look at the treatment of storable goods.

Since the benefits of using the consumer durable extend over more than one period, *it is not appropriate to charge the entire purchase cost of the durable to the initial period of purchase*. If this point of view is taken, then the initial purchase cost must be distributed somehow over the useful life of the asset. This is the *fundamental problem of accounting*.⁴ Hulten (1990) explained the consequences for accountants of the durability of a purchase as follows:

² This treatment of the purchases of durable goods dates back to Alfred Marshall (1898; 594-595) at least: “We have noticed also that though the benefits which a man derives from living in his own house are commonly reckoned as part of his real income, and estimated at the net rental value of his house; the same plan is not followed with regard to the benefits which he derives from the use of his furniture and clothes. It is best here to follow the common practice, and not count as part of the national income or dividend anything that is not commonly counted as part of the income of the individual.”

³ An alternative definition of a durable good is that the good delivers services to its purchaser for a period exceeding three years: “The Bureau of Economic Analysis defines consumer durables as those durables that have an average life of at least 3 years.” Arnold J. Katz (1983; 422).

⁴ “The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple matter of recording completed and fully realized transactions: an act of primitive simplicity.” Stephen Gilman (1939; 26).

“All the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter.” David Solomons (1961; 378). Note that these authors do not mention the additional complications that are due to the fact that future revenues and costs must be discounted to yield values that are equivalent to

“Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting capital in any year and the value of owning the capital asset. This distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year; that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value.” Charles R. Hulten (1990; 120-121).

There are three main methods for dealing with the durability problem:

- Ignore the problem of distributing the initial cost of the durable over the useful life of the good and allocate the entire charge to the period of purchase. This is known as the *acquisitions approach* and it is the present approach used by Consumer Price Index statisticians for all durables with the exception of housing.
- The *rental equivalence* or *leasing equivalence approach*. In this approach, a price is imputed for the durable which is equal to the rental price or leasing price of an equivalent consumer durable for the same period of time.
- The *user cost approach*. In this approach, the initial purchase cost of the durable is decomposed into two parts: one part which reflects an estimated cost of using the services of the durable for the period and another part, which is regarded as an investment, which must earn some exogenous rate of return.

These three major approaches will be discussed more fully in sections 2, 3, 4 and 9 below.⁵ There is a fourth approach that has not been applied but seems conceptually attractive. It will be discussed in section 5: the *opportunity cost approach*. This approach takes the *maximum* of the rental equivalence and user cost as the price for the use of the services of a consumer durable over a period of time. Finally, there is a fifth approach to the treatment of consumer durables that has only been used in the context of pricing owner occupied housing and that is the *payments approach*.⁶ This is a kind of cash flow approach, which will be discussed in section 18 after we have discussed the other approaches in more detail.

The three main approaches to the treatment of durable purchases can be applied to the purchase of any durable commodity. However, historically, it turns out that the rental equivalence and user cost approaches have only been applied to owner occupied housing. In other words, the acquisitions approach to the purchase of consumer durables has been universally used by statistical agencies, with the exception of owner occupied housing. A possible reason for this is tradition; i.e., Marshall (1898) set the standard and statisticians have followed his example for the past century. However, another possible reason is that unless the durable good has a very long useful life, it usually will not make a great deal of difference in the long run whether the

present dollars. For more recent papers on the fundamental problem of accounting, see Diewert (2005a; 480), Cairns (2013; 634) and Diewert and Fox (2016).

⁵ It should be noted that in principle, the user cost and rental equivalence approaches *should* be much the same: the owner of a rental property needs to construct a user cost for the current period (using its opportunity cost of capital as the interest rate that appears in the user cost formula) so that the resulting user cost can be used as the rental price that will just allow the owner to make the target rate of return on the property investment. In practice, the exact equality does not hold due to various market imperfections which will be discussed later.

⁶ This is the term used by Goodhart (2001; F350-F351).

acquisitions approach or one of the two alternative approaches is used. This point is discussed in more detail in section 10 below.

A major component of the user cost approach to valuing the services of Owner Occupied Housing (OOH) is the depreciation component. In section 6, a general model of depreciation for a consumer durable is presented and then it is specialized in sections 7 and 8 to the three models of depreciation that are widely used.

The general model presented in section 6 assumes that homogeneous units of the durable good are produced in each period and it also assumes that used units of the durable trade on secondhand markets so that information on the prices of the various vintages of the durable at any point in time can be used to determine the pattern of depreciation. However, many durables (like housing) are custom produced (i.e., they are unique goods) and thus the methods for determining the form of depreciation explained in section 6 are not immediately applicable. The special problems associated with the measurement of housing services are considered in sections 11-18.

Sections 11 and 12 show how information on the sales of dwelling units can be used to decompose the sales price into land and structure components. This information is required for the country's national balance sheet accounts. The decomposition into land and structure components is also required for the construction of rental prices and user costs and for measures of multifactor productivity for the rental housing sector of the economy.⁷ Section 11 looks at land and structure decompositions for the sale of detached housing units while section 12 does the same for the sales of condominium units. The hedonic regression models which are explained in sections 11 and 12 are basically supply side models while section 13 looks at demand side hedonic regression models for the sales of detached houses. Sections 14 and 15 look at the problems associated with the construction of rent indexes. Section 14 shows how a very simple repeat rents model can be modified in order to deal with depreciation of the structure which causes the quality of a rental unit to decline over time. However, there are some problems with the modified repeat rents model so section 15 considers more general hedonic regression models for rents. Sections 16 and 17 look at the problems associated with valuing the services of Owner Occupied Housing (OOH) in a consumer price index. Section 16 notes that in principle, there are separate user costs for the owned structure and for the land that the structure sits on. Section 17 compares the rental equivalence and user cost approaches for the treatment of OOH. This section also explains why the amount that an owned dwelling unit could rent for is in general different from the user cost that could be used to price the services of the unit to an owner. Section 18 looks at some alternative approaches to measuring housing services in a CPI such as the Payments Approach and the Household Costs Approach.

Section 19 applies the user cost approach to household holdings of monetary balances. The difficult issues associated with defining real monetary balances are also addressed.

Section 20 concludes.

2. The Acquisitions Approach

The net acquisitions approach to the treatment of owner occupied housing is described by Goodhart as follows:

“The first is the net acquisition approach, which is the change in the price of newly purchased owner occupied dwellings, weighted by the net purchases of the reference population. This is an asset based

⁷ Depreciation applies to the structure part of property value but not to the land part.

measure, and therefore comes close to my preferred measure of inflation as a change in the value of money, though the change in the price of the stock of existing houses rather than just of net purchases would in some respects be even better. It is, moreover, consistent with the treatment of other durables. A few countries, e.g., Australia and New Zealand, have used it, and it is, I understand, the main contender for use in the Euro-area Harmonized Index of Consumer Prices (HICP), which currently excludes any measure of the purchase price of (new) housing, though it does include minor repairs and maintenance by home owners, as well as all expenditures by tenants.” Charles Goodhart (2001; F350).

Thus the weights for the net acquisitions approach are the net purchases of the household sector of houses from other institutional sectors in the base period. Note that, in principle, purchases of second-hand dwellings from other sectors are relevant here; e.g., a local government may sell rental dwellings to owner occupiers. However, typically, newly built houses form a major part of these types of transactions. Thus the long term price relative for this category of expenditure will be primarily the price of (new) houses (quality adjusted) in the current period relative to the price of new houses in the base period.⁸ If this approach is applied to other consumer durables, it is extremely easy to implement: the purchase of a durable is treated in the same way as a nondurable or service purchase is treated.

One additional implication of the net acquisition approach is that major renovations and additions to owner occupied dwelling units could also be considered as being in scope for this approach. In practice, major renovations to a house are treated as investment expenditures and not covered as part of a consumer price index. Normal maintenance expenditures on a dwelling unit are usually treated in a separate category in the CPI.

Traditionally, the net acquisitions approach also includes transfer costs related to the buying and selling of secondhand houses as expenditures that are in scope for an acquisitions type consumer price index. These costs are mainly the costs of using a real estate agent’s services and asset transfer taxes. These costs can be measured but the question arises as to what is the appropriate deflator for these costs. An overall property price index is probably a satisfactory deflator.⁹

The major advantage of the acquisitions approach is that it treats durable and nondurable purchases in a completely symmetric manner and thus no special procedures have to be developed by a statistical agency to deal with durable goods.¹⁰ As will be seen in section 10 below, the major disadvantage of this approach is that the expenditures associated with this approach will tend to understate the corresponding expenditures on durables that are implied by the rental equivalence and user cost approaches.

Some differences between the acquisitions approach and the other approaches are:

⁸ This price index may or may not include the price of the land that the new dwelling unit sits on; e.g., a new house price construction index would typically not include the land cost. The acquisitions approach concentrates on the purchases by households of goods and services that are provided by suppliers from outside the household sector. Thus if the land on which a new house sits was previously owned by the household sector, then presumably, the cost of this land would be excluded from an acquisitions type new house price index. In this case, the price index that corresponds to the acquisitions approach is basically a new house price index (excluding land) or a modification of a construction cost index where the modification takes into account builder’s margins.

⁹ See the discussion in section 17 below on transfer costs.

¹⁰ The acquisitions approach is straightforward and simple for most durable goods but not for housing if the land component of property value is regarded as out of scope. Properties are sold with a single price that includes both the land and structure components of housing and so if the land part of property value is regarded as out of scope for the index, then there is a problem in decomposing property value into land and structure components. This decomposition problem can be avoided if information on the construction costs for building a new housing unit are available. In this case, the construction cost index (including builder’s markups) can serve as the price index for newly constructed dwelling units.

- If rental or leasing markets for the durable exist and the durable has a long useful life, then, as mentioned above, the expenditure weights implied by the rental equivalence or user cost approaches will typically be much larger than the corresponding expenditure weights implied by the acquisitions approach; see section 17 below.
- If the base year corresponds to a boom year (or a slump year) for the durable, then the base period expenditure weights may be too large or too small. Put another way, the aggregate expenditures that correspond to the acquisitions approach are likely to be more volatile than the expenditures for the aggregate that are implied by the rental equivalence or user cost approaches.¹¹
- In making comparisons of consumption across countries where the proportion of owning versus renting or leasing the durable varies greatly,¹² the use of the acquisitions approach may lead to misleading cross country comparisons. The reason for this is that opportunity costs of capital are excluded in the net acquisitions approach whereas they are explicitly or implicitly included in the other two approaches.

More fundamentally, whether the acquisitions approach is the right one or not depends on the overall purpose of the index number. If the purpose is to measure the price of current period consumption *services*, then the acquisitions approach can only be regarded as an approximation to a more appropriate approach (which would be either the rental equivalence or user cost approach). If the purpose of the index is to measure monetary (or nonimputed) expenditures by the household sector during the period, then the acquisitions approach is preferable (provided the land component of property value is in scope), since the rental equivalence and user cost approaches necessarily involve imputations.¹³

The details of the acquisitions approach (as applied to OOH) are discussed in great detail in Eurostat (2017).¹⁴ Eurostat is considering the use of the acquisitions approach for the treatment of OOH in its Harmonized Index of Consumer Prices (HICP) but at this date, no decision has been finalized. At present, OOH is simply omitted in the HICP. Eurostat considered the use of the acquisitions approach for OOH because at first sight, it seems that no imputations have to be made in order to implement it. The HICP was created as an index of consumer prices that used actual transactions prices without the use of any imputations.¹⁵ As such, it was thought to be particularly useful for monitoring inflation by central banks. However, the sale of a newly constructed dwelling unit typically includes a land component which the Eurostat methodology excludes but existing methods for excluding the land component involve imputations.¹⁶

3. The Rental Equivalence Approach

¹¹ Hill, Steurer and Wautl (2020) make this point and list other problems with the acquisitions approach.

¹² From Hoffmann and Kurz (2002; 3-4), about 60% of German households lived in rented dwellings whereas only about 11% of Spaniards rented their dwellings in 1999.

¹³ Fenwick (2009) (2012) laid out the case for the use of the acquisitions approach as a useful measure of general inflation. He also argued for the construction of multiple consumer price indexes to suit different purposes.

¹⁴ This very useful publication also discusses the main methods for the treatment of OOH and it also covers the methods used to construct residential property price indexes. The latter topic is also covered in Eurostat (2013).

¹⁵ However, with the passage of time, it became apparent that some imputations for changes in the quality of consumer goods and services had to be made. Thus the current HICP is not completely free from imputations. See Astin (1999) for the methodological foundations of the HICP.

¹⁶ The use of a construction cost index to value the structure component of property value also involves an imputation but it is a reasonably straightforward one.

The rental equivalence approach simply values the services yielded by the use of a consumer durable good for a period by the corresponding market rental value for the same durable for the same period of time (if such a rental value exists).

The most important consumer durable in a consumer price index is housing that is owned (Owner Occupied Housing). The international System of National Accounts: 1993 recommended the use of the rental equivalence approach to measure the services of Owner Occupied Housing (OOH):

“As well-organized markets for rented housing exist in most countries, the output of own-account housing services can be valued using the prices of the same kinds of services sold on the market with the general valuation rules adopted for goods and services produced on own account. In other words, the output of housing services produced by owner-occupiers is valued at the estimated rental that a tenant would pay for the same accommodation, taking into account factors such as location, neighbourhood amenities, etc. as well as the size and quality of the dwelling itself.” Eurostat, IMF, OECD, UN and World Bank (1993; 134).

However, the System of National Accounts 1993 followed Marshall (1898; 595) and did not extend the rental equivalence approach to consumer durables other than housing. This seemingly inconsistent treatment of durables was explained in the SNA 1993 as follows:

“The production of housing services for their own final consumption by owner-occupiers has always been included within the production boundary in national accounts, although it constitutes an exception to the general exclusion of own-account service production. The ratio of owner-occupied to rented dwellings can vary significantly between countries and even over short periods of time within a single country, so that both international and intertemporal comparisons of the production and consumption of housing services could be distorted if no imputation were made for the value of own-account services.” Eurostat, IMF, OECD, UN and World Bank (1993; 126).

Eurostat’s (2001) Handbook on Price and Volume Measures in National Accounts also recommended the rental equivalence approach for the treatment of the dwelling services for owner occupied housing:

“The output of dwelling services of owner occupiers at current prices is in many countries estimated by linking the actual rents paid by those renting similar properties in the rented sector to those of owner occupiers. This allows the imputation of a notional rent for the service owner occupiers receive from their property.” Eurostat (2001; 99).

To summarize the above material, it can be seen that the rental equivalence approach to the treatment of a durable good is conceptually simple: use the current period rental or leasing price for a comparable unit of the consumer durable to measure its service flow. But where will the statistical agency find the relevant rental data to price the services of OOH? There are at least three possible methods:

- Ask home owners what they think the market rent for their dwelling unit is;¹⁷
- Undertake a survey of owners of rental properties or managers of rental properties and ask what rents they charge for their rental properties by type of property or
- Use one of the above two methods to get a rent to value ratio for various types of property for a benchmark period and then link these ratios to indexes of purchase prices for the various types of property.¹⁸

¹⁷ This approach is used by the Bureau of Labor Statistics (1983) in order to determine *expenditure weights* for owner occupied housing; i.e., homeowners are asked to estimate what their house would rent for if it were rented to a third party.

There are some disadvantages associated with the use of the rental equivalence approach to the valuation of OOH services:

- Homeowners may not be able to provide very accurate estimates for the rental value of their dwelling unit.
- On the other hand, if the statistical agency tries to match the characteristics of an owned dwelling unit with a comparable unit that is rented in order to obtain the imputed rent for the owned unit, there may be difficulties in finding such comparable units. Furthermore, even if a comparable unit is found, the rent for the comparable unit may not be an appropriate opportunity cost for valuing the services of the owned unit.¹⁹
- The statistical agency should make an adjustment to these estimated rents over time in order to take into account the effects of depreciation, which causes the quality of the unit to slowly decline over time (unless this effect is completely offset by renovation and repair expenditures).²⁰
- Care must be taken to determine exactly what extra services are included in the homeowner's estimated rent; i.e., does the rent include insurance, electricity and fuel or the use of various consumer durables in addition to the structure? If so, these extra services should be stripped out of the rent if they are covered elsewhere in the consumer price index.²¹

In order to overcome the first difficulty listed above, the Japanese government collects housing rent data from *property management companies* or owners who rent out their dwelling units; i.e., Japan uses the second method to value the services of OOH. However, the characteristics of the owner occupied population of dwelling units are generally quite different from the characteristics of the rental population.²² Thus typically, it is difficult to find rental units that are comparable to owned dwelling units. The use of hedonic regression techniques can mitigate this lack of matching problem. Moreover, the use of hedonic regressions can deal with the depreciation or quality decline problem mentioned above. We will discuss hedonic regression techniques later in this Chapter in sections 11-15.

¹⁸ Lebow and Rudd (2003; 169) note that the US Bureau of Economic Analysis applies a benchmark rent to value ratio for rented properties to the value of the owner occupied stock of housing. It can be seen that this approach is essentially a simplified user cost method where all of the key variables in the user cost formula (to be discussed later) are held constant except the asset price of the property.

¹⁹ We will return to this point after we have discussed the opportunity cost approach to the valuation of OOH services.

²⁰ This issue will be discussed in more detail in section 17 below. Papers which discuss how to strip out utility and insurance costs out of rents include Verbrugge (2012), Coffey, McQuinn and O'Toole (2020) and Adams and Verbrugge (2021). Also, in many countries, there are rent controls. A rent controlled comparable property is not a correct opportunity cost to use to value the services of an owned dwelling unit.

²¹ However, it could be argued that these extra services that might be included in the rent are mainly a weighting issue; i.e., it could be argued that the *trend* in the homeowner's estimated rent would be a reasonably accurate estimate of the trend in the rents after adjusting for the extra services included in the rent.

²² For example, according to the 2013 Japanese Housing and Land Survey, the average floor space (size) of owner occupied housing in Tokyo was 110.64 square meters for single family houses and 82.71 square meters for rental housing, a difference of over 30 square meters. For condominiums, an even greater discrepancy exists: the average floor space is 65.73 square meters for owner-occupied housing and 37.64 square meters for rental housing. Moreover, in addition to the difference in floor space between rented and owned units, the quality of the owned units tends to be higher than the rented units and these quality differences need to be taken into account.

In addition to the above possible biases in using the rental equivalence approach to the valuation of the services of OOH, there are differences between *contract rent* and *market rent*. *Contract rent* or *roll-over rent* refers to the rent paid by a renter who has a long term rental contract with the owner of the dwelling unit and (current) *market rent* is the rent paid by the renter in the first period after a rental contract has been negotiated. In a normal economy which is experiencing moderate or low general inflation, typically market rent will be higher than contract rent. However, if there are rent controls or a temporary glut of rental units, then market rent could be lower than contract rent. In any case, it can be seen that if we value the services of an owner occupied dwelling at its current opportunity cost on the rental market, market rent should be used in the CPI to value the services of OOH rather than contract rent. However, contract rent or rollover rent (adjusted for depreciation and improvements) should be used to estimate the cost of rented dwellings in a CPI.

Finally, it is known that price adjustments are often not made for rollover contracts (i.e. renewed leases). As a result, it is likely that new contract rents determined freely by the market will diverge considerably from rollover contract rents.²³ This is the *stickiness of rents problem*.

In the following section, we provide an introduction to user cost theory for a non-housing durable good. In subsequent sections, we will deal with the problems associated with measuring depreciation and the aggregation of user costs over different ages of the same good. And later yet in sections 11 to 17, we will look at the additional difficulties that are associated with the formation of user costs for housing and the relationship between user costs and rental prices for housing services.

4. The User Cost Approach for Pricing the Services of a Durable Good

The user cost approach to the treatment of durable goods is in some ways very simple: it calculates the cost of purchasing the durable at the beginning of the period, using the services of the durable during the period and then netting off from these costs the benefit that could be obtained by selling the durable good at the end of the period. However, there are several details of this procedure that are somewhat controversial. These details involve the use of opportunity costs (which are usually imputed costs), the treatment of interest and the treatment of capital gains or holding gains.

Another complication with the user cost approach is that it involves making distinctions between current period (flow) purchases within the period under consideration and the holdings of physical stocks of the durable at the beginning and the end of the accounting period. Typically, when constructing a consumer price index, we think of all quantity purchases as taking place at a single point in time, say the middle of the period under consideration, at the (unit value) average prices for the period. In constructing user costs, prices at the beginning and end of an accounting period play an important role.

To determine the net cost of using a durable good during say period 0, it is assumed that one unit of the durable good is purchased at the beginning of period 0 at the price P^0 . The “used” or “second-hand” durable good can be sold at the end of period 0 at the price P_s^1 .²⁴ It might seem

²³ On this point, see Genesove (2003), Shimizu, Nishimura and Watanabe (2010b), Shimizu and Watanabe (2011), Lewis and Restieaux (2015), Gallin and Verbrugge (2019) and Suzuki, Asami and Shimizu (2021).

²⁴ Note that this approach to pricing the services of a durable good assumes the existence of secondhand markets for units of the durable that have aged. This assumption may not be satisfied for many consumer

that a reasonable *net cost* for the use of one unit of the consumer durable during period 0 is its initial purchase price P^0 less its end of period 0 “scrap value”, P_S^1 . However, *money received at the end of the period is not as valuable as money that is received at the beginning of the period*. Thus in order to convert the end of period value into its beginning of the period equivalent value, it is necessary to *discount* the term P_S^1 by the term $1+r^0$ where r^0 is the beginning of period 0 nominal interest rate that the consumer faces. Hence the *period 0 user cost* u^0 for the consumer durable²⁵ is defined as

$$(1) u^0 \equiv P^0 - P_S^1/(1+r^0).$$

There is another way to view the user cost formula (1): the consumer purchases the durable at the beginning of period 0 at the price P^0 and charges himself or herself the rental price u^0 . The remainder of the purchase price, I^0 , defined as

$$(2) I^0 \equiv P^0 - u^0$$

can be regarded as an *investment*, which is to yield the appropriate opportunity cost of capital r^0 that the consumer faces. At the end of period 0, this rate of return could be realized provided that I^0 , r^0 and the selling price of the durable at the end of the period P_S^1 satisfy the following equation:

$$(3) I^0(1+r^0) = P_S^1 .$$

Given P_S^1 and r^0 , (3) determines I^0 , which in turn, given P^0 , determines the user cost u^0 via (2)²⁶.

Thus user costs are not like the prices of nondurables or services because the user cost concept involves pricing the durable at *two* points in time rather than at a single point in time. Because the user cost concept involves prices at two points in time, money received or paid out at the first point in time is more valuable than money paid out or received at the second point in time and so *interest rates* creep into the user cost formula. Furthermore, because the user cost concept involves prices at two points in time, *expected prices* can be involved if the user cost is calculated at the beginning of the period under consideration instead of at the end. With all of these complications, it is no wonder that many price statisticians would like to avoid using user costs as a pricing concept. However, even for price statisticians who would prefer to use the rental equivalence approach to the treatment of durables over the user cost approach, there is some justification for considering the user cost approach in some detail, since this approach gives insights into the economic determinants of the *rental or leasing price* of a durable.

The user cost formula (1) can be put into a more familiar form if the period 0 *economic depreciation rate* δ and the period 0 *ex post asset inflation rate* i^0 are defined. Define δ by:

$$(4) (1 - \delta) \equiv P_S^1/P^1$$

where P_S^1 is the price of a one period old used asset at the end of period 0 and P^1 is the price of a new asset at the end of period 0. Typically, if a new asset and a one period older asset are sold at

durables including unique assets such as dwelling units and works of art, which are not bought and sold every period.

²⁵ This approach to the derivation of a user cost formula was suggested by Diewert (1974), who in turn based it on an approach due to Hicks (1946; 326).

²⁶ This derivation for the user cost of a consumer durable was also made by Diewert (1974; 504).

the same time, then the new asset will be worth more than the used asset and hence δ will be a positive number between 0 and 1. The *period 0 inflation rate* for the new asset, i^0 , is defined by:

$$(5) 1+i^0 \equiv P^1/P^0 .$$

Eliminating P^1 from equations (4) and (5) leads to the following formula for the *end of period 0 used asset price*:

$$(6) P_s^1 = (1 - \delta)(1 + i^0)P^0 .$$

Substitution of (6) into (1) yields the following expression for the *period 0 user cost* u^0 :

$$(7) u^0 = [(1 + r^0) - (1 - \delta)(1 + i^0)]P^0/(1 + r^0) .$$

Note that $r^0 - i^0$ can be interpreted as a period 0 *real interest rate* and $\delta(1+i^0)$ can be interpreted as an *inflation adjusted depreciation rate*.

The user cost u^0 is expressed in terms of prices that are discounted to the *beginning* of period 0. However, it is also possible to express the user cost in terms of prices that are “anti-discounted” or *appreciated* to the *end* of period 0.²⁷ Thus define the *end of period 0 user cost* p^0 as:²⁸

$$(8) p^0 \equiv (1 + r^0)u^0 = [(1 + r^0) - (1 - \delta)(1 + i^0)]P^0$$

where the last equation follows using (7). If the real interest rate r^{0*} is defined as the nominal interest rate r^0 less the asset inflation rate i^0 and the small term δi^0 is neglected, then the end of the period user cost defined by (8) reduces to:

$$(9) p^0 = (r^{0*} + \delta)P^0 .$$

Abstracting from transactions costs and inflation, it can be seen that the end of the period user cost defined by (9) is an *approximate rental cost*; i.e., the rental cost for the use of a consumer (or producer) durable good should equal the (real) opportunity cost of the capital tied up, $r^{0*}P^0$, plus the decline in value of the asset over the period, δP^0 . Formulae (8) and (9) thus cast some light on what are the economic determinants of rental or leasing prices for consumer durables.

²⁷ Thus the beginning of the period user cost u^0 discounts all monetary costs and benefits into their dollar equivalent at the beginning of period 0, whereas p^0 discounts (or appreciates) all monetary costs and benefits into their dollar equivalent at the end of period 0. This leaves open how flow transactions that take place within the period should be treated. Following the conventions used in financial accounting suggests that *flow transactions* taking place within the accounting period be regarded as taking place at the end of the accounting period and hence, following this convention, end of period user costs should be used by the price statistician; see Peasnell (1981).

²⁸ Christensen and Jorgenson (1969) derived a user cost formula similar to (7) in a different way using a continuous time optimization model. If the inflation rate i equals 0, then the user cost formula (7) reduces to that derived by Walras (1954; 269) (first edition 1874). This zero inflation rate user cost formula was also derived by the industrial engineer A. Hamilton Church (1901; 907-908), who perhaps drew on the work of Matheson: “In the case of a factory where the occupancy is assured for a term of years, and the rent is a first charge on profits, the rate of interest, to be an appropriate rate, should, so far as it applies to the buildings, be equal (including the depreciation rate) to the rental which a landlord who owned but did not occupy a factory would let it for.” Ewing Matheson (1910; 169), first published in 1884. Additional derivations of user cost formulae in discrete time have been made by Katz (1983; 408-409) and Diewert (2005a). Hall and Jorgenson (1967) introduced tax considerations into user cost formulae.

If the simplified user cost formula defined by (9) above is used, then, at first glance, forming a price index for the user cost of a durable good is not very much more difficult than forming a price index for the purchase price of the durable good, P^0 . The price statistician needs only to:

- Make a reasonable assumption as to what an appropriate monthly or quarterly real interest rate r^{0*} should be;
- Make an assumption as to what a reasonable monthly or quarterly depreciation rate δ should be;²⁹
- Collect purchase prices P^0 for the durable and use formula (9) to calculate the simplified user cost.³⁰

If it is thought necessary to implement the more complicated user cost formula (8) in place of the simpler formula (9), then the situation is more complicated. As it stands, the end of the period user cost formula (8) is an *ex post* (or after the fact) *user cost*: the asset inflation rate i^0 cannot be calculated until the end of period 0 has been reached. Formula (8) can be converted into an *ex ante* (or before the fact) *user cost* formula if i^0 is interpreted as an *anticipated asset inflation rate*. The resulting formula should approximate a market rental rate for the durable good.³¹

Note that in the user cost approach to the treatment of consumer durables, the *entire* user cost formula (8) or (9) is the period 0 price. Thus in the time series context, it is *not* necessary to deflate each component of the formula *separately*; the period 0 price $p^0 \equiv [r^0 - i^0 + \delta(1+i^0)]P^0$ is compared to the corresponding period 1 price, $p^1 \equiv [r^1 - i^1 + \delta(1+i^1)]P^1$ and so on.

In principle, depreciation rates can be estimated using information on the selling prices of used units of the durable good.³² However, for housing, the situation is more complex, as will be explained later.

²⁹ The geometric model for depreciation to be explained in more detail in section 6 below requires only a single monthly or quarterly depreciation rate. Other models of depreciation may require the estimation of a sequence of vintage depreciation rates. If the estimated annual geometric depreciation rate is δ_a , then the corresponding monthly geometric depreciation rate δ can be obtained by solving the equation $(1 - \delta)^{12} = 1 - \delta_a$. Similarly, if the estimated annual real interest rate is r_a^* , then the corresponding monthly real interest rate r^* can be obtained by solving the equation $(1 + r^*)^{12} = 1 + r_a^*$.

³⁰ Iceland uses a variant of the simplified user cost formula (9) to estimate the services of OOH with a real interest rate approximately equal to 4% and depreciation rate of 1.25%. The depreciation rate is relatively low because it is applied to the entire property value and not to just the structure portion of property value; see Gudnason and Jonsdottir (2011). Eurostat (2005) also uses a simplified user cost formula. Additional simplified user cost formulae have been developed by Verbrugge (2008), Hill, Steurer and Walzl (2020) and many others; see section 17 below.

³¹ Since landlords must set their rent at the beginning of the period (in actual practice, they usually set their rent for an extended period of time) and if the user cost approach is used to model the economic determinants of market rental rates, then the asset inflation rate i^0 should be interpreted as an *expected inflation rate* rather than an after the fact actual inflation rate. This use of *ex ante* prices in this price measurement context should be contrasted with the preference of national accountants to use actual or *ex post* prices in the system of national accounts.

³² For housing, the situation is more complex because typically, a dwelling unit is a *unique good*; its location is a price determining characteristic and each housing unit has a unique location and thus is a unique good. It also changes its character over time due to renovations and depreciation of the structure. Thus the treatment of housing is much more difficult than the treatment of other durable goods. Note that the definitions (4) and (5) of the depreciation rate δ and the asset inflation rate i^0 implicitly assumed that prices for a new asset and a one period old asset were available in both periods 0 and 1. This assumption is not satisfied for a unique asset.

We conclude this introductory section by noting some practical problems that statistical agencies will face when calculating user costs for durable goods:³³

- It is difficult to determine what the relevant nominal interest rate r^0 is for each household. If a consumer has to borrow to finance the cost of a durable good purchase, then this interest rate will typically be much higher than the safe rate of return that would be the appropriate opportunity cost rate of return for a consumer who had no need to borrow funds to finance the purchase.³⁴ It may be necessary to simply use a benchmark interest rate that would be determined by either the government, a national statistical agency or an accounting standards board.³⁵
- It will generally be difficult to determine what the relevant depreciation rate is for the consumer durable.³⁶
- *Ex post user costs* based on formula (8) may be too volatile to be acceptable to users³⁷ (due to the volatility of the ex post asset inflation rate i^0) and hence an *ex ante user cost* concept may have to be used. For most durable goods, the asset inflation rates are smaller than the reference nominal interest rate so that subtracting an ex post asset inflation rate from the sum of the nominal interest rate plus the asset depreciation rate will usually lead to reasonably stable positive user costs. However, for durable goods with very low depreciation rates, like a housing structure or like land (which has a zero depreciation rate), the resulting ex post user costs may turn out to be negative for some periods. This means that the resulting negative user costs are not useful approximations to rental prices for these long-lived durable goods. This creates difficulties in that different national statistical agencies will generally make

³³ For additional material on difficulties with the user cost approach, see Diewert (1980; 475-479) and Katz (1983; 415-422).

³⁴ Katz (1983; 415-416) comments on the difficulties involved in determining the appropriate rate of interest to use: “There are numerous alternatives: a rate on financial borrowings, on savings, and a weighted average of the two; a rate on nonfinancial investments. e.g., residential housing, perhaps adjusted for capital gains; and the consumer’s subjective rate of time preference. Furthermore, there is some controversy about whether it should be the maximum observed rate, the average observed rate, or the rate of return earned on investments that have the same degree of risk and liquidity as the durables whose services are being valued.”

³⁵ One way for choosing the nominal interest rate for period t , r^t , is to set it equal to $(1+r^*)(1+p^t) - 1$ where p^t is a consumer price inflation rate for period t and r^* is a reference real interest rate. The Australian Bureau of Statistics has used this method for determining r^t with $r^* \equiv 0.04$; i.e., a 4 percent real interest rate was chosen. Other methods for determining the appropriate interest rate that should be inserted into user cost formula are discussed by Harper, Berndt and Wood (1989), Schreyer (2001) and Hill, Steurer and Waltl (2020).

³⁶ We will discuss geometric or declining balance depreciation and one hoss shay depreciation below. For references to the depreciation literature and for empirical methods for estimating depreciation rates, see Hulten and Wykoff (1981a) (1981b) (1996), Beidelman (1973) (1976), Jorgenson (1996) and Diewert and Lawrence (2000).

³⁷ Goodhart (2001; F351) commented on the practical difficulties of using ex post user costs for housing as follows: “An even more theoretical user cost approach is to measure the cost foregone by living in an owner occupied property as compared with selling it at the beginning of the period and repurchasing it at the end ... But this gives the absurd result that as house prices rise, so the opportunity cost falls; indeed the more virulent the inflation of housing asset prices, the more negative would this measure become. Although it has some academic aficionados, this flies in the face of common sense; I am glad to say that no country has adopted this method.” As noted above, Iceland and Eurostat have in fact adopted a simplified user cost framework which seems to work well enough. Moreover, the user cost concept is used widely in production function and productivity studies and by national statisticians who construct multifactor productivity accounts for their countries.

different assumptions and use different methods in order to construct anticipated inflation rates for structures and land and hence the resulting ex ante user costs of the durable may not be comparable across countries.³⁸

- The user cost formula (8) should be generalized to accommodate various taxes that may be associated with the purchase of a durable or with the continuing use of the durable.³⁹

Some of the problems associated with estimating depreciation rates will be discussed in section 6 below.

5. The Opportunity Cost Approach

The opportunity cost approach to the valuation of the services of a consumer durable during a time period is very easy to describe: the opportunity cost valuation is simply the *maximum* of the foregone rental or leasing price for the services of the durable during a period of time and the corresponding user cost for the durable.

It is easy to see that when a household has a consumer durable in its possession, the household forgoes the money that one could earn by renting out the services of the durable good for the period of time under consideration. (Such rental markets may not exist, in which case, this opportunity cost is 0). Thus the rental equivalent (at current market rates) is one opportunity cost that the household incurs by continuing to own and use the services of the durable during the period.

However, there is another opportunity cost that is applicable to using the services of the durable good during the period under consideration. By using the services of the durable good, the household also forgoes a *financial opportunity cost*. Thus the durable good could be sold on the secondhand market at the beginning of the period at the price P^0 . This amount of money could be invested in some financial instrument that earns the one period rate of return of r^0 . Thus at the end of the period, the household would have accumulated $P^0(1+r^0)$ dollars as a result of selling the consumer durable at the beginning of the period. Now suppose at the end of the period, the household buys back the consumer durable that it sold at the beginning of the period. The value of the durable good at the end of the period will be $(1+i^0)(1-\delta^0)P^0$ where i^0 is the asset appreciation rate over period 0 and δ^0 is the depreciation rate for the durable good. Thus the net opportunity cost of using the services of the durable for period 0 from the financial perspective is

³⁸ For additional material on the difficulties involved in constructing ex ante user costs, see Diewert (1980; 475-486) and Katz (1983; 419-420). For empirical comparisons of different user cost formulae, see Harper, Berndt and Wood (1989), Diewert and Lawrence (2000) and Hill, Steurer and Wautl (2020). In Diewert and Fox (2018), the authors calculated Jorgensonian (ex post) user costs for US land used in residential housing for the years 1960-2014 and found that negative user costs occurred. Diewert and Fox then replaced the ex post capital gains term in the user cost for land with the long term inflation rate for land over the previous rolling window of 25 years (as an approximation to the ex ante or expected asset inflation rate) and this substitution led to positive user costs for land that were relatively smooth. Hill, Steurer and Wautl (2020) also recommend the use of long run asset inflation rates to avoid chain drift in housing indexes based on user costs.

³⁹ For example, property taxes are associated with the use of housing services and hence should be included in the user cost formula; see section 16 below. As Katz (1983; 418) noted, taxation issues also impact the choice of the interest rate: "Should the rate of return be a before or after tax rate?" From the viewpoint of a household that is not borrowing to finance the purchase of the durable, an after tax rate of return seems appropriate but from the point of a leasing firm, a before tax rate of return seems appropriate. This difference helps to explain why rental equivalence prices for the durable might be higher than user cost prices; see also section 16 below.

$P^0(1+r^0) - (1+i^0)(1-\delta^0)P^0$ which is exactly the *end of period user cost* for the durable good that was derived earlier; see equation (8) above.

A true opportunity cost for using the services of a durable good should equal the *maximum* of the benefits that are foregone by not using these services. Thus the opportunity cost approach to pricing the services of a consumer durable is equivalent to taking the maximum of the rent and user cost that the durable could generate over the period under consideration.⁴⁰

6. A General Model of Depreciation for Consumer Durables

In this section, a “general” model of depreciation for durable goods that appear on the market each period without undergoing quality change will be presented. In the following two sections, this general model will be specialized to the three most common models of depreciation that appear in the literature.

The main tool that can be used to identify depreciation rates for a durable good is the *cross sectional sequence of asset prices classified by their age* that units of the good sell for on the secondhand market at any point of time.⁴¹ Thus in order to apply this method for the measurement of depreciation, it is necessary that such secondhand markets exist.

Some notation is required. Let P_0^t be the price of a newly produced unit of the durable good at the *beginning* of period t . Let P_v^t be the secondhand market price at the beginning of period t of a unit of the durable good that is v periods old.⁴² The *beginning of period t cross sectional depreciation rate* for a brand new unit of the durable good, δ_0^t , is defined as follows:

$$(10) \quad 1 - \delta_0^t \equiv P_1^t/P_0^t .$$

Once δ_0^t has been defined by (10), the *period t cross sectional depreciation rate* for a unit of the durable good that is one period old at the beginning of period t , δ_1^t , can be defined using the following equation:

$$(11) \quad (1 - \delta_1^t)(1 - \delta_0^t) \equiv P_2^t/P_0^t .$$

Note that P_2^t is the beginning of period t asset price of a unit of the durable good that is 2 periods old and it is compared to the price of a brand new unit of the durable, P_0^t .

⁴⁰ The opportunity cost approach to pricing the services of Owner Occupied Housing was first proposed by Diewert (2008). It was further developed by Diewert and Nakamura (2011) and Diewert, Nakamura and Nakamura (2011). There have been at least two studies that implemented the opportunity cost approach to the valuation of the services of OOH; see Shimizu, Diewert, Nishimura and Watanabe (2012) and Aten (2018).

⁴¹ Another information source that could be used to identify depreciation rates for the durable good is the sequence of vintage rental or leasing prices that might exist for some consumer durables. In the closely related capital measurement literature, the general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten (1990; 127-129) (1996; 152-160).

⁴² If these secondhand vintage prices depend on how intensively the durable good has been used in previous periods, then it will be necessary to further classify the durable good not only by its vintage v but also according to the intensity of its use. In this case, think of the sequence of vintage asset prices P_v^t as corresponding to the prevailing market prices of the various vintages of the good at the beginning of period t for assets that have been used at “average” intensities.

Given that the period t cross sectional depreciation rates for units of the durable that are $0, 1, 2, \dots, v-1$ periods old at the beginning of period 0 are defined (these are the depreciation rates $\delta_0^t, \delta_1^t, \delta_2^t, \dots, \delta_{v-1}^t$), then the *period t cross sectional depreciation rate for units of the durable that are v periods old* at the beginning of period t , δ_v^t , can be defined using the following equation:

$$(12) (1 - \delta_v^t)(1 - \delta_{v-1}^t) \dots (1 - \delta_1^t)(1 - \delta_0^t) \equiv P_{v+1}^t/P_0^t.$$

Thus it is clear how the sequence of *period 0 vintage asset prices* P_v^t can be converted into a sequence of *period t vintage depreciation rates*, δ_v^t . In the depreciation literature, it is usually assumed that the sequence of vintage depreciation rates, δ_v^t , is independent of the period t so that:

$$(13) \delta_v^t = \delta_v \quad \text{for all periods } t \text{ and all ages } v.$$

The above material shows how the sequence of vintage or used durable goods prices at a point in time can be used in order to estimate depreciation rates. This method for estimating depreciation rates using data on secondhand assets, with a few extra modifications to account for differing ages of retirement, was pioneered by Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) (1996).⁴³

Recall the user cost formula for a new unit of the durable good under consideration which was defined by (1) above. The same approach can be used in order to define a sequence of period 0 user costs for all vintages v of the durable. Thus suppose that P_{v+1}^{1a} is the *anticipated end of period 0 price* of a unit of the durable good that is v periods old at the beginning of period 0 and let r^0 be the consumer's opportunity cost of capital for period 0. Then the discounted to the beginning of period 0 *user cost* of a unit of the durable good that is v periods old at the beginning of period 0, u_v^0 , is defined as follows:

$$(14) u_v^0 \equiv P_v^0 - P_{v+1}^{1a}/(1 + r^0); \quad v = 0, 1, 2, \dots$$

It is now necessary to specify how the *end* of period 0 anticipated vintage asset prices P_v^{1a} are related to their counterpart *beginning* of period 0 vintage asset prices P_v^0 . The assumption that is made now is that the entire sequence of vintage asset prices at the end of period 0 is equal to the corresponding sequence of asset prices at the beginning of period 0 times a general anticipated period 0 inflation rate factor, $(1+i^0)$, where i^0 is the *anticipated period 0 (general) asset inflation rate*. Thus it is assumed that⁴⁴

$$(15) P_v^{1a} = (1 + i^0)P_v^0; \quad v = 0, 1, 2, \dots$$

Substituting (15) and (10)-(13) into (14) leads to the following beginning of *period 0 sequence of vintage user costs*:⁴⁵

⁴³ See also Jorgenson (1996) for a review of the empirical literature on the estimation of depreciation rates.

⁴⁴ More generally, we assume that assumptions (15) hold for subsequent periods t as well; i.e., it is assumed that $P_v^{t+1a} = (1 + i^t)P_v^t$ for $v = 0, 1, 2, \dots$ and $t = 0, 1, 2, \dots$ where P_v^{t+1a} is the anticipated price of a unit of the durable good that is v periods old at the end of period t , i^t is a period t expected asset inflation rate for all ages of the durable and P_v^t is the secondhand market price for a unit of the durable good that is v periods old at the beginning of period t .

⁴⁵ When $v = 0$, define $\delta_{-1} \equiv 1$; i.e., the terms in front of the square brackets on the right hand side of (16) are set equal to 1.

$$(16) u_v^0 = (1 - \delta_{v-1})(1 - \delta_{v-2}) \dots (1 - \delta_0)[(1 + r^0) - (1 - \delta_v)(1 + i^0)]P_0^0/(1 + r^0) \\ = (1 - \delta_{v-1})(1 - \delta_{v-2}) \dots (1 - \delta_0)[r^0 - i^0 + \delta_v(1 + i^0)]P_0^0/(1 + r^0); \quad v = 1, 2, \dots$$

If $v = 0$, then $u_0^0 \equiv [r^0 - i^0 + \delta_0(1 + i^0)]P_0^0/(1 + r^0)$ and this agrees with the user cost formula for a new purchase of the durable u^0 that was derived earlier in (7) (with our changes in notation; i.e., P^0 is now called P_0^0).

The sequence of vintage user costs u_v^0 defined by (16) is expressed in terms of prices that are discounted to the *beginning* of period 0. However, as was done in section 4 above, it is also possible to express the user costs in terms of prices that are “anti-discounted” to the *end* of period 0. Thus define the sequence of vintage *end of period 0 user cost* p_v^0 as follows:

$$(17) p_v^0 \equiv (1+r^0)u_v^0 = (1-\delta_{v-1})(1-\delta_{v-2}) \dots (1-\delta_0)[r^0 - i^0 + \delta_v(1 + i^0)]P_0^0; \quad v = 1, 2, \dots$$

with p_0^0 defined as follows:

$$(18) p_0^0 \equiv (1+r^0)u_0^0 = [r^0 - i^0 + \delta_0(1 + i^0)]P_0^0.$$

Thus if the price statistician has estimates for the vintage depreciation rates δ_v , the nominal interest rate r^0 , the expected asset inflation rate and is also able to collect a sample of prices for new units of the durable good P_0^0 , then the sequence of vintage user costs defined by (17) can be calculated. To complete the model, the price statistician should gather information on the stocks held by the household sector of each vintage of the durable good and then normal index number theory can be applied to these p 's and q 's, with the p 's being vintage user costs and the q 's being the vintage stocks pertaining to each period. For some worked examples of this methodology under various assumptions about depreciation rates and the calculation of expected asset inflation rates, see Diewert and Lawrence (2000) and Diewert (2005a).⁴⁶

In the following two sections, the general methodology described above is specialized by making additional assumptions about the form of the vintage depreciation rates δ_v .⁴⁷

7. Geometric or Declining Balance Depreciation

The *declining balance method of depreciation* dates back to Matheson (1910; 55) at least.⁴⁸ In terms of the algebra presented in the previous section, the method is very simple: all of the cross sectional vintage depreciation rates δ_v^t defined by (10)-(12) are assumed to be equal to the same rate δ , where δ is a positive number less than one; i.e., for all time periods t and all vintages v , it is assumed that

$$(19) \delta_v^t = \delta; \quad v = 0, 1, 2, \dots$$

⁴⁶ Additional examples and discussion can be found in two OECD Manuals on productivity measurement and the measurement of capital; see Schreyer (2001) (2009).

⁴⁷ In the case of one hoss shay depreciation, assumptions are made about the sequence of user costs, u_v^t , as the asset age v increases.

⁴⁸ A case for attributing the method to Walras (1954; 268-269) could be made but he did not lay out all of the details. Matheson (1910; 91) used the term “diminishing value” to describe the method. Hotelling (1925; 350) used the term “the reducing balance method” while Canning (1929; 276) used the term the “declining balance formula”. For a more recent exposition of the geometric model of depreciation, see Jorgenson (1989).

Substitution of (19) into (17) leads to the following formula for the sequence of *end of period 0 vintage user costs*:

$$(20) \quad p_v^0 = (1 - \delta)^v [r^0 - i^0 + \delta(1 + i^0)] P^0; \quad v = 0, 1, 2, \dots \\ = (1 - \delta)^v p_0^0$$

where the second equation follows using definition (18). The second set of equations in (20) says *that all of the vintage user costs are proportional to the user cost for a new asset*. This proportionality means that it is not necessary to use an index number formula to aggregate over vintages to form a durable services aggregate. To see this, it is useful to calculate the aggregate value of services yielded by all vintages of the consumer durable at the beginning of period 0. Let q^{-v} be the quantity of the new durable purchased by the household sector v periods ago for $v = 1, 2, \dots$ and let q^0 be the new purchases of the durable during period 0. The beginning of period 0 user cost for the holdings of the durable of age v will be p_v^0 defined by (20) above. Thus the aggregate value of services over all vintages of the good, including those purchased in period 0, will have the value v^0 defined as follows:

$$(21) \quad v^0 = p_0^0 q^0 + p_1^0 q^{-1} + p_2^0 q^{-2} + \dots \\ = p_0^0 q^0 + (1 - \delta) p_0^0 q^{-1} + (1 - \delta)^2 p_0^0 q^{-2} + \dots \quad \text{using (20)} \\ = p_0^0 [q^0 + (1 - \delta)q^{-1} + (1 - \delta)^2 q^{-2} + \dots] \\ = p_0^0 Q^0$$

where the period 0 aggregate (quality adjusted) quantity of durable services consumed in period 0, Q^0 , is defined as

$$(22) \quad Q^0 \equiv q^0 + (1 - \delta)q^{-1} + (1 - \delta)^2 q^{-2} + \dots$$

Thus the *period 0 services quantity aggregate* Q^0 is equal to new purchases of the durable in period 0, q^0 , plus one minus the depreciation rate δ times the purchases of the durable in the previous period, q^{-1} , plus the square of one minus the depreciation rate times the purchases of the durable two periods ago, q^{-2} , and so on. The service price that can be applied to this quantity aggregate is p_0^0 , the imputed rental price or user cost for a new unit of the durable purchased in period 0.

The above result greatly simplifies the valuation of consumer durables. Normally, the price statistician would have to keep track of all new purchases of the durable good by the reference population by period, calculate the relevant user costs p_v^0 and p_v^t for periods 0 and t , and apply the relevant index number formula (Laspeyres, Paasche, Fisher or whatever formula is being used in the CPI) to these age specific prices and quantities for periods 0 and t . But because under assumptions (13), (15) and (19), *all vintage user costs vary in a proportional manner over time*,⁴⁹ and thus any reasonable index number formula will find that the price index going from period 0 to t is equal to p_0^t/p_0^0 , the ratio of user costs for a new unit of the durable good. Moreover the

⁴⁹ Equations (20) for period t are the following ones: $p_v^t = (1 - \delta)^v p_0^t$ for $v = 1, 2, \dots$ and so the entire sequence of user costs by age of asset vary in a proportional manner over time under our assumptions. Thus an aggregate period t price for the entire group of assets of varying ages is p_0^t and the corresponding aggregate quantity will be Q^t defined by (23). This is an application of Hicks' (1946; 312-313) Aggregation Theorem: "Thus we have demonstrated mathematically the very important principle, used extensively in the text, that if the prices of a group of goods change in the same proportion, that group of goods behaves just as if it were a single commodity."

corresponding *aggregate quantity index* will be equal to Q^t/Q^0 , where Q^0 is defined by (22) and Q^t is defined by

$$(23) \quad Q^t \equiv q^t + (1 - \delta)q^{t-1} + (1 - \delta)^2q^{t-2} + \dots \\ = q^t + (1 - \delta)Q^{t-1}.$$

Note that the second equation simplifies the calculation of the period t aggregate service flow (in real terms) over all vintages of the consumer durable: the period t aggregate flow, Q^t , is equal to period t new purchases of the durable, q^t , plus $(1-\delta)$ times the aggregate flow of services in the previous period, Q^{t-1} .

If the depreciation rate δ and the purchases of the durable in prior periods are known, then the aggregate service quantity Q^0 can readily be calculated using (22). Then using (21), it can be seen that the period 0 value of the services of the durable (over all vintages), v^0 , decomposes into the price term p_0^0 times the quantity term Q^0 . Hence, it is not necessary to use an index number formula to aggregate over vintages using this depreciation model.

The stock of consumer durables held by the household sector of a country should appear in the balance sheets of the country.⁵⁰ Using the geometric model of depreciation, it is very easy to calculate the nominal and real value of the stock of consumer durables held by households. At time t , the stocks held by the household sector for the particular type of consumer durable under consideration are $q^t, q^{t-1}, q^{t-2}, \dots$ and the corresponding asset prices by age of asset are $P_0^t, P_1^t, P_2^t, \dots$. Assumptions (12), (13) and (19) imply that these period t asset prices satisfy the following equations:

$$(24) \quad P_v^t = (1-\delta)^v P_0^t; \quad v = 1, 2, \dots$$

Equations (24) can be used to define period t *aggregate asset value* for the stocks held by households for the durable good over all ages of the durable good, V^t :

$$(25) \quad V^t \equiv P_0^t q^t + P_1^t q^{t-1} + P_2^t q^{t-2} + P_3^t q^{t-3} + \dots \\ = P_0^t [q^t + (1-\delta)q^{t-1} + (1-\delta)^2 q^{t-2} + \dots] \quad \text{using (24)} \\ = P_0^t Q^t$$

where Q^t is defined by (23). Thus Q^t serves as a measure of the real capital stock of the consumer durable at the end of period t *and* it also serves as a measure of the real consumption services provided by this capital stock during period t .

The above algebra explains why the geometric model of depreciation is used so widely in production function studies and in the measurement of Total Factor Productivity or Multifactor Productivity in the production accounts of countries: it is very simple to work with!⁵¹

8. Alternative Depreciation Models

⁵⁰ However, for many countries, stocks of consumer durables will not be present in the country's balance sheets and so it will be necessary to use historical data on the purchases of durables along with estimated depreciation rates in order to form estimated stocks for consumer durables.

⁵¹ See Jorgenson (1989) who popularized the use of the geometric model of depreciation in production function and Total Factor Productivity studies. For an application of his methodology to valuing the services of consumer durables in the US, see Christensen and Jorgenson (1969).

Another very common model of depreciation is the *straight line model*.⁵² In this model, the most probable length of life for the durable is somehow determined, say L periods, so that after being used for L periods, the durable is scrapped. In the straight line depreciation model, it is assumed that the period 0 cross sectional vintage asset prices P_v^0 decline in a linear fashion relative to the period 0 price of a new asset P_0^0 :

$$(26) P_v^0/P_0^0 = [L - v]/L \quad \text{for } v = 0, 1, 2, \dots, L-1.$$

For $v = L, L+1, \dots$, it is assumed that $P_v^0 = 0$. Now use definitions (14) and (17) along with assumptions (15) in order to obtain the following sequence of *end of period 0 vintage user costs* for a unit of the durable good of age v at the beginning of period 0:

$$(27) p_v^0 = P_v^0(1 + r^0) - (1 + i^0)P_{v+1}^0 \quad \text{for } v = 0, 1, 2, \dots, L-1$$

$$= [1/L][(L - v)(1+r^0) - (L - v - 1)(1+i^0)]P_0^0 \quad \text{using assumptions (26)}$$

$$= [(r^0 - i^0)(L - v)L^{-1} + (1+i^0)L^{-1}]P_0^0.$$

The user costs for units of the durable good that are older than L periods are zero; i.e., $p_v^0 \equiv 0$ for $v \geq L$. Looking at the terms in square brackets on the right hand side of (27), it can be seen that the first term $(r^0 - i^0)(L - v)L^{-1}P_0^0$ is a real interest opportunity cost for holding and using the unit of the durable that is v periods old (and this imputed real interest cost declines as the durable good ages; i.e., as the age v increases) and the second term $(1+i^0)(1/L)P_0^0$ is an inflation adjusted depreciation term that is equal to the constant straight line depreciation rate $1/L$ times the adjustment factor for asset inflation over the period, $(1+i^0)$, times the price of a new unit of the durable good P_0^0 . In period t , the corresponding end of period user cost for a unit of the durable good that is v periods old is defined as $p_v^t \equiv [(r^t - i^t)(L - v)L^{-1} + (1+i^t)L^{-1}]P_0^t$ for $v = 0, 1, 2, \dots, L-1$. Thus in both periods 0 and t , the sequences of end of period user costs by age, $\{p_v^0\}$ and $\{p_v^t\}$ for $v = 0, 1, 2, \dots, L-1$, are proportional to the price of a new unit of the durable for periods 0 and t , P_0^0 and P_0^t respectively⁵³ but if r^0 and/or i^0 change to a different r^t or i^t , then the factors of proportionality will change as we go from period 0 to t and so we cannot apply Hicks' Aggregation Theorem in this case.

In the case of changing nominal interest rates r and/or changing expected or actual asset price inflation rates, i^t , we cannot assume that the overall inflation rate between periods 0 and t for all ages of the durable good is equal to P_0^t/P_0^0 as was the case with the geometric model of depreciation. Thus for the straight line model of depreciation, it is necessary to keep track of household purchases of the durable for L periods and weight up each vintage quantity q^{-v} of these purchases by the corresponding end of period user costs vintage user cost p_v^0 defined by (27) for period 0 and a similar calculation will have to be made for period t . Once these vectors of prices and quantities have been calculated for both periods, then normal index number theory can be applied to get the overall price index for the household holdings of the durable good and this

⁵² This model of depreciation dates back to the late 1800's; see Matheson (1910; 55), Garcke and Fells (1893; 98) or Canning (1929; 265-266).

⁵³ Thus as the price of a new unit of the durable good changes over time, the value of depreciation will also change in line with the change in the price of the new unit. Thus economic depreciation as we have defined it is different from historical cost accounting depreciation which does not adjust depreciation allowances for changes in the levels of asset prices over time. Put another way, historical cost depreciation does not reflect current opportunity costs of using the services of consumer durable.

index can be used to deflate the user cost aggregate values to get an appropriate volume index.⁵⁴ It can be seen that the straight line model of depreciation is considerably more complicated to implement than the geometric model of depreciation explained in the previous section.⁵⁵

The final model of depreciation that is in common use is the “light bulb” or *one hoss shay model of depreciation*.⁵⁶ In this model, the durable delivers the *same* services for each vintage: a chair is a chair, no matter what its age is (until it falls to pieces and is scrapped). Thus this model also requires an estimate of the most probable life L of the consumer durable.⁵⁷ In this model, it is assumed that the sequence of vintage beginning of the period user costs u_v^0 defined by (14) and (15) is *constant* for all vintages younger than the asset lifetime L ; i.e., it is assumed that

$$(28) u_v^0 \equiv P_v^0 - (1 + i^0)P_{v+1}^0/(1 + r^0) = u^0; \quad v = 0, 1, 2, \dots, L-1$$

where $u^0 > 0$ is a constant. Equations (28) can be rewritten in the following form:

$$(29) u^0 = P_v^0 - \gamma P_{v+1}^0; \quad v = 0, 1, 2, \dots, L-1$$

where the *discount factor* γ is defined as

$$(30) \gamma \equiv (1 + i^0)/(1 + r^0) \equiv 1/(1 + r^{0*}).$$

The interest rate r^{0*} can be regarded as an *asset specific real interest rate*; i.e., $1+r^{0*} \equiv (1+r^0)/(1+i^0)$ so that one plus the nominal interest rate r^0 is deflated by one plus the expected asset price inflation rate, i^0 . Note that equations (29) can be rewritten as follows:

⁵⁴ Diewert and Lawrence (2000) noted this problem with the straight line model of depreciation; i.e., that in general, an *index number formula* should be used to aggregate over the different ages of the asset in order to obtain an aggregate of the capital services of the different vintages of the asset.

⁵⁵ However, if one is willing to assume that the reference interest rate for period t , r^t , and the expected asset inflation rate over all ages of the asset, i^t , both remain constant, then all reasonable index number formula will estimate the overall rate of user cost inflation between periods 0 and t as the new consumer good purchase price ratio, P_0^t/P_0^0 . However, the assumption that r^t and i^t remain constant over time is only a rough approximation to reality. Note that in order to calculate real and nominal consumption of the durable (over all ages of the durable), it will be necessary to use the vintage user costs defined by (27) for a constant r and i to weight up past purchases of the durable good. Thus define the constants $\alpha_v \equiv [(r - i)(L - v)L^{-1} + (1+i)L^{-1}]$ for $v = 0, 1, 2, \dots, L-1$ and $\alpha_v \equiv 0$ for $v \geq L$. Then the period t *nominal value of durable services* is defined as $v^t \equiv p_0^t q^t + p_1^t q^{t-1} + p_2^t q^{t-2} + \dots + p_{L-1}^t q^{t-L+1} = \alpha_0 P_0^t q^t + \alpha_1 P_0^t q^{t-1} + \alpha_2 P_0^t q^{t-2} + \dots + \alpha_{L-1} P_0^t q^{t-L+1} = P_0^t Q^t$ where Q^t is the real value or volume of durable services defined as $Q^t \equiv \alpha_0 q^t + \alpha_1 q^{t-1} + \alpha_2 q^{t-2} + \dots + \alpha_{L-1} q^{t-L+1}$. Define $\beta_v \equiv (L - v)/L$ for $v = 0, 1, 2, \dots, L - 1$. The period t *asset value of consumer holdings of the durable good* is defined as $V^t \equiv P_0^t q^t + P_1^t q^{t-1} + P_2^t q^{t-2} + \dots + P_{L-1}^t q^{t-L+1} = P_0^t [\beta_0 q^t + \beta_1 q^{t-1} + \beta_2 q^{t-2} + \dots + \beta_{L-1} q^{t-L+1}] = P_0^t Q^{t*}$ where we have used assumptions (26) applied to period t and the real value of durable stocks held by households at the end of period t is defined as $Q^{t*} \equiv \beta_0 q^t + \beta_1 q^{t-1} + \beta_2 q^{t-2} + \dots + \beta_{L-1} q^{t-L+1}$. The decomposition of V^t into $P_0^t Q^{t*}$ does not require the assumption of constant r^t and i^t .

⁵⁶ This model can be traced back to Böhm-Bawerk (1891; 342). For a more comprehensive exposition, see Hulten (1990; 124) or Diewert (2005a).

⁵⁷ The assumption of a single life L for a durable can be relaxed using a methodology due to Hulten: “We have thus far taken the date of retirement T to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate T assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort’s useful life T_i .” Charles R. Hulten (1990; 125). For more details on how this methodology works, see Schreyer (2009).

$$(31) P_v^0 = u^0 + \gamma P_{v+1}^0; \quad v = 0, 1, 2, \dots, L-1.$$

Use equation (31) with $v = 0$ to express P_0^0 in terms of u^0 and P_1^0 . Now use (31) with $v = 1$ to express P_2^0 in terms of u^0 and P_1^0 and then substitute out P_1^0 using the previous expression that expressed P_1^0 in terms of P_0^0 and u^0 . Continue this substitution process until finally it ends after L such substitutions when P_L^0 is reached and of course, P_L^0 equals zero. The following equation is obtained:

$$(32) P_0^0 = u^0 + \gamma u^0 + \gamma^2 u^0 + \dots + \gamma^{L-1} u^0 \\ = u^0 [1 + \gamma + \gamma^2 + \dots + \gamma^{L-1}] \\ = \{u^0/(1 - \gamma)\} - \{u^0 \gamma^L/(1 - \gamma)\} \quad \text{provided that } \gamma < 1 \text{ }^{58} \\ = u^0 (1 - \gamma^L)/(1 - \gamma).$$

Now use the last equation in (32) in order to solve for the constant over vintages (beginning of the period) *user cost* for this model, u^0 , in terms of the period 0 price for a new unit of the durable, P_0^0 , and the discount factor γ defined by (31):

$$(33) u^0 = (1 - \gamma)P_0^0/(1 - \gamma^L) = u_v^0; \quad v = 0, 1, 2, \dots, L-1.$$

The sequence of *end of period 0 user cost*, p_v^0 , is as usual, equal to the corresponding beginning of the period 0 user cost, u_v^0 , times the period 0 nominal interest rate factor, $1+r^0$:

$$(34) p_v^0 \equiv (1 + r^0)u_v^0 = [1+r^0][1-\gamma^0][1-(\gamma^0)^L]^{-1}P_0^0 = p_0^0; \quad v = 0, 1, 2, \dots, L-1$$

and $p_v^0 = 0$ for $v = L, L+1, \dots$ and $\gamma^0 \equiv (1+i^0)/(1+r^0)$.

The *aggregate services of all vintages* of the good for period 0, including those purchased in period 0, will have the following value, v^0 :

$$(35) v^0 = p_0^0 q^0 + p_1^0 q^{-1} + p_2^0 q^{-2} + \dots + p_{L-1}^0 q^{-(L-1)} \\ = p_0^0 [q^0 + q^{-1} + q^{-2} + \dots + q^{-(L-1)}] \\ = p_0^0 Q^0$$

where the *period 0 aggregate (quality adjusted) quantity of durable services* consumed in period 0, Q^0 , is defined as follows for this depreciation model:

$$(36) Q^0 \equiv q^0 + q^{-1} + q^{-2} + \dots + q^{-(L-1)}.$$

Thus in this model of depreciation, the service quantity aggregate is the simple sum of household purchases over the last L periods.⁵⁹ As was the case with the geometric depreciation model, the one hoss shay model does not require index number aggregation over vintages when calculating aggregate services from all vintages of the durable: there is a constant service price p_0^0 for all assets that are less than L periods old and the associated period 0 quantity Q^0 is the simple sum defined by (36) over the purchases of the last L periods.⁶⁰

⁵⁸ If $\gamma \geq 1$, then use the second equation in (32) to express u^0 in terms of P_0^0 and the various powers of γ .

⁵⁹ In the national income accounting literature, this measure is sometimes called the gross capital stock.

⁶⁰ Using equations (31), it can be shown that $P_v^0 = u^0[1 + (\gamma^0) + (\gamma^0)^2 + \dots + (\gamma^0)^{L-1-v}]$ for $v = 0, 1, 2, \dots, L-1$ where $\gamma^0 \equiv (1+i^0)/(1+r^0)$ and $P_v^0 = 0$ for $v \geq L$. Thus the period 0 value of the stock of consumer durables is

The first two models of depreciation considered earlier (the geometric and straight line models) made assumptions about the pattern of depreciation rates for durables of different ages. The light bulb model made assumptions about the pattern of user costs for a durable good by its age. For a more general model of depreciation that allows for an arbitrary pattern of user costs by age of asset, see Diewert and Wei (2017).

How can the different models of depreciation be distinguished empirically? For durable goods that do not change in quality over time, there are *three possible methods* for determining the sequence of vintage depreciation rates:⁶¹

- By making a rough estimate of the average length of life L for the durable good and then by *assuming* a depreciation model that seems most appropriate.⁶²
- By using cross sectional information on the sales of used durable prices at a single point in time and then using equations (10)-(12) above to determine the corresponding sequence of vintage depreciation rates.⁶³
- By using cross sectional information on the rental or leasing prices of the durable as a function of the age of the durable and then equations (17) and (18), along with information on the appropriate nominal interest rate r^0 and expected durables inflation rate i^0 along with information on the price of a new unit of the durable good P^0 , can be used to determine the corresponding sequence of vintage depreciation rates.

Which one of the three models of depreciation presented in this chapter should be used in empirical applications? It is not possible to give a universally valid answer to this question but it is worth mentioning that the geometric model of depreciation is probably the most useful at the macro level. A problem with the models of depreciation considered in this section is that they assume that all assets in the asset class under consideration are retired at the same age. In real life, this is not the case. Thus Hulten and Wykoff (1981a) and Schreyer (2009) generalized these models to allow for the assets to be retired at different ages and they showed under these conditions, aggregate depreciation followed the geometric model to a reasonably high degree of approximation. The resulting geometric depreciation rates reflect the sum of wear and tear depreciation of unretired assets plus the average amount of additional depreciation that is due to premature retirement of the assets.

9. The Relationship Between User Costs and Acquisition Costs

In this section, the user cost approach to the treatment of consumer durables will be compared to the acquisitions approach. Obviously, in the short run, the value flows associated with each approach could be very different. For example, if real interest rates, $r^0 - i^0$, are very high and the economy is in a severe recession or depression, then purchases of new consumer durables, q^0 say,

$\sum_{v=0}^{L-1} P_v^0 q^{-v}$. The corresponding asset prices for period t are equal to $P_v^t = u^t [1 + (\gamma^t) + (\gamma^t)^2 + \dots + (\gamma^t)^{L-1-v}]$ for $v = 0, 1, 2, \dots, L-1$ where $u^t \equiv [1 - (\gamma^t)] P_0^t / [1 - (\gamma^t)^L]$, $\gamma^t \equiv (1+i^t)/(1+r^t)$ and $P_v^t = 0$ for $v \geq L$. The period t value of the stock of consumer durables is $\sum_{v=0}^{L-1} P_v^t q^{t-v}$. An index number formula will have to be used to form aggregate price and quantity indexes for the *stocks* of consumer durables using the one hoss shay model of depreciation.

⁶¹ These three classes of methods were noted in Malpezzi, Ozanne and Thibodeau (1987; 373-375) in the housing context.

⁶² A length of life L can be converted into an equivalent geometric depreciation rate δ by setting δ equal to a number between $1/L$ and $2/L$.

⁶³ This method will be pursued in sections 11-15 for housing depreciation rates.

could be very low and even approach 0 for very long lived assets, like houses. On the other hand, using the user cost approach, existing stocks of consumer durables would be carried over from previous periods and priced out at the appropriate user costs and the resulting consumption value flow could be quite large. Thus in the short run, the monetary values of consumption under the two approaches could be vastly different. Hence, in what follows, a (hypothetical) longer run comparison is considered where real interest rates are held constant.⁶⁴

Suppose that in period 0, the reference population of households purchased q^0 units of a consumer durable at the purchase price P^0 . Then *the period 0 value of consumption from the viewpoint of the acquisitions approach* is:

$$(37) V_A^0 \equiv P^0 q^0 .$$

Recall that the end of period user cost for one new unit of the asset purchased at the beginning of period 0 was p^0 defined by (8) above. In order to simplify the analysis, the *geometric model of depreciation* is assumed; i.e., at the beginning of period 0, a one period old asset is worth $(1-\delta)P^0$, a two period old asset is worth $(1-\delta)^2 P^0$, ... , a t period old asset is worth $(1-\delta)^t P^0$, etc. Under these hypotheses, the corresponding end of period 0 user cost for a new asset purchased at the beginning of period 0 is p^0 ; the end of period 0 user cost for a one period old asset at the beginning of period 0 is $(1-\delta)p^0$; the corresponding user cost for a two period old asset at the beginning of period 0 is $(1-\delta)^2 p^0$; ... ; the corresponding user cost for a t period old asset at the beginning of period 0 is $(1-\delta)^t p^0$; etc. The final simplifying assumption is that household purchases of the consumer durable have been growing at the geometric rate g into the indefinite past. This means that if household purchases of the durable were q^0 in period 0, then in the previous period they purchased $q^0/(1+g)$ new units, two periods ago, they purchased $q^0/(1+g)^2$ new units, ... , t periods ago, they purchased $q^0/(1+g)^t$ new units, etc. Putting all of these assumptions together, it can be seen that *the period 0 value of consumption services from the viewpoint of the user cost approach* is:

$$(38) V_U^0 \equiv p^0 q^0 + [(1-\delta)p^0 q^0/(1+g)] + [(1-\delta)^2 p^0 q^0/(1+g)^2] + \dots$$

$$= (1+g)p^0 q^0/(g+\delta) \quad \text{summing the infinite series}$$

$$= (1+g)[(1+r^0) - (1-\delta)(1+i^0)]P^0 q^0/(g+\delta) \quad \text{using (8).}$$

Equation (38) can be simplified by letting the asset inflation rate i^0 be 0 (or by replacing $r^0 - i^0$ by the real interest rate r^{0*} and by ignoring the small term δi^0) and under these conditions, the ratio of the user cost flow of consumption (38) to the acquisitions measure of consumption in period 0, (37) is:

$$(39) V_U^0/V_A^0 = (1+g)(r^{0*} + \delta)/(g+\delta) .$$

Using formula (39), it can be seen that if $1+g > 0$ and $\delta + g > 0$, then V_U^0/V_A^0 will be greater than unity if $r^{0*} > g(1-\delta)/(1+g)$, a condition that will usually be satisfied. Thus under normal conditions and over a longer time horizon, *household expenditures on consumer durables using the user cost approach will tend to exceed the corresponding expenditures on new purchases of the consumer durable*. Since the value of consumption services using the rental equivalence approach will tend to approximate the value of consumption services using the user cost approach, it can be seen that the acquisitions approach to household expenditures will tend to understate the value of consumption services estimated by the user cost and rental equivalence

⁶⁴ The following material is based on Diewert (2002).

approaches. The difference between the user cost and acquisitions approach will tend to grow as the depreciation rate δ decreases.

To get a rough idea of the possible magnitude of the value ratio for the two approaches, V_U^0/V_A^0 , equation (39) is evaluated for a “housing” example using annual data where the depreciation rate is 2% (i.e., $\delta = .02$), the real interest rate is 3% (i.e., $r^{0*} = .03$) and the growth rate for the production of new houses is 1% (i.e., $g = .01$). In this base case, the ratio of user cost expenditures on housing to the purchases of new housing in the same period, V_U^0/V_A^0 , is 1.68. If the depreciation rate is *decreased* to 1%, then V_U^0/V_A^0 *increases* to 2.02. If the real interest rate is *decreased* to 2% (with $\delta = .02$ and $g = .01$), then V_U^0/V_A^0 *decreases* to 1.35 while if the real interest rate is *increased* to 4%, then V_U^0/V_A^0 *increases* to 2.02. Thus an acquisitions approach to housing in the CPI is likely to give a substantially smaller weight to housing services than a user cost approach would give.

However, for shorter lived consumer durables like clothing, the difference between the acquisitions approach and the user cost approach will not be so large and hence, the acquisitions approach can be justified as being approximately “correct” as a measure of consumption services for these high depreciation rate durable goods.⁶⁵

For longer lived durables such as houses, automobiles and household furnishings, it would be useful for a national statistical agency to produce user costs for these goods and for the national accounts division to produce the corresponding consumption flows as “analytic series”. This would extend the present national accounts treatment of housing to other long lived consumer durables. Note also that this revised treatment of consumption in the national accounts would tend to make rich countries richer, since poorer countries hold fewer long lived consumer durables on a per capita basis.

10. User Costs for Storable Goods

A *storable good* is similar to a durable good in that it can be purchased in one period and then consumed in a subsequent period. However, the services of a durable good can be utilized in multiple periods, whereas a storable good (such as a can of beans) can only be consumed in a single period. Stocks of storable goods that are held at the beginning of an accounting period tie up financial capital in a manner that is similar to the holdings of durable goods at the beginning of the period. Thus the implicit (or explicit) interest cost of inventories of storable goods should be recognized in the household accounts. Furthermore, stocks of storable goods should be included in the balance sheets or wealth accounts of households.⁶⁶

The *user cost* for a unit of a storable good held at the beginning of an accounting period can be formed using the same methodology that was used in section 4 where the user cost of a durable good was set equal to its purchase cost less the discounted value of its price at the end of the accounting period.

⁶⁵ Let $r^{0*} = .03$, $g = .01$ and $\delta = .2$. Under these assumptions, using (39), we find that $V_U^0/V_A^0 = 1.11$; i.e., using a geometric depreciation rate of 20%, the user cost approach leads to an estimated value of consumption that is 11% higher than the acquisitions approach under the conditions specified. Thus the acquisitions approach for consumer durables with high depreciation rates is probably satisfactory.

⁶⁶ The response of households to the lockdown restrictions prevailing at the time of writing has been to dramatically increase inventories of storable goods. Health authorities have encouraged households to make fewer trips to retail outlets and this advice has led to increased inventories of storables.

Suppose that there are N storable goods that a household (or group of households) can purchase during an accounting period t . Denote the vector of period t purchases of storable goods by the household group in scope as $q^t \equiv [q_{t1}, \dots, q_{tN}] > 0_N$ and denote the corresponding period t (unit value) price vector by $p^t \equiv [p_{t1}, \dots, p_{tN}] \gg 0_N$ with $p^t \cdot q^t > 0$. However, the household group also holds some inventories of the N storable goods. Denote the vector of household inventory holdings of the N storable goods at the beginning of period t by $Q^t \equiv [Q_{t1}, \dots, Q_{tN}] \geq 0_N$.⁶⁷ Denote the corresponding vector of storable goods prices at the beginning of period t by $P^t \equiv [P_{t1}, \dots, P_{tN}] \gg 0_N$. Typically, these prices would be the market prices for the storable goods that prevail at the beginning of period t .⁶⁸ In any case, the beginning of period t value of inventories of storable goods is equal to $P^t \cdot Q^t = \sum_{n=1}^N P_{tn} Q_{tn}$.

The *period t user cost for storable good n* , U_{tn}^* , is defined as the cost of purchase of a unit of the good at the beginning of the accounting period less the discounted price of a similar unit sold at the end of the accounting period; i.e., U_{tn}^* , is defined as follows:

$$(40) U_{tn}^* \equiv P_{tn} - P_{t+1,n}/(1+r_t); \quad n = 1, \dots, N$$

where r_t is the beginning of period t household cost of financial capital for the group of households under consideration; i.e., r_t is an appropriate nominal interest rate.⁶⁹

The user cost defined by (40) is a beginning of the period t user cost; i.e., costs and benefits are discounted to the *beginning of period* t . If we anti-discount prices to the *end of period* t , the resulting user cost, U_{tn} , is defined as follows:

$$(41) U_{tn} \equiv (1+r_t)U_{tn}^* = (1+r_t)P_{tn} - P_{t+1,n} = r_t P_{tn} - (P_{t+1,n} - P_{tn}); \quad n = 1, \dots, N.$$

Thus the end of period user cost for holding a unit of the n th storable good during period t , U_{tn} , is the imputed or actual interest cost of tying up financial capital during the period, $r_t P_{tn}$, less the actual or imputed capital gain the household would make on selling the unit of the storable good at the end of the period.⁷⁰

Define the period t vector of user costs of storable products, U^t , as $[U_{t1}, \dots, U_{tN}]$. Using definitions (41), U^t is equal to the following vector:

$$(42) U^t = r_t P^t - (P^{t+1} - P^t); \quad t = 1, 2, \dots$$

⁶⁷ It should be noted that most countries do not have estimates for inventories of household storable items. For Japan, some information on food inventories is collected by the Lifescape Marketing Company. This information was used in a study on storable goods by Ueda, Watanabe and Watanabe (2020). This study has many references to the literature on the treatment of storable commodities in a CPI.

⁶⁸ If beginning of the period t prices for storable goods are not available, P^t could be approximated by $(\frac{1}{2})p^{t-1} + (\frac{1}{2})p^t$ or by p^{t-1} .

⁶⁹ As usual, it is difficult to determine this reference interest rate. If the household is borrowing money, then r_t is the appropriate borrowing rate or mortgage interest rate; if the household is loaning financial capital to others, then the appropriate interest rate is the expected rate of return on investments.

⁷⁰ The user costs U_{tn} are the counterparts to the end of period user cost for a durable good defined by (8) in section 4 above. If the depreciation rate δ in equation (8) is equal to 0, then the user costs defined by (41) are exactly the same as the user costs defined by (8) using different notation. The user costs of inventories defined by definitions (41) are frequently used to value the services of business inventories; e.g., see Christensen and Jorgenson (1969) and Diewert and Fox (2018). However, the valuation of the services of storable inventories in the household context has not been widespread.

Thus far, the treatment of inventories of storable products in the consumer context seems to be a straightforward extension of the earlier treatment of durable products in section 4. But the situation is a bit more complicated than the above algebra would indicate. When dealing with storable products in the consumer context, there is an extra set of equations that does not occur when dealing with inventory items in the producer context. The extra equations are the following ones:

$$(43) \quad q^t = c^t + [Q^{t+1} - Q^t] = c^t + \Delta Q^t \quad t = 1, 2, \dots$$

where $c^t \equiv [c_{t1}, \dots, c_{tN}] > 0_N$ is the period t vector of actual consumption of the N storable commodities and $\Delta Q^t \equiv Q^{t+1} - Q^t$ is the period t vector of change in inventories of storable goods. Equation (43) says that household period t purchases of storable commodities, q^t , equals household period t actual consumption of the commodities, c^t , plus the net change in inventories of the storable products, $Q^{t+1} - Q^t$, which in turn is equal to the end of period t stock of inventories, Q^{t+1} , less the beginning of period t stock of inventories, Q^t . Of course, equation t in equations (43) can be rearranged to give us the following supply equals demand equations:

$$(44) \quad Q^t + q^t = c^t + Q^{t+1}; \quad t = 1, 2, \dots$$

Thus the beginning of period t stock of storable goods, Q^t , plus new purchases of storable goods, q^t , equals consumption of the storable goods in period t , c^t , plus the end of period t stocks of storable goods, Q^{t+1} .

Recall that period t price and quantity vectors for household purchases of storable goods are p^t and q^t . Thus the value v^t of household purchases of storable goods during period t is defined as follows:

$$(45) \quad \begin{aligned} v^t &\equiv p^t \cdot q^t && t = 1, 2, \dots \\ &= p^t \cdot c^t + p^t \cdot [Q^{t+1} - Q^t] && \text{using (43)} \\ &= p^t \cdot c^t + p^t \cdot \Delta Q^t. \end{aligned}$$

Thus the period t value of household *purchases* of storables, $p^t \cdot q^t$, is equal to the period t value of household *consumption* of storable goods, $p^t \cdot c^t$, plus period t *net investment* in storables, $p^t \cdot \Delta Q^t$. All of these value aggregates use the vector of *average period t purchase prices* p^t to value q^t , c^t and ΔQ^t .

When a product goes on sale, typically households will dramatically increase their purchases of it. However, not all of the purchased storable good will be consumed in the period of purchase, so inventories of the storable product will greatly increase. Basically, changes in inventory will tend to smooth purchases of storable goods so that consumption is relatively stable over time. Thus adjusting purchases of storable goods for changes in inventory will lead to estimates of household actual consumption of storables that are much smoother than household purchases of storables. Constructing consumer price indexes using p^t and c^t as the basic price and quantity data to be used in an index number formula (rather than using p^t and q^t) will greatly mitigate the chain drift problem that will arise if household purchase data are used in place of household consumption data.

The length of the accounting period will affect the severity of the chain drift problem. If the accounting period length is a day, inventory changes may be large relative to daily consumption, leading to a big chain drift problem if daily purchase price and quantity data are used in an index

number formula with variable weights. Furthermore, if the household data pertain to a single household or a small number of households, the vector of daily purchases of storable goods may have many zero components, leading to a lack of matching problem which affects the reliability of the resulting daily price index. On the other hand, the changes in storable inventories for an annual CPI will be small relative to the annual consumption of storables and thus the difference between q^t and c^t in the case of an annual index will be small. Thus for annual CPIs, it is probably not necessary to collect data on inventories of storables.⁷¹ However, if a daily or weekly CPI is to be produced, it will be important to collect inventory data on storable goods and to adjust purchase data for changes in inventories.

The above accounting treatment for storable goods does not give any insight into why large changes in storable inventories might occur. In order to provide an analytic framework for the treatment of storable goods in a cost of living index, it is necessary to introduce the concept of intertemporal cost minimization. The basic idea is that the consumer or household tries to minimize the discounted cost of consumption over a number of discrete time periods subject to attaining a certain level of (intertemporal) utility.⁷² In order to minimize conceptual and notational complexity, we will look at the household's intertemporal cost minimization problem over a horizon that consists of just two periods.⁷³

Suppose that the household's opportunity cost of capital at the beginning of period t is the interest rate r . As usual, let p^t and q^t be the price and quantity vectors for household *purchases* of storable goods for period t for $t = 1, 2, 3$. Define the household's (expected) discounted value of *household purchases* of storable goods $W > 0$ over the two period horizon (discounted to the end of period 1) as follows:⁷⁴

$$\begin{aligned}
 (46) \quad W &\equiv p^1 \cdot q^1 + (1+r_2)^{-1} p^2 \cdot q^2 \\
 &= p^1 \cdot q^1 + p^{2*} \cdot q^2 && \text{defining } p^{2*} \equiv (1+r_2)^{-1} p^2 \\
 &= p^1 \cdot [c^1 + Q^2] + p^{2*} \cdot [c^2 - Q^2] && \text{using } Q^1 \equiv 0_N, q^1 = c^1 + Q^2, q^2 = c^2 - Q^2 \text{ and } Q^3 \equiv 0_N \\
 &= p^1 \cdot c^1 + p^{2*} \cdot c^2 + [p^1 - p^{2*}] \cdot Q^2 \\
 &= p^1 \cdot c^1 + p^{2*} \cdot c^2 + u^2 \cdot Q^2
 \end{aligned}$$

where

$$(47) \quad u^2 \equiv p^1 - p^{2*} = p^1 - (1+r_2)^{-1} p^2$$

is the vector of *user costs of storable products* for the beginning of period 2 stocks of inventories.⁷⁵ In the above model of consumer expenditures on storable goods, we are assuming

⁷¹ However, if the national statistical agency also constructs measures of household wealth, it will be necessary to undertake periodic surveys of household inventories of storables.

⁷² The framework for intertemporal consumer theory is basically the consumer theory counterpart to Hicks' (1946; 325-328) intertemporal producer theory; see Diewert (1974) (1977).

⁷³ It is straightforward to extend the number of time periods under consideration to an arbitrary finite number.

⁷⁴ As was the case for our analysis of user costs in section 4 above, we are following the conventions used in financial accounting that suggest that *flow transactions* taking place within the accounting period be regarded as taking place at the *end* of the accounting period and hence the period t cost of household purchases of storables, $p^t \cdot q^t$, is regarded as taking place at the end of period t ; see Peasnell (1981). Thus the period 2 and 3 purchase costs, $p^2 \cdot q^2$ and $p^3 \cdot q^3$, in definition (46) are discounted to the end of period 1 which is the beginning of period 2.

⁷⁵ Compare these new user costs to our previous definition for the vector beginning of period 2 user costs given by equations (40) for $t = 2$. These definitions imply that $U^{2*} \equiv P^2 - (1+r_2)^{-1} P^3$ where P^t is the vector of

that the household has no inventories of storables at the beginning of period 1 and at the end of period 2. Thus inventories Q^2 are only held at the beginning of period 2.

In order to apply classical economic theory to the problem on deciding the level of inventories for storables, it is useful to regard the W on the left hand side of definition (46) as an exogenous amount of money that the household plans to spend on purchases of storable goods over the two period horizon. Thus we assume that the household is subject to a partial “wealth” constraint of the form $W \geq p^1 \cdot c^1 + p^{2*} \cdot c^2 + u^2 \cdot Q^2 = p^1 \cdot q^1 + p^{2*} \cdot q^2$ where the household’s decision variables are purchases of storables over the two periods, q^1 , q^2 , consumption of storables over the two periods, c^1 , c^2 , and holdings of inventories of storables at the beginning of period 2, Q^2 .

If holdings of storables are not valued, except that they allow consumers to transfer purchases of consumption goods from one period where they are relatively cheap to another period where they are relatively expensive,⁷⁶ then if any component of the user cost vector u^2 is positive, say $u_{2n} = p_{1n} - (1+r_2)^{-1}p_{2n} > 0$, then it does not make sense to purchase storable good n in period 1 in order to consume it in period 2 because it will be cheaper to purchase the good in period 2, taking into account the fact that the household will tie up financial capital if it holds the good as an inventory item. Thus good n will be held as an inventory item (so that $Q_{2n} > 0$) only if $u_{2n} \leq 0$. If $u_{2n} < 0$, then it definitely will be worthwhile to hold some inventories of storable good n . But how much inventory will be held? Furthermore, can we apply the usual exact index number theory that relies on static utility maximizing behavior to the household’s purchases of storable goods? In order to provide answers to these questions, we will look at an economic model of consumer behavior.

When modeling consumer behavior over a time horizon, economists assume that households have intertemporal utility functions to measure the relative worth of consuming the services of various commodities. A general utility function to model the relative value of storables over a two period horizon is a function of the form $U(c^1, c^2)$. However, for producers of consumer price indexes who might want to apply the simple exact index number theory explained in Chapter 5 to produce a price index for each separate period, it is necessary to assume a more restrictive functional form for the intertemporal utility function. Thus we assume that $U(c^1, c^2) = F(f(c^1), f(c^2))$ where $F(C^1, C^2)$ is a “macro” utility function that describes the tradeoffs in consuming aggregate consumption in period 1, $C^1 \equiv f(c^1)$, against consuming aggregate consumption in period 2, $C^2 = f(c^2)$ where $f(c)$ is a within the period static utility function of the type studied in Chapter 5. As usual, we assume that the one period utility function $f(c)$ is a differentiable, concave, linearly homogeneous, increasing function of the nonnegative consumption vector c . We assume that the macro utility function, $F(C^1, C^2)$ is a differentiable,⁷⁷ increasing and concave function of C^1 and C^2 .

The household’s intertemporal utility maximization problem is the problem of maximizing $F(f(c^1), f(c^2))$ subject to: (i) the intertemporal budget constraint $W - [p^1 \cdot q^1 + p^{2*} \cdot q^2] \geq 0$; (ii) the household material balance equations that relate purchases to consumption and inventory change for both periods, $q^1 = c^1 + Q^2$ and $q^2 + Q^2 = c^2$ and (iii) the nonnegativity constraints $q^1 \geq 0_N$, $q^2 \geq 0_N$, $Q^2 \geq 0_N$, $c^1 \geq 0_N$ and $c^2 \geq 0_N$. The decision variables for this constrained utility maximization

purchase prices for storable goods at the beginning of period t . It can be seen that $u^2 \equiv p^1 - (1+r_2)^{-1}p^2$ has a similar form except in place of P^2 and P^3 , the new definition uses the *average purchase prices* for storable goods for periods 1 and 2, p^1 and p^2 .

⁷⁶ The user cost theory developed at the beginning of this section essentially assumed that holdings of storable goods increased household utility; i.e., holding inventories of storable goods was assumed to be desirable even if the goods were never consumed. Our present perspective assumes that inventories are only valuable when they are consumed.

⁷⁷ We assume that the first order partial derivatives of $F(C^1, C^2)$ are positive.

problem are q^1 , q^2 , Q^2 , c^1 and c^2 . Use the material balance equations to eliminate q^1 and q^2 from the intertemporal budget constraint. After eliminating q^1 and q^2 from the constraints, the household's utility maximization problem becomes maximize $F(f(c^1), f(c^2))$ subject to: (i) the intertemporal budget constraint $W - [p^1 \cdot c^1 + p^2 \cdot c^2 + u^2 \cdot Q^2] \geq 0$; (ii) $c^2 - Q^2 \geq 0_N$ and (iii) the nonnegativity constraints $Q^2 \geq 0_N$, $c^1 \geq 0_N$ and $c^2 \geq 0_N$. The decision variables for this constrained utility maximization problem are Q^2 , c^1 and c^2 . The *Lagrangian* for this constrained maximization problem is defined as follows:

$$(48) L(c^1, c^2, Q^2, \lambda, \kappa, \mu) \equiv F(f(c^1), f(c^2)) + \lambda \{W - [p^1 \cdot c^1 + p^2 \cdot c^2 + u^2 \cdot Q^2]\} + \mu \cdot [c^2 - Q^2] + \kappa \cdot Q^2$$

where λ is a nonnegative scalar Lagrange or Kuhn-Tucker (1951) multiplier and μ and κ are nonnegative vectors of Lagrange multipliers.

Suppose $c^{1*} \gg 0_N$, $c^{2*} \gg 0_N$ and $Q^{2*} \geq 0_N$ is a solution to the household's intertemporal constrained maximization problem. Then there exist $\lambda^* > 0$,⁷⁸ $\mu^* \geq 0_N$ and $\kappa^* \geq 0_N$ such that the following *Kuhn-Tucker conditions* are satisfied:⁷⁹

$$(49) \begin{array}{lll} \text{(i)} & F_1^* \nabla f(c^{1*}) = \lambda^* p^1; & c^{1*} \gg 0_N; \\ \text{(ii)} & F_2^* \nabla f(c^{2*}) = \lambda^* p^{2*} - \mu^*; & c^{2*} \gg 0_N; \\ \text{(iii)} & -\lambda^* u^2 - \mu^* + \kappa^* \leq 0_N; & Q^{2*} \geq 0_N; \quad [-\lambda^* u^2 - \mu^* + \kappa^*] Q^{2*} = 0; \\ \text{(iv)} & W = p^1 \cdot c^{1*} + p^{2*} \cdot c^{2*} + u^2 \cdot Q^{2*}; & \lambda^* > 0; \\ \text{(v)} & c^{2*} - Q^{2*} \geq 0_N; & \mu^* \geq 0_N; \quad \mu^* \cdot [c^{2*} - Q^{2*}] = 0; \\ \text{(vi)} & Q^{2*} \geq 0_N; & \kappa^* \geq 0_N; \quad \kappa^* \cdot Q^{2*} = 0 \end{array}$$

where $F_1^* \equiv \partial F(f(c^{1*}), f(c^{2*})) / \partial C^1$, $F_2^* \equiv \partial F(f(c^{1*}), f(c^{2*})) / \partial C^2$ are the first order partial derivatives of the macro utility function with respect to aggregate consumption C^t in each period t and $\nabla f(c^{t*})$ is the vector of first order partial derivatives of the micro utility function $f(c^t)$ with respect to the components of the period t consumption of storables vector c^t for $t = 1, 2$. Conditions (49) are more complicated than the usual first order necessary conditions that economists use when solving constrained optimization problems because, usually, we can assume that an interior solution to the optimization problem occurs and hence we can ignore nonnegativity constraints. But for this particular intertemporal utility maximization problem, nonnegativity constraints cannot be ignored; i.e., usually, the solution to the problem will require that some decision variables be equal to zero. The conditions defined by (49) allow for zero decision variables.

Suppose the above solution to the household's intertemporal utility maximization problem satisfies conditions (49) and in addition, $\mu^* = 0_N$. Then conditions (49) (ii) become $F_2^* \nabla f(c^{2*}) = \lambda^* p^{2*}$, which are the period 2 counterparts to conditions (49) (i): $F_1^* \nabla f(c^{1*}) = \lambda^* p^1$. Using these

⁷⁸ The Kuhn-Tucker conditions only imply the existence of a $\lambda \geq 0$ whereas we assumed the existence of a $\lambda > 0$. Our stronger assumption is justified if the first order partial derivatives of the utility function are positive.

⁷⁹ See Kuhn and Tucker (1951) or Karlin (1959; 204). In the original constrained utility maximization problem that involved q^1 , q^2 , c^1 , c^2 and Q^2 , all of these decision variables were restricted to be nonnegative. Recall that $q^1 = c^1 + Q^2$. Thus if $c^1 \geq 0_N$ and $Q^2 \geq 0_N$, then we also have $q^1 \geq 0_N$. However, $q^2 = c^2 - Q^2$ and so even though the simplified constrained utility maximization problem involved only the decision variables c^1 , c^2 and Q^2 , we still need to impose the restriction $q^2 \geq 0_N$ which implies the restriction (49) (v).

two sets of equations and the linear homogeneity of $f(c)$, we can establish the following equations:⁸⁰

$$(50) \quad p^t/p^t \cdot c^{t*} = \nabla f(c^{t*})/f(c^{t*}); \quad t = 1, 2.$$

But equations (50) are the equations for *Wold's* (1944; 69-71) *Identity*; see equations (15) in Chapter 5. Thus if the vector of Kuhn-Tucker multipliers μ^* turns out to be a vector of zeros, then we can apply the exact index number theory that was explained in Chapter 5 to the consumer's demand for storable goods in our highly simplified model of inventory behavior.

The question that now needs to be addressed is: under what conditions will $\mu^* = 0_N$? An answer is provided below. We will look at each component μ_n^* of μ^* in turn.

Case (i): Suppose that user cost for storable good n for beginning of period 2 inventories is positive; i.e., suppose that $u_n^2 \equiv p_n^1 - p_n^{2*} = p_n^1 - (1+r_2)^{-1}p_n^2 > 0$. Thus we have $p_n^1 > (1+r_2)^{-1}p_n^2$ so that the price of storable good n in period 1 is greater than its discounted period 2 expected price. Under these conditions, it makes no sense to purchase storable good n in period 1 to use in period 2 so that under these conditions, there will be no accumulation of inventories so that Q_n^{2*} will equal 0. To see that $Q_n^{2*} = 0$ follows from conditions (49), *suppose* that $Q_n^{2*} > 0$. Using conditions (49) (vi), it can be seen that our supposition implies that $\kappa_n^* = 0$. Using $\kappa_n^* = 0$ and $Q_n^{2*} > 0$, condition (49) (iii) implies that $-\lambda^* u_n^2 - \mu_n^* = 0$ or $\mu_n^* = -\lambda^* u_n^2 < 0$ using $\lambda^* > 0$ and $u_n^2 > 0$. But $\mu_n^* < 0$ contradicts conditions (49) (v) which implies $\mu_n^* \geq 0$. This contradiction means that our *supposition* that $Q_n^{2*} > 0$ is false and hence $Q_n^{2*} = 0$. Using $Q_n^{2*} = 0$ along with (49) (ii) which implies $c_n^{2*} > 0$ means that the equations $0 = \mu_n^* [c_n^{2*} - Q_n^{2*}] = \mu_n^* c_n^{2*}$ will hold using (49) (v) which in turn implies that $\mu_n^* = 0$. This algebra can be summarized as follows: if the user cost of storable good n at the beginning of period 2, u_n^2 , is positive, then no inventories of good n will be accumulated (so that $Q_n^{2*} = 0$) and the Lagrange multiplier for the nonnegativity constraints pertaining to purchases of good n will also be equal to zero (so that $\kappa_n^* = \mu_n^* = 0$). Thus if all N user costs of storables, u_n^2 , are positive, then there will be no purchases of inventories so that actual consumption in period t , c^{t*} , will equal market purchases for period t , q^{t*} , for periods $t = 1, 2$.⁸¹

Case (ii): Suppose that user cost for storable good n for beginning of period 2 inventories is negative; i.e., suppose that $u_n^2 \equiv p_n^1 - (1+r_2)^{-1}p_n^2 < 0$. In this case, it makes sense to accumulate inventories of good n in period 1 because the period 1 price is less than the discounted period 2 price for storable good n . It turns out that our simple model will imply that all of the purchases of good n are made in period 1; i.e., we will have $c_n^{2*} = Q_n^{2*}$. Thus there is a maximal amount of inventory accumulation that takes place in period 1.⁸² We explain how conditions (49) can be used to explain this result. *Suppose* $0 \leq Q_n^{2*} < c_n^{2*}$. Conditions (49) (v) and our supposition imply that $\mu_n^* = 0$. Conditions (49) (iii) and $\mu_n^* = 0$ imply that $-\lambda^* u_n^2 + \kappa_n^* \leq 0$ and this condition along

⁸⁰ Premultiply both sides of $\nabla f(c^{t*}) = [\lambda^*/F_t^*] p^t$ by c^{t*} for $t = 1, 2$. Using Euler's Theorem on linearly homogeneous functions, $f(c^{t*}) = c^{t*} \cdot \nabla f(c^{t*})$ for $t = 1, 2$. Use the equations $c^{t*} \cdot \nabla f(c^{t*}) = f(c^{t*}) = [\lambda^*/F_t^*] p^t \cdot c^{t*}$ to solve for $[\lambda^*/F_t^*] = f(c^{t*})/p^t \cdot c^{t*}$ for $t = 1, 2$. Thus we obtain the equations $\nabla f(c^{t*}) = [\lambda^*/F_t^*] p^t = p^t f(c^{t*})/p^t \cdot c^{t*}$ for $t = 1, 2$, which are equivalent to equations (50).

⁸¹ This very simple economic approach to the accumulation of inventories of storable goods *neglects the costs of shopping* which will imply some short term inventory accumulation even if all user costs are positive.

⁸² Our simple model of inventory accumulation *neglects any costs of inventory storage*, which helps to explain our all or none results.

with $u_n^2 < 0$ and $\lambda^* > 0$ imply that $0 < -\lambda^* u_n^2 \leq -\kappa_n^*$ which in turn implies that $\kappa_n^* < 0$. This contradicts part of conditions (49) (vi). Thus our *supposition* is false. Since we also have the constraint $q_n^{2*} \equiv c_n^{2*} - Q_n^{2*} \geq 0$, we see that we must have $q_n^{2*} \equiv c_n^{2*} - Q_n^{2*} = 0$. In this case, we also have $\kappa_n^* = 0$ and $\mu_n^* \geq 0$.

It can be seen that using an economic approach to model household purchases of storable goods is a difficult task. More realistic models of inventory accumulation need to take into account the costs of storing the inventories and they need to take into account the costs of shopping, which would include not only the transportation costs to the retail outlets but also the expenditure of *time* during the shopping process. A more realistic model of inventory accumulation would require a great deal of household information; information which is unlikely to be available to national statistical agencies in the near future.

What practical implications for statistical agencies can be drawn from the above analysis?

- The simplest strategy would be to just apply the acquisitions approach to purchases of storable goods; i.e., simply assume that purchases of storables over a month are equal to the actual consumption of the goods over the month. Over the course of a year, the value of average inventory holdings of storable goods to total household consumption of storables will typically be a small stable fraction⁸³ and thus the overall accuracy of the CPI will not be greatly affected.
- If periodic surveys of household inventories of storable goods are made and if the statistical agency target index is a cost of living index, then it would be useful to treat holdings of storable goods in the same manner as holdings of durable goods are treated; i.e., a user cost approach should be applied to storable goods.⁸⁴ If monthly surveys for household inventories of storable goods could be undertaken, then estimates for the actual consumption of storables could be made along with estimates for the user cost value for the household holdings of storable inventories. Users could decide to use the estimates for actual consumption or for actual consumption plus the services of household inventories of storables, depending on their needs.
- For the country's Balance Sheet accounts, household inventories of storable goods are part of household wealth. Thus for the construction of the Balance Sheet accounts, it is necessary for the national statistical agency to provide quarterly or annual estimates of household holdings of storable goods.⁸⁵

In the following eight sections of this chapter, the focus will be on the special problems that are associated with both measuring the value of the housing stock as well as on valuing the services of Owner Occupied Housing (OOH).

11. Decomposing Residential Property Prices into Land and Structure Components

In this section, the problems associated with the construction of constant quality residential property price indexes will be studied. The user cost approach to valuing the services of a durable good discussed in section 4 above cannot be applied directly to the construction of user costs for Owner Occupied Housing (OOH) because a residential property has two main components: a

⁸³ This assumption will not be satisfied if the country is under a COVID-19 lockdown. Inventories of storable goods will be much larger than usual and may be quite variable.

⁸⁴ See (47) which defines the vector of user costs for storable goods.

⁸⁵ If Balance Sheet estimates are made at a quarterly frequency, approximate monthly estimates for holdings of storable goods could be constructed using various interpolation methods.

structure (which depreciates) and a *land plot* (which does not depreciate).⁸⁶ In this section, we will look at the resulting problems associated with the construction of *constant quality indexes* for the *stock of residential housing units*; in subsequent sections, we will look at the problems associated with pricing the services of a residential dwelling unit.

There are two difficult measurement problems associated with the construction of a constant quality house price index:

- A dwelling unit is a *unique consumer durable good*; i.e., the location of a housing unit is a price determining characteristic of the unit and each house or apartment has a unique location.
- As mentioned above, there are two main components of a dwelling unit: (i) *the size of the structure* (measured in square meters of floor space) and (ii) *the size of the land plot* that the structure sits on (also measured in square meters). However, the purchase or selling price of a dwelling unit is for the entire property and thus *the decomposition of property price into its two main components will involve imputations*.

The first problem area listed above might not be a problem if the same dwelling unit sold at market prices at a frequent rate so that the location would be held constant and it would seem that the usual matched model methodology that is used in constructing price indexes could be applied. But houses do not transact all that frequently; typically, a house is held for 10-20 years by the same owner before it is resold. Moreover, the structure is not constant over time; depreciation of the structure occurs over time and owners renovate and replace aging components of the structure. For example, the roofing materials for many dwellings are replaced every 20 or 30 years. Thus depreciation and renovation constantly change the quality of the structure.

The second problem area is associated with the difficulty of decomposing the transaction price for a housing unit into *separate components* representing the structure value and the land value; i.e., the single property price is for both components of the housing unit but for many purposes, we require separate valuations for the two components. The international System of National Accounts, requires separate valuations for the land and structure components of residential housing in the National Balance Sheets of the country. Many countries construct estimates for the Total Factor Productivity or Multifactor Productivity of the various sectors in the economy and the methodology used to construct these estimates requires separate price and quantity information on both structures and the land that the structures sit on. In this section, we will indicate a possible method that can be used to accomplish this decomposition of property value into constant quality land and structure components.

The *builder's model* for valuing a detached dwelling unit postulates that the value of the property is the sum of two components: the value of the land which the structure sits on plus the value of the structure. This model can be justified in two situations:

⁸⁶ It is important to recognize that a residential property is a bundle of two important components: a land component and a structure component. Knoll, Schularick and Steger (2017; 331) summarize their study of house prices in 14 countries over the period 1870-2012 as follows: "Land prices, not replacement costs, are the key to understanding the trajectory of house prices. Rising land prices explain about 80 percent of the global house price boom that has taken place since World War II."

- A household purchases a residential land plot with no structure on it (or if there are structures on the land plot, they are immediately demolished).⁸⁷
- A household purchases a land plot and immediately builds a new dwelling unit on the property.

In the first case, it is clear that the property value is equal to the land value. In the second case, the total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S square meters, times the building cost per square meter β_t during period t , plus the cost of the land, which will be equal to the cost per square meter α_t times the area of the land site, say L square meters. Now think of a sample of properties of the same general type in the same general location, which have prices or values V_{tn} in period t (where $t = 1, \dots, T$) and structure floor space areas S_{tn} and land areas L_{tn} for $n = 1, \dots, N(t)$ where $N(t)$ is the number of observations in period t . Assume that these prices are equal to the sum of the land and structure costs plus error terms ε_{tn} which we assume are independently normally distributed with zero means and constant variances. This leads to the following *hedonic regression model* for period t where the α_t and β_t are the parameters to be estimated in the regression:⁸⁸

$$(51) V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

The hedonic regression model defined by (51) applies to new structures and to purchases of vacant residential lots in the neighbourhood under consideration where $S_{tn} = 0$. Note that there are some strong simplifying assumptions built into the model defined by (51): (i) the period t land price α_t (per m^2) is assumed to be constant across all properties in the neighbourhood under consideration and (ii) the construction cost (per m^2) is also assumed to be constant across all housing units built in the neighbourhood during period t . The above model applies to raw land purchases and the purchases of new dwelling units during period t in the neighbourhood under consideration. It is likely that a model that is similar to (51) applies to sales of older structures as well. Older structures will be worth less than newer structures due to the *depreciation* of the structure. Assuming that we have information on the age of the structure n at time t , say $A(t, n)$, and assuming a geometric (or declining balance) depreciation model, a more realistic hedonic regression model than that defined by (51) above is the following *basic builder's model*:

$$(52) V_{tn} = \alpha_t L_{tn} + \beta_t (1 - \delta)^{A(t, n)} S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t)$$

where the parameter δ reflects the *net geometric depreciation rate* as the structure ages one additional period. Thus if the age of the structure is measured in years, we would expect an annual *net* depreciation rate to be around 1 to 3 percent per year.⁸⁹ Note that (52) is now a nonlinear regression model whereas (51) was a simple linear regression model. The period t

⁸⁷ The cost of the demolition should be added to the purchase price for the land to get the overall land price for the land plot.

⁸⁸ Other papers that have suggested hedonic regression models that lead to additive decompositions of property values into land and structure components include Clapp (1980; 257-258), Davis and Heathcote (2007), Bostic, Longhofer and Redfearn (2007; 184), Francke and Vos (2004), Diewert (2008; 19-22) (2010), Francke (2008; 167), Koev and Santos Silva (2008), Rambaldi, McAllister, Collins and Fletcher (2010), Diewert, Haan and Hendriks (2011) (2015), Eurostat (2013), Diewert and Shimizu (2015) (2016) (2020), Diewert, Huang and Burnett-Isaacs (2017) and Burnett-Isaacs, Huang and Diewert (2021).

⁸⁹ This estimate of depreciation is regarded as a *net depreciation rate* because it is equal to a “true” gross structure depreciation rate less an average renovations appreciation rate. Since typically information on renovations and major repairs to a structure is not available, the age variable will only pick up average gross depreciation less average real renovation expenditures.

constant quality price of land will be the estimated coefficient for the parameter α_t and the price of a unit of a newly built structure for period t will be the estimate for β_t . The period t quantity of land for property n is L_{tn} and the period t quantity of structure for property n , expressed in equivalent units of a new structure, is $(1 - \delta)^{A(t,n)}S_{tn}$ where S_{tn} is the floor space area of the structure for property n in period t .

Note that the above model can be viewed as a *supply side model* as opposed to a *demand side model*.⁹⁰ Basically, we are assuming a valuation of a housing structures that is equal to the cost per unit floor space area of a new unit times the floor space area times an adjustment for structure depreciation. The corresponding land value of the property is determined residually as total property value minus the imputed value of structures quality adjusted for the age of the structure. This assumption is justified for the case of newly built houses and sales of vacant lots but it is less well justified for sales of properties with older structures where a demand side model may be more relevant.

There is a major practical problem with the hedonic regression model defined by (52): The *multicollinearity problem*. Experience has shown that it is usually not possible to estimate sensible land and structure prices in a hedonic regression like that defined by (52) due to the multicollinearity between lot size and structure size.⁹¹ Thus in order to deal with the multicollinearity problem, the parameter β_t in (52) is replaced by p_{St} , an *exogenous period t construction cost price* for houses in the area under consideration.⁹² The exogenous construction price index may be an official construction price index estimated by the national statistical agency or a relevant commercially available residential construction price index. Thus the new model that replaces (52) is the following nonlinear hedonic regression model:

$$(53) V_{tn} = \alpha_t L_{tn} + p_{St}(1 - \delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

This model has T land price parameters (the α_t) and one (net) geometric depreciation rate δ . Note that the replacement of the β_t by the exogenous construction price level, p_{St} , means that we have saved T degrees of freedom as well as eliminated the multicollinearity problem.

In order to allow for a finer structure of local land prices, the sales data may be further classified into a finer classification of locations. For example, the initial regression (53) may be applied to say city wide sales of residential properties. Suppose that the postal code of each sale is also available and there are J postal codes. Then one can introduce the following *postal code dummy variables*, $D_{PC,tn,j}$, into the hedonic regression (53). These J dummy variables are defined as follows: for $t = 1, \dots, T; n = 1, \dots, N(t); j = 1, \dots, J$:

$$(54) D_{PC,tn,j} \equiv 1 \text{ if observation } n \text{ in period } t \text{ is in Postal Code } j; \\ \equiv 0 \text{ if observation } n \text{ in period } t \text{ is not in Postal Code } j.$$

⁹⁰ We will pursue a demand side model in Section 13 below.

⁹¹ See Schwann (1998) and Diewert, de Haan and Hendriks (2011) (2015) on the multicollinearity problem.

⁹² This formulation follows that of Diewert (2010), Diewert, Haan and Hendriks (2011) (2015), Eurostat (2013), Diewert and Shimizu (2015) (2016) (2020), Diewert, Huang and Burnett-Issacs (2017) and Burnett-Issacs, Huang and Diewert (2021). These authors assume that property value is the sum of land and structure components but movements in the price of structures are proportional to an exogenous structure price index. Note that the index p_{St} should be a levels price that gives the period t cost of building one square meter of structure.

We now modify the model defined by (53) to allow the *level* of land prices to differ across the J postal codes. The new nonlinear regression model is the following one:

$$(55) V_{tn} = \alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})L_{tn} + p_{st}(1 - \delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Comparing the models defined by equations (53) and (55), it can be seen that we have added an additional J *neighbourhood relative land value parameters*, $\omega_1, \dots, \omega_J$, to the model defined by (53). However, looking at (55), it can be seen that the T land time parameters (the α_t) and the J location parameters (the ω_j) cannot all be identified. Thus it is necessary to impose at least one identifying normalization on these parameters. The following normalization is a convenient one:⁹³

$$(56) \omega_1 \equiv 1.$$

Thus Model 2 is defined by equations (55) and (56) has $J-1$ additional parameters compared to Model 1 defined by (53). Note that if we initially set *all* of the ω_j equal to unity, Model 2 collapses to Model 1. It is useful to make use of this fact in running a sequence of nonlinear hedonic regressions. The models that are proposed in this section are *nested* so that the final parameter estimates from a previous model can be used as starting parameter values in the next model's nonlinear regression.⁹⁴

Model 2 makes the price of residential land a nonsmooth function of the postal code or local neighbourhood area; i.e., the estimated price of land will exhibit discrete jumps as we move from one local area to an adjacent local area that has a different ω_j . If it is possible to collect spatial coordinate information for the properties in the sample, then it is possible to estimate a continuous land price surface for the hedonic regression model in place of the discrete plateau model that is defined by (55). These continuous surface models are very complex and not easy to estimate. However, Hill and Scholz (2018) and Diewert and Shimizu (2019) showed that for their particular samples of Australian and Japanese properties, the continuous surface models generated very similar price indexes to their counterpart discrete models. Thus if the purpose of the hedonic regressions is to generate residential land or property price indexes, it is not necessary to estimate complex continuous surface models.

In the next model, some nonlinearities in the pricing of the land area for each property are introduced. The land plot areas in a typical sample of properties can vary 5 or 10 fold.⁹⁵ Up to this point, we have assumed that land plots in the same neighbourhood sell at a constant price per

⁹³ Equivalently, one could make the normalization $\alpha_1 = 1$ and not normalize the ω_j . The resulting estimated α_t for $t = 2, 3, \dots, T$ can then be interpreted as a constant quality land price index for the entire region relative to period 1 where $\alpha_1 \equiv 1$. In this section, we are drawing on Diewert, Huang and Burnett-Isaacs (2017) and using the normalization used in that paper.

⁹⁴ In order to obtain sensible parameter estimates in our final (quite complex) nonlinear regression model, it is absolutely necessary to follow our procedure of sequentially estimating gradually more complex models, using the final coefficients from the previous model as starting values for the next model. The models that are being described in this section were implemented in Diewert, Huang and Burnett-Isaacs (2017) where the econometric software Shazam was used to perform the nonlinear regressions; see White (2004).

⁹⁵ This brings up an important point that has not been mentioned up to now. Panel data on the selling prices of properties and on the characteristics of the properties are subject to tremendous variations in the ratio of the highest price property to the lowest price property, to the largest lot size to the smallest lot size, to the largest floor space area to the smallest floor space area and so on. The observations that appear in the tails of the distribution of prices and in the distributions of property characteristics are inevitably sparse and subject to measurement error. Thus in order to obtain sensible estimates in running these hedonic regressions, it is typically necessary to delete the observations that are in the tails of these distributions.

square meter of lot area. However, it is likely that there is some nonlinearity in this pricing schedule; for example, it is likely that large lots sell at a per m² price that is well below the per m² price of medium sized lots. In order to capture this nonlinearity, divide up the total number of observations into K groups of observations based on their lot size. The Group 1 properties have lot size less than L₁ m², the Group 2 properties L_{tn} have lot sizes which satisfy the inequalities L₁ ≤ L_{tn} < L₂; the Group 3 properties L_{tn} have lot sizes which satisfy the inequalities L₂ ≤ L_{tn} < L₃; ...; the Group K properties L_{tn} have lot sizes which satisfy the inequalities L_{K-1} ≤ L_{tn}. The break points L₁ < L₂ < ... < L_{K-1} should be chosen so that the sample probability that any property in the sample will fall into any one of the groups is approximately equal. For each observation n in period t, the K *land dummy variables*, D_{L,tn,k}, for k = 1,...,K are defined as follows:

$$(57) D_{L,tn,k} \equiv 1 \text{ if observation } tn \text{ has land area that belongs to group } k; \\ \equiv 0 \text{ if observation } tn \text{ has land area that does not belong to group } k.$$

These dummy variables are used in the definition of the following piecewise linear function of L_{tn}, f_L(L_{tn}), defined as follows:

$$(58) f_L(L_{tn}) \equiv D_{L,tn,1}\lambda_1 L_{tn} + D_{L,tn,2}[\lambda_1 L_1 + \lambda_2(L_{tn} - L_1)] + D_{L,tn,3}[\lambda_1 L_1 + \lambda_2(L_2 - L_1) + \lambda_3(L_{tn} - L_2)] \\ + \dots + D_{L,tn,K}[\lambda_1 L_1 + \lambda_2(L_2 - L_1) + \dots + \lambda_K(L_{tn} - L_{K-1})]$$

where the λ_k are unknown parameters. The function f_L(L_{tn}) defines a *relative valuation function for the land area of a house* as a function of the plot area, L_{tn}. The new nonlinear regression model is the following one:

$$(59) V_{tn} = \alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})f_L(L_{tn}) + p_{St}(1 - \delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Comparing the models defined by equations (55) and (59), it can be seen that we have added an additional K *land plot size parameters*, λ₁, ..., λ_K, to the model defined by (55). However, looking at (59), it can be seen that the T land time parameters (the α_t), the J postal code parameters (the ω_j) and the K land plot size parameters (the λ_k) cannot all be identified. Thus the following identification normalizations on the parameters for Model 3 defined by (59) and (60) are imposed:

$$(60) \omega_1 \equiv 1; \lambda_1 \equiv 1.$$

Note that if all of the λ_k are set equal to unity, Model 3 collapses to Model 2. Typically, the log likelihood for Model 3 will be considerably higher than for Model 2.⁹⁶ Land prices as functions of lot size do not always decline monotonically but for very large land plots, the marginal price of an extra square foot of land is typically quite low.

The next model is similar to Model 3 except that now the marginal price of adding an extra amount of structure is allowed to vary as the size of the structure increases. It is likely that the quality of the structure increases as the size of the structure increases. In order to capture this nonlinearity, divide up the sample observations into M groups of observations based on their structure size. The Group 1 properties have structures with floor space area S_{tn} less than S₁ m², the Group 2 properties have structure areas S_{tn} satisfying the inequalities S₁ ≤ S_{tn} < S₂, ..., the Group

⁹⁶ For the example in Diewert, Huang and Burnett-Isaacs (2017) where the models described in this section were estimated, the log likelihood increased by 1762 log likelihood points and the R² jumped from 0.7662 for Model 2 to 0.8283 for Model 3 for the addition of 6 new λ_k parameters.

M-1 properties have structure areas S_{tn} satisfying the inequalities $S_{M-2} \leq S_{tn} < S_{M-1}$, and the Group M properties have structure areas S_{tn} satisfying the inequalities $S_{M-1} \leq S_{tn}$ where the M-1 break points satisfy the inequalities $S_1 < S_2 < \dots < S_{M-1}$. Again, the break points should be chosen so that the sample probability that any property in the sample will fall into any one of the groups is approximately equal. For each observation n in period t , we define the M *structure dummy variables*, $D_{S,tn,m}$, for $m = 1, \dots, M$ as follows:

$$(61) \quad D_{S,tn,m} \equiv 1 \text{ if observation } tn \text{ has structure area that belongs to structure group } m; \\ \equiv 0 \text{ if observation } tn \text{ has structure area that does not belong to group } m.$$

These dummy variables are used in the definition of the following piecewise linear function of S_{tn} , $g_S(S_{tn})$, defined as follows:

$$(62) \quad g_S(S_{tn}) \equiv D_{S,tn,1}\mu_1 S_{tn} + D_{S,tn,2}[\mu_1 S_1 + \mu_2(S_{tn} - S_1)] + D_{S,tn,3}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_{tn} - S_2)] \\ + D_{S,tn,4}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_3 - S_2) + \mu_4(S_{tn} - S_3)] + \dots \\ + D_{S,tn,M}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_3 - S_2) + \dots + \mu_M(S_{tn} - S_{M-1})].$$

where the μ_m are unknown parameters. The function $g_S(S_{tn})$ defines a *relative valuation function for the structure area of a house* as a function of the structure area.

The new nonlinear regression model is the following Model 4:

$$(63) \quad V_{tn} = \alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})f_L(L_{tn}) + p_{St}(1 - \delta)^{A(t,n)} g_S(S_{tn}) + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Comparing the models defined by equations (59) and (63), it can be seen that an additional M *structure floor space parameters*, μ_1, \dots, μ_M , have been added to the model defined by (59).⁹⁷ Again, we add the normalizations (60) in order to identify all of the parameters in the model. Note that if all of the μ_m are set equal to unity, Model 4 collapses down to Model 3. Typically, the log likelihood for Model 4 will be considerably higher than for Model 3.⁹⁸

At this stage, it is often the case that an acceptable model has been estimated. How can the estimated parameters from the final model be used in order to form price and quantity indexes?

The sequence of price levels for the land component of residential property sales is defined to be $\alpha_1, \alpha_2, \dots, \alpha_T$ and the corresponding sequence of price levels for the structure component of residential property sales in the T periods is defined to be the exogenous sequence of indexes, $p_{S1}, p_{S2}, \dots, p_{ST}$. The land and structure values of properties transacted in period t , V_{Lt} and V_{St} , are defined by using the estimated land and structure additive components of transacted properties in period t , $\alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})f_L(L_{tn})$ and $p_{St}(1 - \delta)^{A(t,n)} g_S(S_{tn})$ respectively, and summing over properties that were sold in period t :

⁹⁷ At this stage of the sequential estimation procedure, it is usually not necessary to impose a normalization on the parameters $\mu_1 - \mu_M$. This lack of a normalization means that the scale of the exogenous structure price levels p_{St} is allowed to change; i.e., essentially, allowance is now made to quality adjust the exogenous index to a certain extent. However, if the resulting estimated structure values turn out to be unreasonably large or small, then it will be necessary to set one of the μ_m to equal 1.

⁹⁸ For the example in Diewert, Huang and Burnett-Isaacs (2017), the log likelihood increased by 935 log likelihood points and the R^2 jumped from 0.8283 for Model 3 to 0.8520 for Model 4 for the addition of 5 new μ_m parameters.

$$(64) V_{Lt} \equiv \sum_{n \in N(t)} \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) f_L(L_{tn}) ; \quad t = 1, \dots, T;$$

$$(65) V_{St} \equiv \sum_{n \in N(t)} p_{St} (1 - \delta)^{A(t,n)} g_S(S_{tn}) ; \quad t = 1, \dots, T.$$

Using the prices $\alpha_1, \alpha_2, \dots, \alpha_T$, the corresponding estimated land values V_{L1}, \dots, V_{LT} , the prices $p_{S1}, p_{S2}, \dots, p_{ST}$ and the corresponding estimated structure values V_{S1}, \dots, V_{ST} , one can just apply normal index number theory using these data to construct Laspeyres, Paasche, Fisher or whatever index formula is being used by the statistical agency in order to construct constant quality price and quantity overall property indexes for the sales of residential properties in the area under consideration for the T periods.

However, constant quality land and structure price indexes for *sales* of Owner Occupied Residential houses is not what is needed for most purposes; what is required are constant quality price and quantity indexes for the *stock* of residential houses. In order to accomplish this task, it is necessary to have a census of the housing stock in the country which would include information on the characteristics that are used in the hedonic regression model that is defined by (63). The information that is required in order to estimate (63) is information on the following variables:

- The selling price of the residential properties (P_{tn});
- The age of the structure on the property (A_{tn});
- The area of the land plot (L_{tn});
- The floor space area of the structure (S_{tn});
- The neighbourhood of the property (or the postal code) and
- An exogenous structure price index which provides the construction cost of a new structure per meter squared or per square foot (p_{St}).

If a national housing Census has information on the above property characteristics (excluding the information on selling prices P_{tn} and on the exogenous structure price index p_{St})⁹⁹, then it will be possible to insert the characteristics of each residential dwelling unit into the right hand side of (63) and then using appropriate modifications of definitions (64) and (65), it will be possible to obtain estimates for the land and structure value for each dwelling unit in the area covered by the regression. If there is no national housing census information or the required characteristics are not included in the census, then it will be very difficult to form estimates for the value of residential land.

Additional information on house and property characteristics will lead to more accurate land and structure decompositions of property value. Examples of useful additional structure price determining characteristics are: (i) the number of bathrooms; (ii) the number of bedrooms; (iii) the type of construction material; (iv) the number of stories; etc. Examples of useful additional land price determining characteristics are: (i) the distance to the nearest subway station; (ii) the distance to the city core; (iii) the quality of neighbourhood schools; (iv) the existence of various neighbourhood amenities; etc. For examples of how these characteristics can be integrated into the builder's model, see Diewert, de Haan and Hendriks (2011) (2015), Eurostat (2013) (2017), Diewert and Shimizu (2015) and Diewert, Huang and Burnett-Isaacs (2017).¹⁰⁰

⁹⁹ Every country will have a national residential construction deflator because this deflator is required in order to form estimates of real investment in residential structures. However, this national deflator may not be entirely appropriate for the type of buildings in a particular neighbourhood.

¹⁰⁰ It is also possible to estimate more general models of depreciation using the builder's model; see Diewert and Shimizu (2017) and Diewert, Huang and Burnett-Isaacs (2017).

The estimates for the geometric depreciation rate generated by the application of the builder's model are useful for national income accountants because they facilitate the accurate estimation of structure depreciation, which is required for the national accounts. However, the depreciation estimates that are generated by the builder's model are *wear and tear depreciation* estimates that apply to structures that continue in existence over the sample period. The estimated depreciation rate measures (net) depreciation¹⁰¹ of a structure that has survived from its birth to the period of its sale. However, there is another form of structure depreciation that the estimated depreciation rate misses; namely the loss of residual structure value that results from the *early demolition* of the structure. This problem was noticed and addressed by Hulten and Wykoff (1981a; 377-379) (1981b) (1996). Wear and tear depreciation is often called *deterioration* depreciation and *demolition* or *early retirement depreciation* is sometimes called *obsolescence* depreciation.¹⁰² Methods for estimating this form of depreciation have been proposed by Hulten and Wykoff as mentioned above and by Diewert and Shimizu (2017; 512-516). Both methods require information on the distribution of the ages of retirement for the asset class. The Hulten and Wykoff method absorbs demolition depreciation into the wear and tear depreciation rate, whereas the Diewert and Shimizu method uses the wear and tear depreciation rate that is generated by sales of surviving buildings but adds a separate depreciation rate that is due to early demolition of the structures in the asset class. Both methods require information on the age of structures when they are demolished.¹⁰³

The above paragraph simply warns the reader that wear and tear depreciation¹⁰⁴ for surviving buildings is not the entire depreciation story: there is also a loss of asset value that results from the early retirement of a building that needs to be taken into account when constructing national income accounting estimates of depreciation.

There is one additional complication that needs to be taken into account when running a hedonic regression on the sales of houses; i.e., what happens when the sales information for an additional period becomes available? The simplest way of dealing with this problem dates back to Court (1939). His method works as follows: set $T = 2$ and run a hedonic regression that has a time dummy variable in it. In the context of the hedonic regression model defined by (63), estimates for the price of land for periods 1 and 2 would be obtained, say α_1^1 and α_2^1 . The price index for land for periods 1 and 2 is defined as $P_L^1 = 1$ and $P_L^2 = \alpha_2^1/\alpha_1^1$. Now run a new hedonic regression using (63) for $t = 2, 3$ and obtain new estimates for the price of land in periods 2 and 3,

¹⁰¹ It is a net estimate since renovation and replacement investments in the building tend to extend the life of the building or augment its value. Thus the gross wear and tear depreciation rate for the structure will tend to be larger than the estimated net depreciation rate.

¹⁰² Crosby, Devaney and Law (2012; 230) distinguish the two types of depreciation and in addition, they provide a comprehensive survey of the depreciation literature as it applies to commercial properties.

¹⁰³ The Hulten and Wykoff method estimates the age of retirement in a somewhat arbitrary fashion whereas the Diewert and Shimizu method relies on mortality distributions on the age of buildings at the time they are demolished. Over long periods of time and using country wide data, the two methods should be equivalent. However, the Diewert and Shimizu method should give more accurate results at the firm and regional levels since their method is consistent with the hedonic estimation of structure depreciation rates as explained in this section.

¹⁰⁴ What has been labeled as wear and tear depreciation could be better described as *anticipated amortization of the structure* rather than wear and tear depreciation. Once a structure is built, it becomes a fixed asset which cannot be transferred to alternative uses (like a truck or machine). Thus amortization of the cost of the structure should be proportional to the cash flows or to the service flows of utility that the building generates over its expected lifetime. However, technical progress, obsolescence or unanticipated market developments can cause the building to be demolished before it is fully amortized. See Diewert and Fox (2016) for a more complete discussion of the fixity problem.

say α_2^2 and α_3^2 . The price index for land in period 3 is defined as $P_L^3 = P_L^2(\alpha_3^2/\alpha_2^2)$; i.e., we update the price index value for period 2, P_L^2 , by the rate of change in land prices going from period 2 to 3, (α_3^2/α_2^2) . Thus the previously estimated index is updated each period as new information becomes available. This *adjacent period time dummy model* has the advantage that it does not revise the previously estimated indexes as the new information becomes available.¹⁰⁵

The above method does not always work well in the context of estimating property price indexes due to the sparseness of sales in a neighbourhood and the multiplicity of parameters that are required to adequately control for differences in housing characteristics. Thus Shimizu, Nishimura and Watanabe (2010a; 797) suggested extending the number of periods from 2 to a longer window of T consecutive periods, leading to the *rolling window time dummy hedonic regression model*. Thus for the model defined by (63), the land price parameters that are estimated by the first regression using the data for periods 1 to T are $\alpha_1^1, \alpha_2^1, \dots, \alpha_T^1$ and the corresponding land price indexes for periods 1 to t are $P_L^t \equiv \alpha_t^1/\alpha_1^1$ for $t = 1, \dots, T$. The second hedonic regression uses the data for periods 2, 3, ..., $T, T+1$ and the estimated land price parameters are $\alpha_2^2, \alpha_3^2, \dots, \alpha_T^2, \alpha_{T+1}^2$. The price index for land in period $T+1$ is defined as $P_L^{T+1} = P_L^T(\alpha_{T+1}^2/\alpha_T^2)$; i.e., the price index for period T , P_L^T , is updated by the rate of change in land prices going from period T to $T+1$, $\alpha_{T+1}^2/\alpha_T^2$.

There are two additional issues that need to be addressed when using a rolling window time dummy hedonic regression model:

- How long should the window length be? A longer window length will usually lead to more stable estimates for the unknown parameters in the hedonic regression. A shorter window length will allow for taste changes to take place more quickly. A window length of one year plus one period will allow for seasonal effects. At this stage of our knowledge, it is difficult to give definitive advice on the length of the window.
- When a new window is computed, how should the index results from the new window be linked to the previous index values? The same issue applies when a multilateral method is used in the time series context. Ivancic, Diewert and Fox (2011) along with Shimizu, Nishimura and Watanabe (2010a) and Shimizu, Takatsuji, Ono and Nishimura (2010) suggested that the movement of the indexes for the last two periods in the new window be linked to the last index value generated by the previous window. However, Krsinich (2016) suggested that the movement of the indexes generated by the new window over the entire new window period be linked to the window index value for the second period in the previous window. Krsinich called this a *window splice* as opposed to the *movement splice* explained above. De Haan (2015; 27) suggested that perhaps the linking period should be in the middle of the old window which the Australian Bureau of Statistics (2016; 12) termed a *half splice*. Ivancic, Diewert and Fox (2011; 33) suggested that the average of all possible links of the new window to the old window be used and they called this a *mean splice* method for linking the results of the new window to the previous

¹⁰⁵ The two period time dummy variable hedonic regression (and its extension to many periods) was first considered explicitly by Court (1939; 109-111) as his hedonic suggestion number two. Court used adjacent period time dummy hedonic regressions as links in a longer chain of comparisons extending from 1920 to 1939 for US automobiles: "The net regressions on time shown above are in effect price link relatives for cars of constant specifications. By joining these together, a continuous index is secured." If the two periods being compared are consecutive years, Griliches (1971; 7) coined the term "adjacent year regression" to describe this method for updating the index as new information becomes available. Diewert (2005b) looked at the axiomatic properties of adjacent year time dummy hedonic regressions.

window.¹⁰⁶ Again, there is no consensus at this time on which linking method is “best”. However, it is likely that all of these linking methods will generate much the same results.

It can be seen that estimating price indexes for houses (or detached dwelling units) is not a straightforward task, particularly if one wants separate constant quality indexes for the land and structure components of property value.¹⁰⁷ In the following section, it will be seen that it is even more complicated to obtain separate indexes for the land and structure components for condominium sales.

12. Decomposing Condominium Sales Prices into Land and Structure Components

A starting point for applying the builder’s model to condominium sales is the hedonic regression model defined by equations (53) in the previous section.¹⁰⁸ For convenience, equations (53) are repeated below as equations (66):

$$(66) V_{tn} = \alpha_t L_{tn} + p_{St}(1 - \delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t)$$

where V_{tn} is the selling price of a condominium property in a neighbourhood in period t , α_t is the price of the land that the structure sits on (per m^2), L_{tn} is the land area that can be attributed to the condo unit, p_{St} is an exogenous period t construction cost for the type of condo under consideration (per m^2), δ is the one period wear and tear geometric depreciation rate for the structure, $A_{tn} = A(t,n)$ is the age of the structure in periods, S_{tn} is the floor space of unit n that is sold in period t (in m^2) and ε_{tn} is an error term.

A problem with the above model is that it is not appropriate to allocate the entire land value of the condominium property to any particular unit that is sold in period t . Thus each condo unit in the building should be allocated a *share* of the total land value of the property. The problem is: how exactly should this imputed land share be calculated? There are two simple methods for constructing an appropriate land share: (i) Use the unit’s share of floor space to total structure floor space or (ii) simply use $1/N$ as the share, where N is the total number of units in the building. Thus define the following two land share imputations for unit n in period t :

$$(67) L_{Stn} \equiv (S_{tn}/TS_{tn})TL_{tn}; L_{Ntn} \equiv (1/N_{tn})TL_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t)$$

where S_{tn} is the floor space area of unit n which is sold in period t , TS_{tn} is the total building floor space area, TL_{tn} is the total land area of the building and N_{tn} is the total number of units in the building for unit n sold in period t . The first method of land share imputation is used by the Japanese land tax authorities. The second method of imputation implicitly assumes that each unit can enjoy the use of the entire land area and so an equal share of land for each unit seems “fair”.

There is a problem with the definition of L_{Stn} in (67): the floor space “share” of unit n , S_{tn}/TS_{tn} if summed over all units in the building would be less than 1 because the privately held floor space of each unit in the building does not account for shared building floor spaces such as halls,

¹⁰⁶ For the details on how the mean splice method works, see Diewert and Fox (2020).

¹⁰⁷ For additional hedonic regression models for detached houses, see Verbrugge (2008), Garner and Verbrugge (2011), Eurostat (2013) (2017), Hill (2013), Hill, Scholz, Shimizu and Steurer (2018), Rambaldi and Fletcher (2014) and Silver (2018).

¹⁰⁸ The analysis in this section follows that of Diewert and Shimizu (2016) and Burnett-Isaacs, Huang and Diewert (2021).

elevators, storage spaces, furnace rooms and other “public” floor spaces, which are included in total building floor space, TS_{tn} . Thus the “share” S_{tn}/TS_{tn} must be adjusted upward by some percentage to account for these shared building facilities.¹⁰⁹ In what follows, it is assumed that this adjustment has been made to S_{tn} (so that S_{tn} is now interpreted as adjusted condo floor space area).

In order to obtain sensible decompositions of the condominium selling price into land and structure components, it may be necessary to assume a structure value and focus on the determinants of land value at the initial stages of the sequential estimation procedure. Thus following Diewert and Shimizu (2016), assume that the *imputed structure value* for unit n in period t , $V_{S_{tn}}$, is defined as follows:

$$(68) V_{S_{tn}} \equiv p_{St}(1 - \delta)^{A_{(t,n)}} S_{tn} ; \quad t = 1, \dots, T; n = 1, \dots, N(t)$$

where δ is an assumed geometric depreciation rate.¹¹⁰ Once the imputed value of the structure has been defined by (68), the *imputed land value* for condo n in period t , $V_{L_{tn}}$, is defined by subtracting the imputed structure value from the total value of the condo unit, which is V_{tn} :

$$(69) V_{L_{tn}} \equiv V_{tn} - V_{S_{tn}} ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

In the hedonic regressions which follow immediately, the imputed value of land for the condominium unit, $V_{L_{tn}}$, is used as the dependent variable in a hedonic regression. The following regressions explain variations in these imputed land values in terms of the property characteristics.

Suppose that the postal code of each sale is also available and there are J postal codes. Then one can introduce the following *postal code dummy variables*, $D_{PC,tn,j}$, as explanatory variables into a hedonic regression. Define these J dummy variables using definitions (54) in the previous section and estimate the following hedonic regression which is a *land counterpart* to the hedonic regression defined by (55) in the previous section:

$$(70) V_{L_{tn}} = \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) L_{S_{tn}} + \varepsilon_{tn} ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Note that the imputed value of land, $V_{L_{tn}}$ defined by (69), replaces total property value V_{tn} which was the dependent variable in (55).¹¹¹

¹⁰⁹ Diewert and Shimizu (2016; 303) constructed estimates of Tokyo total building private floor space to total building floor space for each observation nt as $N_{tn} S_{tn}/TS_{tn}$, where N_{tn} is the number of units in the building which contained condo sale n in period t , S_{tn} is the private floor space of the sold unit and TS_{tn} is the total floor space of the building. The sample wide average of these ratios was 0.899. Thus the first imputation method in definitions (67) was changed from $L_{S_{tn}} \equiv (S_{tn}/TS_{tn}) TL_{tn}$ to $L_{S_{tn}} \equiv (1/0.899)(S_{tn}/TS_{tn}) TL_{tn} = (1.1)(S_{tn}/TS_{tn}) TL_{tn}$. Burnett-Issacs, Huang and Diewert (2021) estimated a similar condo model and consulted with construction experts and determined that on average, the ratio of total space to private space for Ottawa condominium apartments was approximately 1.33. Thus they changed $L_{S_{tn}} \equiv (S_{tn}/TS_{tn}) TL_{tn}$ to $L_{S_{tn}} \equiv (1.33)(S_{tn}/TS_{tn}) TL_{tn}$.

¹¹⁰ Diewert and Shimizu (2016) assumed $\delta = 0.03$ and Burnett-Isaacs, Huang and Diewert (2021) assumed $\delta = 0.02$ where the age variable A_{tn} is measured in years. Later, δ will be estimated.

¹¹¹ As usual, we need a normalization on the parameters such as $\alpha_1 = 1$ in order to identify all of the remaining parameters, $\alpha_2, \dots, \alpha_T, \omega_1, \dots, \omega_J$. Note that this regression uses the first method of land imputation defined by (67). Later, the second method will also be considered.

It is likely that the height of the building (number of stories) increases the value of the land plot supporting the building, all else equal. Thus define the number of stories dummy variables, $D_{NS,tn,s}$, as follows: $t = 1, \dots, T$; $n = 1, \dots, N(t)$; $s = 1, \dots, NS$:

$$(71) D_{NS,tn,s} \equiv 1 \text{ if observation } n \text{ in period } t \text{ is in a building with } s \text{ stories ;} \\ \equiv 0 \text{ if observation } n \text{ in period } t \text{ is } \textit{not} \text{ in building with } s \text{ stories.}$$

The new nonlinear regression model is the following one:

$$(72) V_{Ltn} = \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) (\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}) L_{Stn} + \varepsilon_{tn} ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Comparing the models defined by equations (70) and (72), it can be seen that an additional NS *building height parameters*, χ_1, \dots, χ_{NS} , have been added to the model defined by (70).¹¹² As usual, the models defined by (70) and (72) are nested so that the finishing parameter values from the nonlinear regression (70) can be used as starting values for (72) along with the starting values $\chi_1 = \chi_2 = \dots = \chi_{NS} = 1$.

The higher up a unit is, the better is the view on average and so it could be expected that the price of the unit increases as its height increases. The quality of the structure probably does not increase as the height of the unit increases, so it seems reasonable to impute the height premium as an adjustment to the land price component of the unit.

It is possible to introduce the height of the unit (the H variable) as a categorical variable (like the number of stories NS in the last hedonic regression model). However, both Diewert and Shimizu (2016) (hereafter DS) and Burnett-Isaacs, Huang and Diewert (2021) (hereafter BHD) found that this dummy variable approach could be replaced by using H as a continuous variable with little change in the fit of the model. Thus the new nonlinear regression model is the following one where $t = 1, \dots, T$; $n = 1, \dots, N(t)$:

$$(73) V_{Ltn} = \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) (\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}) (1 + \gamma(H_{tn} - 3)) L_{Stn} + \varepsilon_{tn} ;$$

where H_{tn} is the height of the sold unit n in period t (measured in number of stories from ground level) and γ is a height of the unit parameter to be estimated.¹¹³ The above model assumes that the lowest height for the units sold in the sample was $H_{tn} = 3$. Thus for all the observations that correspond to the sold unit being located on the third floor of the building, the new parameter γ in (73) will not affect the predicted value in the regression. However, for heights of the sold units that were greater than 3, the regression implies that the land value will increase by γ for each story that is above 3.¹¹⁴

As was mentioned earlier, there are two simple methods for imputing the share of the building's total land area to the sold unit. Up until now, we have used the first method of imputation defined by (67) which set the share of total land imputed to unit n in period t , L_{Stn} , equal to $(S_{tn}/TS_{tn})TL_{tn}$ whereas the second method set L_{Ntn} equal to $(1/N_{tn})TL_{tn}$. In the next model, the land imputation

¹¹² Again normalizations like $\alpha_1 \equiv 1$; $\chi_1 \equiv 1$ are required in order to identify the remaining parameters. If all $\chi_s = 1$, then the model defined by (72) collapses to the model defined by (70).

¹¹³ Normalizations like $\alpha_1 \equiv 1$; $\chi_1 \equiv 1$ need to be imposed in order to identify the remaining parameters.

¹¹⁴ The studies that have implemented this model found that the estimated γ was in the 2-4% range. Thus the imputed land value of a unit increases by 2 to 4% for each story above the threshold level of 3.

for unit n in period t is set equal to a *weighted average* of the two imputation methods and the best fitting weight, λ , is estimated. Thus define:

$$(74) L_{tn}(\lambda) = [\lambda(S_{tn}/TS_{tn}) + (1-\lambda)(1/N_{tn})]TL_{tn} ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

The new nonlinear regression model is the following one, where $t = 1, \dots, T$; $n = 1, \dots, N(t)$ and $L_{tn}(\lambda)$ is defined by (74).¹¹⁵:

$$(75) V_{Ltn} = \alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s})(1+\gamma(H_{tn}-3))L_{tn}(\lambda) + \varepsilon_{tn}.$$

Conditional on the land area of the building, one would expect the sold unit's land imputation value to increase as the number of units in the building increases. Thus one could use the total number of units in the building, N_{tn} , as a quality adjustment variable for the imputed land value of a condo unit. DS introduced this variable as a continuous variable. The smallest number of units in the buildings in their sample was 11. Thus they introduced the term $1+\kappa(N_{tn}-11)$ as an explanatory term in the nonlinear regression. The new parameter κ is the percentage increase in the unit's imputed value of land as the number of units in the building grows by one unit. The new nonlinear regression model is the following one where $t = 1, \dots, T$; $n = 1, \dots, N(t)$ and $L_{tn}(\lambda)$ is defined by (74):

$$(76) V_{Ltn} = \alpha_t(\sum_{j=1}^J \omega_j D_{PC,tn,j})(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s})(1+\gamma(H_{tn}-3))(1+\kappa(N_{tn}-11))L_{tn}(\lambda) + \varepsilon_{tn} .$$

where $L_{tn}(\lambda)$ is defined by (74).

The next explanatory variable to be introduced into the hedonic regression model is one which is not obvious but turned out to be very significant in the regressions run by DS and BHD. The *footprint* of a building is the area of the land that directly supports the structure. An approximation to the footprint land for unit n in period t is the total structure area TS_{tn} divided by the total number of stories in the structure TH_{tn} . If footprint land is subtracted from the total land area, TL_{tn} , the resulting variable is *excess land*,¹¹⁶ EL_{tn} , defined as follows:

$$(77) EL_{tn} \equiv TL_{tn} - (TS_{tn}/TH_{tn}) ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

In the Tokyo data used by DS, excess land ranged from 47 m² to 2912 m². Now group the sample observations into M categories, depending on the amount of excess land that pertained to each observation. Group 1 consists of observations tn where EL_{tn} is less than some number EL_1 ; Group 2: observations such that $EL_1 \leq EL_{tn} < EL_2$; ... ; Group M : $EL_{M-1} \leq EL_{tn}$. The break points, $EL_1, EL_2, \dots, EL_{M-1}$ should be chosen so that the number of observations in each group is approximately equal. Define the *excess land dummy variables*, $D_{EL,tn,m}$, as follows for $t = 1, \dots, T$; $n = 1, \dots, N(t)$; $m = 1, \dots, M$:

$$(78) D_{EL,tn,m} \equiv 1 \text{ if observation } n \text{ in period } t \text{ is in excess land group } m; \\ \equiv 0 \text{ if observation } n \text{ in period } t \text{ is not in excess land group } m.$$

¹¹⁵ For the DS Tokyo condo data, the estimated λ turned out to be $\lambda^* = 0.3636$ ($t = 9.84$) so that the very simple land imputation method that just divided the total land plot size by the number of units in the building got a higher weight (0.6364) than the weight for the floor space allocation method (0.3636). For the Ottawa condo data, the estimated λ turned out to be $\lambda^* = 0.2525$ ($t = 12.10$).

¹¹⁶ This is land that is usable for purposes *other* than the direct support of the structure on the land plot.

The new regression model is the following one:

$$(79) V_{Ltn} = \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) (\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}) (\sum_{m=1}^M \mu_m D_{EL,tn,m}) \times \\ (1 + \gamma(H_{tn} - 3)) (1 + \kappa(N_{tn} - 11)) L_{tn}(\lambda) + \varepsilon_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Not all of the parameters in (79) can be identified, so the following normalizations on the parameters in (79) are imposed:

$$(80) \alpha_1 \equiv 1; \chi_1 \equiv 1; \mu_1 \equiv 1.$$

Introducing the excess land dummy variables led to huge jumps in the log likelihoods for the hedonic regressions run by DS and BHS: 1020 for DS and 2652 for BHS.¹¹⁷ Both studies found that the estimated μ_m were positive but their magnitudes decreased monotonically as the excess land variable increased.

There are three additional explanatory variables that were used by DS that may affect the price of land. Define TW as the walking time in minutes to the nearest subway station; TT as the subway running time in minutes to the Central Tokyo station from the nearest station and the SOUTH dummy variable is set equal to 1 if the sold condo unit faces south and 0 otherwise. Let $D_{S,tn,2}$ equal the SOUTH dummy variable for sale n in period t . Define $D_{S,tn,2} = 1 - D_{S,tn,1}$. In the Tokyo data set used by DS, TW ranged from 1 to 19 minutes, while TT ranged from 12 to 48 minutes. These new variables are inserted into the previous nonlinear regression model (79) in the following manner for $t = 1, \dots, T; n = 1, \dots, N(t)$:

$$(81) V_{Ltn} = \alpha_t (\sum_{j=1}^J \omega_j D_{PC,tn,j}) (\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}) (\sum_{m=1}^M \mu_m D_{EL,tn,m}) (\phi_1 D_{S,tn,1} + \phi_2 D_{S,tn,2}) \\ \times (1 + \gamma(H_{tn} - 3)) (1 + \kappa(N_{tn} - 11)) (1 + \eta(TW_{tn} - 1)) (1 + \theta(TT_{tn} - 12)) L_{tn}(\lambda) + \varepsilon_{tn};$$

where $L_{tn}(\lambda)$ is defined by (74). Not all of the parameters in (81) can be identified, so the following normalizations (82) are imposed on the parameters in (81):

$$(82) \alpha_1 \equiv 1; \chi_1 \equiv 1; \mu_1 \equiv 1; \phi_1 \equiv 1.$$

Using the DS Tokyo data, the R^2 for this model turned out to be 0.6308 and the log likelihood increased by 406 points over the log likelihood of the previous model defined by (79) for the addition of 3 new parameters. The estimated parameters had the expected signs and had reasonable magnitudes.

At this point, DS concluded that the imputed land value for each condominium in their sample was predicted reasonably well by the hedonic regression model defined by (81) and (82). Thus in the following regression, they switched from using the imputed land value V_{Ltn} defined by (69) as the dependent variable in the regressions to using the actual selling price of the property, V_{tn} . They used the specification for the land component of the property that is defined by (81) and (82) but they also added the structure term $p_{S_t}(1 - \delta)^{A(t,n)} S_{tn}$ to account for the structure component of the value of the condo unit. Note that the annual depreciation rate δ is now estimated by the new hedonic regression model, rather than assuming that it was equal to 3%. Thus the number of

¹¹⁷ Recall the hedonic regression model defined by (59) in the previous section which introduced linear splines on the valuation of the land area of a stand alone housing unit. This introduction also greatly increased the log likelihood of the regression. In the present context, the excess land dummy variables take the place of the linear spline functions in (59).

unknown parameters in the new model increased by 1. They used the estimated values for the coefficients in (81) as starting values in this new nonlinear regression.¹¹⁸

Using their Tokyo data, DS found that the R^2 for this new model was 0.8190 and the estimated depreciation rate was $\delta^* = 0.0367$ ($t = 27.1$). Note that the R^2 is satisfactory; i.e., the new model explains a substantial fraction of the variation in condo prices.

DS and BHD introduced some additional explanatory variables as quality adjusting variables for the imputed value of structures. DS introduced the number of bedrooms and the type of building as quality adjusters for the value of the structure. BHD introduced the number of bedrooms, the number of bathrooms, the presence of balconies, the use of natural gas as the heating fuel and whether there was commercial space in the building as additional variables that could determine the value of the structure. These variables were significant explanatory variables but the overall R^2 for the final hedonic regression did not increase by a large amount with the addition of these variables to the regression. The details may be found in Diewert and Shimizu (2016) and Burnett-Isaacs, Huang and Diewert (2021).

Once the final hedonic regression has been run, the sequence of land prices is given by $\alpha_1, \alpha_2, \dots, \alpha_T$ and the sequence of condo structure prices is given by the exogenous structure price indexes, $p_{S1}, p_{S2}, \dots, p_{ST}$. To obtain overall property price indexes for sales of condos, form the following counterparts to equations (64) and (65) in the previous section to obtain an estimate of period t condo land value, V_{Lt} , and estimated period t structure value, V_{St} , for $t = 1, \dots, T$:

$$(83) V_{Lt} \equiv \sum_{n \in N(t)} \alpha_t (\sum_{j=1}^J \omega_j D_{PC,t,n,j}) (\sum_{s=1}^{NS} \chi_s D_{NS,t,n,s}) (\sum_{m=1}^M \mu_m D_{EL,t,n,m}) \\ \times (\phi_1 D_{S,t,n,1} + \phi_2 D_{S,t,n,2}) (1 + \gamma(H_{t,n} - 3)) (1 + \kappa(N_{t,n} - 1)) (1 + \eta(TW_{t,n} - 1)) (1 + \theta(TT_{t,n} - 12)) L_{t,n}(\lambda);$$

$$(84) V_{St} \equiv \sum_{n \in N(t)} p_{St} (1 - \delta)^{A(t,n)} S_{t,n}.$$

Using the prices $\alpha_1, \alpha_2, \dots, \alpha_T$, the corresponding estimated land values V_{L1}, \dots, V_{LT} , the prices $p_{S1}, p_{S2}, \dots, p_{ST}$ and the corresponding estimated structure values V_{S1}, \dots, V_{ST} , one can again apply normal index number theory using these data to construct Laspeyres, Paasche, Fisher or whatever index formula is being used by the statistical agency in order to construct constant quality price and quantity overall property indexes for the sales of condominium units in the area under consideration for the T periods.

In summary: the builder's model can be modified to apply to the sales of condominium units and reasonable decompositions of property value into land and structure components can be obtained. However, the nonlinear regressions that are required in order to implement the model end up being rather complex. In addition, information on more characteristics of the condominium properties needs to be collected in order to implement the models. The information that is required in order to estimate the final model and calculate (83) and (84) is as follows:

- The selling prices of the condominium properties in the sample ($P_{t,n}$);
- The age of the structure on the property ($A_{t,n}$);
- The total area of the land plot ($TL_{t,n}$);

¹¹⁸ Attempting to estimate the parameters in (83) without good starting values for the nonlinear regression will not lead to sensible parameter estimates. Thus it is necessary to obtain good starting values for (83) by estimating the rather long sequence of regressions explained above, starting with a very simple model and gradually introducing additional explanatory variables. Each regression in the sequence contains the previous one as a special case so that the final estimates of one regression can be used as starting values for the subsequent one.

- The floor space area of the condo unit (S_{tn});
- The total floor space area of the entire building (TS_{tn});
- The neighbourhood of the property (or the postal code);
- An exogenous structure price index which provides the construction cost of a new structure per meter squared or per square foot (p_{st});
- The number of stories of the building (NS_{tn});
- The height of the sold unit (the number of stories from ground level) (H_{tn});
- The number of units in the building (N_{tn});
- The walking time in minutes to the nearest subway station (TW_{tn}) and
- The subway running time in minutes to the city center from the nearest station (TT_{tn}).

The last two variables are not essential (and are not relevant in small towns and cities). Other non-essential variables which could be useful are the number of bedrooms, the number of bathrooms, the existence of balconies, the type of construction, the number of parking spaces and so on.

The hedonic regression models that were considered in the last two sections are essentially modified supply side models. In the following section, demand side hedonic regressions are considered.

13. Demand Side Property Price Hedonic Regressions

A way of rationalizing the traditional log price time dummy hedonic regression model for properties with varying amounts of land area L and constant quality structure area S^* is that the utility that these properties yield to consumers is proportional to the Cobb-Douglas utility function $L^\alpha S^{*\beta}$ where α and β are positive parameters (which do not necessarily sum to one).¹¹⁹ Initially, assume that the constant quality structure area S^* is equal to the floor space area of the structure, S , times an age adjustment, $(1-\delta)^A$, where A is the age of the structure in years and δ is a positive depreciation rate that is less than 1. Thus S^* is related to S as follows:

$$(85) S^* \equiv S(1-\delta)^A.$$

In any given time period t , assume that the sale price of transacted property n , V_{tn} , with the amount of land L_{tn} and the amount of quality adjusted structure S_{tn}^* is equal to the following expression:

$$(86) \begin{aligned} V_{tn} &= p_t L_{tn}^\alpha [S_{tn}^*]^\beta \\ &= p_t L_{tn}^\alpha [S_{tn}(1-\delta)^{A(t,n)}]^\beta && \text{using (85)} \\ &= p_t L_{tn}^\alpha S_{tn}^\beta (1-\delta)^{\beta A(t,n)} \\ &= p_t L_{tn}^\alpha S_{tn}^\beta \phi^{A(t,n)} \end{aligned}$$

where $A(t,n) = A_{tn}$ is the age of house n sold in period t , p_t can be interpreted as a *period t property price index* and the constant ϕ is defined as follows:

¹¹⁹ The early analysis in this section follows that of McMillen (2003; 289-290), Shimizu, Nishimura and Watanabe (2010a; 795) and Diewert, Huang and Burnett-Isaacs (2017). McMillen assumed that $\alpha + \beta = 1$. We follow Shimizu, Nishimura and Watanabe in allowing α and β to be unrestricted. Knoll, Schularick and Steger (2017; 344-345) assumed a Cobb-Douglas production function in order to decompose house prices into land and structure components; i.e., they applied a production side model in their decomposition instead of a demand side decomposition as will be done in this section.

$$(87) \phi \equiv (1-\delta)^\beta.$$

Thus if V_{tn} is deflated by the period t property price index p_t , the real value or utility u_{tn} of the property with characteristics L_{tn} and S_{tn}^* is obtained:

$$(88) V_{tn}/p_t = L_{tn}^\alpha S_{tn}^{*\beta} \equiv u_{tn}.$$

Thus $u_{tn} \equiv q_t$ is the *aggregate real value of the property* with characteristics L_{tn} and S_{tn}^* .¹²⁰

Define ρ_t as the logarithm of p_t and γ as the logarithm of ϕ ; i.e.,

$$(89) \rho_t \equiv \ln p_t ; \gamma \equiv \ln \phi.$$

After taking logarithms of both sides of the first equation in (88), using definitions (85) and (89) and adding error terms, the following system of estimating equations is obtained:¹²¹

$$(90) \ln V_{tn} = \rho_t + \alpha \ln L_{tn} + \beta \ln S_{tn} + \gamma A_{tn} + \varepsilon_{tn} ; \quad t = 1, \dots, T; n = 1, \dots, N(t)$$

where the ε_{tn} are independently distributed error terms with 0 means and constant variances. It can be seen that (90) is a traditional log price time dummy hedonic regression model with a minimal number of characteristics. The unknown parameters in (90) are the constant quality log property prices, ρ_1, \dots, ρ_T , the taste parameters α , β and the transformed depreciation rate γ . Once these parameters have been determined, the geometric depreciation rate δ which appears in equations (86) can be recovered from the regression parameter estimates as follows:

$$(91) \delta \equiv 1 - e^{\gamma/\beta}.$$

We now explain how the hedonic pricing model defined by (86) can be manipulated to provide a decomposition of property value in period t into land and quality adjusted structure components.

Once estimates for α , β and δ have been obtained, define period t value of a property with characteristics L_{tn} and S_{tn}^* is given by the following *period t property valuation function* by the right hand side of (86); i.e., define $V(p_t, L_{tn}, S_{tn}^*) \equiv p_t L_{tn}^\alpha S_{tn}^{*\beta}$. In empirical applications of the hedonic regression model defined by (90), it will often happen that estimates for α and β are such that $\alpha + \beta$ is less than 1.¹²² This means that a property in a given period that has double the land and quality adjusted structure than another property will sell for less than double the price of the smaller property. This follows from the fact that the Cobb-Douglas hedonic utility function, $u(L, S^*) \equiv L^\alpha S^{*\beta}$, exhibits diminishing returns to scale when $\alpha + \beta < 1$; i.e., we have:

¹²⁰ For each property n in scope for period t , equations (88) can be rearranged to read as follows: $V_{tn}/u_{tn} = p_t$. Thus the model assumes that purchasers of the type of property in scope for the sales index have the same property preferences over alternative properties n in period t (with land and quality adjusted structure quantities defined by L_{tn} and S_{tn}^*) given by the utility function $L_{tn}^\alpha S_{tn}^{*\beta}$. Competition between purchasers forces the price of the properties in scope per unit utility to equalize in period t ; i.e., we obtain the equations $V_{tn}/L_{tn}^\alpha S_{tn}^{*\beta} = p_t$. Of course, these assumptions will only be approximately correct so equations (88) will only hold approximately. If the R^2 obtained for the hedonic regression (90) is low, then the underlying economic model will provide only a poor approximation to reality.

¹²¹ Log price hedonic regressions for property prices date back to Bailey, Muth and Nourse (1963).

¹²² See for example the estimated model in Diewert, Huang and Burnett-Isaacs (2017).

$$(92) u(\lambda L, \lambda S^*) = \lambda^{\alpha+\beta} u(L, S^*)$$

for all $\lambda > 0$. This behavior is roughly consistent with our builder's Models 5-7 where there was a tendency for property prices to increase less than proportionally as L and S^* increased.

The *marginal prices of land and constant quality structure* in period t for a property with characteristics L and S^* , $\pi_L(p_t, L, S^*)$ and $\pi_{S^*}(p_t, L, S^*)$, are defined by partially differentiating the property valuation function with respect to L and S^* respectively:

$$(93) \pi_L(p_t, L_{tn}, S_{tn}^*) \equiv \partial V(p_t, L_{tn}, S_{tn}^*) / \partial L \equiv p_t \alpha L_{tn}^{\alpha} S_{tn}^{*\beta} / L_{tn} = \alpha V(p_t, L_{tn}, S_{tn}^*) / L_{tn} ;$$

$$(94) \pi_{S^*}(p_t, L_{tn}, S_{tn}^*) \equiv \partial V(p_t, L_{tn}, S_{tn}^*) / \partial S^* \equiv p_t \beta L_{tn}^{\alpha} S_{tn}^{*\beta} / S_{tn}^* = \beta V(p_t, L_{tn}, S_{tn}^*) / S_{tn}^* .$$

Multiply the marginal price of land by the amount of land in the property and add to this value of land the product of the marginal price of constant quality structure by the amount of constant quality structure on the property in order to obtain the following identity:

$$(95) (\alpha+\beta)V(p_t, L_{tn}, S_{tn}^*) = \pi_L(p_t, L_{tn}, S_{tn}^*)L_{tn} + \pi_{S^*}(p_t, L_{tn}, S_{tn}^*)S_{tn}^* .$$

If $\alpha+\beta$ is less than one, then using marginal prices to value the land and constant quality structure in a property will lead to a property valuation that is less than its selling price. Thus to make the land and structure components of property value add up to property value, divide the marginal prices defined by (93) and (94) by $\alpha+\beta$ in order to obtain the following *adjusted prices of land and structures for property n sold in period t* , $p_{tL}(p_t, L_{tn}, S_{tn}^*)$ and $p_{tS^*}(p_t, L_{tn}, S_{tn}^*)$:

$$(96) p_{tL}(p_t, L_{tn}, S_{tn}^*) \equiv \pi_L(p_t, L_{tn}, S_{tn}^*) / (\alpha+\beta) = \alpha(\alpha+\beta)^{-1} V(p_t, L_{tn}, S_{tn}^*) / L_{tn} ;$$

$$(97) p_{tS^*}(p_t, L_{tn}, S_{tn}^*) \equiv \pi_{S^*}(p_t, L_{tn}, S_{tn}^*) / (\alpha+\beta) = \beta(\alpha+\beta)^{-1} V(p_t, L_{tn}, S_{tn}^*) / S_{tn}^* .$$

The above material outlines a theoretical framework that can generate a decomposition of property value into land and structure components using the results of a traditional log price time dummy hedonic regression model. To complete the analysis, it is necessary to fill in the details of how the individual property land and structure prices that are generated by the model can be aggregated into period t overall land and structure price indexes.

Run the hedonic regression model defined by (90). Define the *constant quality property price index* p_t for period t as follows:

$$(98) p_t \equiv \exp(\rho_t) ; \quad t = 1, \dots, T.$$

Define the geometric depreciation rate δ by (91). Once δ has been defined, the amount of quality adjusted structure for property n in period t , S_{tn}^* is defined as follows:

$$(99) \ln(S_{tn}^*) \equiv \ln(S_{tn}) + A_{tn} \ln(1-\delta) ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Now that p_t , L_{tn} , S_{tn}^* , α and β have all been defined, we use these data in order to define the predicted prices for property n sold in period t , V_{tn}^* :

$$(100) V_{tn}^* \equiv p_t (L_{tn})^{\alpha} (S_{tn}^*)^{\beta} ; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Use equations (96) and (97) in order to define *constant quality land and structure prices* for sold property n in period t , p_{tL} and p_{tS^*} , as follows:

$$(101) p_{tL} \equiv \alpha(\alpha+\beta)^{-1}V_{tn}^*/L_{tn}; \quad t = 1, \dots, T; n = 1, \dots, N(t);$$

$$(102) p_{tS^*} \equiv \beta(\alpha+\beta)^{-1}V_{tn}^*/S_{tn}^*; \quad t = 1, \dots, T; n = 1, \dots, N(t).$$

Finally, *unit value constant quality land and structure prices* for all properties sold in period t , p_{tL} and p_{tS^*} , are defined as follows:

$$(103) p_{tL} \equiv \sum_{n=1}^{N(t)} p_{tLn} L_{tn} / \sum_{n=1}^{N(t)} L_{tn}; \quad t = 1, \dots, T;$$

$$(104) p_{tS^*} \equiv \sum_{n=1}^{N(t)} p_{tSn^*} S_{tn}^* / \sum_{n=1}^{N(t)} S_{tn}^*; \quad t = 1, \dots, T.$$

The period t land and structure prices that are defined by (103) and (104) are reasonable summary statistic prices for land and structures sold in period t that are generated by the log price time dummy hedonic regression model defined by (90).

If the price of land grows at a different rate than the price of a constant quality structure, then the time dummy log price hedonic regression model defined by (90) will generate very different constant quality land and structure subindexes when compared to the corresponding indexes estimated by the builder's model. To see this, suppose the same house n sold in period t and sold again in the following period $t+1$. The period t data for this house are V_{tn}^* , L_{tn} and S_{tn}^* , while the period $t+1$ data are V_{t+1n}^* , $L_{t+1n} = L_{tn}$ and $S_{t+1n}^* = (1-\delta)S_{tn}^*$. Use definitions (101) and (102) for this house for periods t and $t+1$ and calculate the following land and structure inflation rates for this house going from period t to period $t+1$:

$$(105) p_{t+1nL}/p_{tLn} = [\alpha(\alpha+\beta)^{-1}V_{t+1n}^*/L_{tn}] / [\alpha(\alpha+\beta)^{-1}V_{tn}^*/L_{tn}] = V_{t+1n}^*/V_{tn}^*;$$

$$(106) p_{t+1nS^*}/p_{tLn} = [\beta(\alpha+\beta)^{-1}V_{t+1n}^*/(1-\delta)S_{tn}^*] / [\beta(\alpha+\beta)^{-1}V_{tn}^*/S_{tn}^*] = (1-\delta)^{-1}(V_{t+1n}^*/V_{tn}^*).$$

Thus (one plus) the imputed land inflation rate, p_{t+1nL}/p_{tLn} , will equal (one plus) the growth in property value, V_{t+1n}^*/V_{tn}^* , and (one plus) the imputed constant quality structure inflation rate, p_{t+1nS^*}/p_{tLn} , will equal $(1-\delta)^{-1}(V_{t+1n}^*/V_{tn}^*)$. Hence if δ is small, then the land and structure inflation rates will be almost identical and approximately equal to (one plus) the growth rate for overall property value. Thus the constant quality price indexes for land and structures will move in an almost proportional manner. In most countries, the price of land will grow much more rapidly than the price of structures so the hedonic regression model defined by (90) is not suitable for finding usable land price indexes for residential housing.

However, the hedonic regression model defined by (90) (and its generalizations) can generate very reasonable overall constant quality property price indexes, provided that the model generates a plausible estimate for the structure depreciation rate. To see why this result might occur, a highly simplified comparison of a builder's model and the log price traditional hedonic regression model studied in this section will be undertaken below.

Consider the valuation of a representative property in periods 1 and 2 using both the builder's model and the traditional hedonic regression model explained in this section. In period 1, the quantity of land and constant quality structure is L_1 and S_1^* with total property value equal to V_1 . In period 2, the quantity of land and constant quality structure is $L_2 = (1+g_L)L_1$ and $S_2^* = (1+g_S)S_1^*$ with total property value equal to V_2 . The L_t and S_t^* are known and hence the growth rates g_L and g_S are also known. Using the property valuation function defined by (100), the two properties have the following value decompositions, where p_1 and p_2 are the constant quality property price levels for periods 1 and 2:

$$(107) V_1 = p_1 L_1^\alpha S_1^{*\beta};$$

$$(108) V_2 = p_2 L_2^\alpha S_2^{*\beta}$$

$$= p_1(1+\rho)[L_1(1+g_L)]^\alpha [S_1^*(1+g_S)]^\beta \quad \text{where } 1+\rho = p_2/p_1$$

$$= V_1(1+\rho)(1+g_L)^\alpha(1+g_S)^\beta$$

$$\approx V_1(1+\rho)[\alpha(1+g_L) + \beta(1+g_S)]$$

where the last approximate equality follows if $\alpha + \beta = 1$ and the geometric mean $(1+g_L)^\alpha(1+g_S)^\beta$ is approximated by the corresponding arithmetic mean, $\alpha(1+g_L) + \beta(1+g_S)$.

Now use the builder's model to value the same properties. Let p_{L1} and p_{L2} be the price levels for land in periods 1 and 2 and let p_{S1} and p_{S2} be the constant quality price levels for structures in periods 1 and 2. The builder's model imputes the following values for the properties in the two periods:

$$(109) V_1 = p_{L1}L_1 + p_{S1}S_1^* ;$$

$$(110) V_2 = p_{L2}L_2 + p_{S2}S_2^*$$

$$= p_{L1}(1+\rho_L)(1+g_L)L_1 + p_{S1}(1+\rho_S)(1+g_S)S_1^*$$

where the land and structure constant quality price indexes are defined as $1+\rho_L = p_{L2}/p_{L1}$ and $1+\rho_S = p_{S2}/p_{S1}$. Define the land and structure share of property value in period 1 as $s_{L1} \equiv p_{L1}L_1/V_1$ and $s_{S1} \equiv p_{S1}S_1^*/V_1$ respectively. The *Laspeyres quantity* and *Paasche price indexes* for properties, Q_L and P_P , are defined as follows:

$$(111) Q_L \equiv s_{L1}(L_2/L_1) + s_{S1}(S_2^*/S_1^*)$$

$$= s_{L1}(1+g_L) + s_{S1}(1+g_S) ;$$

$$(112) P_P \equiv [V_2/V_1]/Q_L$$

$$= [V_2/V_1]/[s_{L1}(1+g_L) + s_{S1}(1+g_S)]$$

where the last equality follows using (111). Using (108), we have the following approximate expression for $1+\rho$, which is the property price index generated by the traditional hedonic regression model:

$$(113) 1+\rho \approx [V_2/V_1]/[\alpha(1+g_L) + \beta(1+g_S)].$$

Comparing (112) to (113), it can be seen that the Paasche property price index that is generated by the builder's model, P_P , will be approximately equal to the property price index $1+\rho$ that is generated by a traditional log price time dummy hedonic regression model provided that α is approximately equal to the land share s_{L1} and β is approximately equal to structure share s_{S1} .¹²³ Since the hedonic utility function for the traditional model is Cobb Douglas, this approximate equality is likely to hold. Thus the traditional model is likely to generate approximately the same overall property price indexes as would be generated by the builder's model.¹²⁴

The approximation result in the previous paragraph opens up another possible method for obtaining aggregate land values for residential housing. There are residential property price indexes for many countries that are based on traditional hedonic regression models. Consider

¹²³ To obtain this approximation result, it is also necessary that the depreciation rate that is estimated by the log price time dummy model be reasonable.

¹²⁴ For examples of studies where it was found that this approximate equality held, see Diewert (2010; 21), Diewert and Shimizu (2015; 1692) and Diewert, Huang and Burnett-Isaacs (2017; 32).

such a country that also conducts periodic censuses of housing where owners of residential dwelling units are asked to value their properties. Let the estimated value of housing in periods 1 and t be V_1 and V_t . Suppose the aggregate housing price index levels for these two periods are p_1 and p_t . Using these data, one can form aggregate volume estimates for residential housing as $q_1 \equiv V_1/p_1$ and $q_t \equiv V_t/p_t$. From the country's system of national accounts, it should be possible to obtain estimates for the aggregate price and quantity or volume of residential structures which we denote by p_{S1} and q_{S1} for period 1 and p_{St} and q_{St} for period t . With these data in hand, aggregate Laspeyres, Paasche and Fisher (1922) price and quantity indexes for residential land can be formed using (p_1, p_{S1}) and (p_t, p_{St}) as period 1 and t price vectors and using $(q_1, -q_{S1})$ and $(q_t, -q_{St})$ as period 1 and t quantity vectors. The resulting land prices (p_{L1}, p_{Lt}) and volumes (q_{L1}, q_{Lt}) would fill a gap in the System of National Accounts for the country. Real household wealth accounts could be constructed that had household land and household structures as separate assets.

For data series on residential property prices for either the sales of properties or the stock of properties, see the European Central Bank (2018) (which lists 228 series for European countries) and the Bank for International Settlements (2018), which lists long series for 18 advanced economies. For additional information on alternative approaches for the measurement of residential property price indexes for sales of properties and for making estimates for the stock of residential properties, see Statistics Portugal (2009), Eurostat (2013) (2017), Hill (2013), Silver (2018) and Hill, Scholz, Shimizu and Steurer (2020).

14. Price Indexes for Rental Housing: The Modified Repeat Rents Approach

At first sight, it would seem that the construction of price indexes for rental housing should be fairly straightforward, since typically, rents are paid to owners every month. Thus all that seems to be necessary is to collect information on rents paid (from either the tenants or from the owners), say R_{tn} and R_{t+1n} for rental unit n in periods t and $t+1$, form the price ratios, R_{t+1n}/R_{tn} , and take a suitable average of these ratios to form a rent index.

Specifically, suppose we have data on rents R_{tn} for a group of “somewhat homogeneous” rental dwelling units for $N(t)$ properties in period t for consecutive months $t = 0, 1$. Denote the set of available properties in period t by $S(t)$ for $t = 0, 1$. Assume that there is a large overlap of properties between the two periods; i.e., assume that the intersection set of properties $S(0) \cap S(1)$ consists of many properties. By “somewhat homogeneous” properties, we mean that the properties are similar in type (either detached, semidetached or high or medium rise apartments), located in a local area where a separate rent index could be produced (a postal code area or a neighbourhood), either furnished or unfurnished and the rental properties in scope have roughly similar ratios of land value to structure value. Later in this section, it will be assumed that estimates for the floor space of the structure of property n in period t , S_{tn} , and for the corresponding land area of the property, L_{tn} , are known. Typically, the floor space area and the land area of a specific rental property n will remain constant from period to period so that $S_{tn} = S_n$ and $L_{tn} = L_n$ for all time periods t that property n is in scope for the index. We also assume that the age of property n in period t , $A_{tn} = A(t, n)$, (in months if the index is a monthly index) is known (at least approximately).

Each rental property provides a unique service, since the location of each rental property will in general be different and the location of the property is an important price determining characteristic of each rental property. The quantity associated with each rent observation could be considered to be unity. Since periods 0 and 1 are close to each other, the characteristics which describe each rental property will not change much. Thus a useful preliminary rent index going

from period 0 to 1 is the following *repeat rents index*, P_{RR} , defined as the sum of rents paid in period 1 divided by the sum of rents paid in period 0 for all properties that are common to the two periods:

$$(114) P_{RR} \equiv \sum_{n \in S(0) \cap S(1)} R_{1n} / \sum_{n \in S(0) \cap S(1)} R_{0n}.$$

Thus this preliminary index P_{RR} is equal to the *maximum overlap rent value ratio*. The above index can be interpreted as a Dutot index but it can also be interpreted as a Laspeyres, Paasche, Lowe or Fisher index since the quantity associated with rental property n in period t is 1 and the corresponding price is the rent R_{tn} .

Since rents usually do not change much from month to month, P_{RR} will be close to unity if months 0 and 1 are close to each other. Thus the construction of rental property indexes seems to be very straightforward!

However, there are *three problems* with the above maximum overlap rental index:

- The quantity (or utility) associated with each property does not remain constant from period to period *due to depreciation of the structure*. This depreciation can be offset by increased maintenance and renovation of the structure. But in general, there will be (on average) a *net depreciation rate* associated with the structure on the rental property.
- *New rental properties* may come into the location in scope during period 1. These properties are excluded from the continuing unit rent index defined by (114). Newly renovated properties also have the character of a new commodity which is not directly comparable to the corresponding rental property in period 0. If these properties can be identified, they should be excluded from the matched model index defined by (114) and they should be treated as a “new” property.
- Some rental properties which were rented out in period 0 may become *vacant* in period 1 and thus no household is getting utility from the vacant rental property in period 1 and hence these vacant properties should not be included in the CPI. Similarly, rental properties which were *demolished* in period 1 should be excluded from the matched model index.

Some solutions to the above problems can be implemented at the cost of making additional assumptions.

To deal with the depreciation problem, assume that the statistical agency has an estimate for the annual structure geometric depreciation rate for the type of structure in scope for the local area rent index. This annual structure depreciation rate should be converted into a monthly rate. Thus suppose the annual structure geometric depreciation rate is 1% or 2%. The corresponding monthly rate is 0.083% or 0.165% respectively. But this monthly *structure depreciation rate*, δ , needs to be converted into a *property depreciation rate*; i.e., it needs to be further reduced by the ratio of structure value to total property value (which includes land value). Suppose that the reduced value depreciation rate is known and is equal to the small fraction $\Delta > 0$.¹²⁵

¹²⁵ Later in this section, we will indicate how this property depreciation rate could be estimated using a hedonic regression. Malpezzi, Ozanne and Thibodeau (1987; 382) found that for their US sample of rental properties, annual rent declined about 0.6% per year. This corresponds to a monthly Δ equal to 0.050% per month.

The estimated depreciation rate δ could equal 0. In this case, renters do not experience any reduction in the quality of the rented structure as the structure ages. This corresponds to one hoss shay or light bulb depreciation. If this case were to occur, it would imply that the aging bias adjustments made in the above two models are not warranted and the estimating equations for those two models would need to be changed to reflect the one hoss shay depreciation of the structures. However, the available empirical evidence indicates that depreciation rates are positive.¹²⁶

The next assumption that we make is that the *utility or real value of a rental property* declines at a geometric rate as the structure on the property ages. Thus the utility of a rental property with a new structure on it in period t is set equal to one and then its utility declines at a geometric rate as it ages. Thus for rental property n in period t that has a structure on it of age $A_{tn} = A(t,n)$ on it, its utility or *real quantity* q_{tn} as a function of the structure age is defined as follows:

$$(115) \quad q_{tn} \equiv (1-\Delta)^{A(t,n)} ; \quad t = 0,1; n \in S(t)$$

where Δ is the assumed geometric property depreciation rate that is due to structure depreciation. Thus in order to measure the rental property quantity and adjust it for the change in the quality of the structure over time, it is necessary to have an estimate for Δ . We will address this problem later in this section.¹²⁷

The rent for property $n \in S(t)$ in period t is R_{tn} and the corresponding quantity q_{tn} is defined by (115) so the *constant quality price* for property $n \in S(t)$ in period t is p_{tn} defined as the following value to quantity ratio:

$$(116) \quad p_{tn} \equiv R_{tn}/q_{tn} = R_{tn}/(1-\Delta)^{A(t,n)} ; \quad t = 0,1; n \in S(t).$$

Assuming that an estimate for the property depreciation rate Δ is available and the ages of the structures on the rental properties in scope are available, the prices and quantities defined by (116) and (115) can be used to form many indexes, depending on statistical agency preferences. Thus the *Maximum Overlap Laspeyres price index* is defined as follows:

$$(117) \quad \begin{aligned} P_{MOL} &\equiv \frac{\sum_{n \in S(0) \cap S(1)} p_{1n} q_{0n}}{\sum_{n \in S(0) \cap S(1)} p_{0n} q_{0n}} && \text{using (115) and (116) for } t = 0 \\ &= \frac{\sum_{n \in S(0) \cap S(1)} p_{1n} q_{0n}}{\sum_{n \in S(0) \cap S(1)} R_{0n}} && \text{using (116) for } t = 1 \text{ and (115) for } t = 0 \\ &= \frac{\sum_{n \in S(0) \cap S(1)} [R_{1n}/(1-\Delta)]}{\sum_{n \in S(0) \cap S(1)} R_{0n}} && \text{using definition (114)} \\ &= P_{RR}/(1-\Delta) \\ &> P_{RR} \end{aligned}$$

where the inequality follows since $0 < 1 - \Delta < 1$. Thus the simple repeat rents index P_{RR} defined by (114) will *understate* constant quality maximum overlap Laspeyres rental price inflation, P_{MOL} , defined by the first line in (117) by the factor $1/(1-\Delta)$ where Δ is the one period geometric property depreciation rate.

The *Maximum Overlap Paasche price index* is defined as follows:

¹²⁶ See Malpezzi, Ozanne and Thibodeau (1987) and the literature cited in thier paper.

¹²⁷ As indicated earlier, it may be possible to form an estimate for the property depreciation rate from a knowledge of the structure depreciation rate (obtained from national accounts information) and estimates of the relative value of the land and structure components of the rental properties in scope.

$$\begin{aligned}
(118) \ P_{MOP} &\equiv \sum_{n \in S(0) \cap S(1)} p_{1n} q_{1n} / \sum_{n \in S(0) \cap S(1)} p_{0n} q_{1n} \\
&= \sum_{n \in S(0) \cap S(1)} R_{1n} / \sum_{n \in S(0) \cap S(1)} [R_{0n}(1-\Delta)] && \text{using (115) and (116)} \\
&= P_{MO}/(1-\Delta) && \text{using definition (114).} \\
&> P_{MO}.
\end{aligned}$$

Using (117) and (118), we see that $P_{MOP} = P_{MOL}$. Define the *maximum overlap Fisher index* P_{MOF} as the geometric mean of the maximum overlap Laspeyres and Paasche indexes:

$$(119) \ P_{MOF} \equiv [P_{MOP}P_{MOL}]^{1/2} = P_{MOL} = P_{MOP} = P_{MO}/(1-\Delta)$$

where P_{MO} is the rent to value ratio for the properties that are in the sample for periods 0 and 1, $\sum_{n \in S(0) \cap S(1)} R_{1n} / \sum_{n \in S(0) \cap S(1)} R_{0n}$.

As a point of interest, define the *maximum overlap unit value price index*, P_{MOUV} , as follows:

$$\begin{aligned}
(120) \ P_{MOUV} &\equiv [\sum_{n \in S(0) \cap S(1)} R_{1n} / \sum_{n \in S(0) \cap S(1)} q_{1n}] / [\sum_{n \in S(0) \cap S(1)} R_{0n} / \sum_{n \in S(0) \cap S(1)} q_{0n}] \\
&= [\sum_{n \in S(0) \cap S(1)} R_{1n} / \sum_{n \in S(0) \cap S(1)} (1-\Delta)^{A(0,n)+1}] / [\sum_{n \in S(0) \cap S(1)} R_{0n} / \sum_{n \in S(0) \cap S(1)} (1-\Delta)^{A(0,n)}] \\
& && \text{using definitions (115)} \\
&= [\sum_{n \in S(0) \cap S(1)} R_{1n} / (1-\Delta)] / [\sum_{n \in S(0) \cap S(1)} R_{0n}] \\
&= P_{MO}/(1-\Delta).
\end{aligned}$$

Thus under the geometric property depreciation assumptions, the maximum overlap unit value price index P_{MOUV} is also equal to the string of indexes in (119) that are all equal to each other.

The above analysis indicates a way forward to deal with the depreciation of the structure problem. With an appropriate estimate for the average *property* depreciation rate Δ , we need only apply a simple adjustment to the aggregate rent ratio for properties present in both periods under consideration. Of course, our assumptions about the form of depreciation may not be very accurate *but making some adjustment for depreciation is better than making no adjustment at all*.

In order to deal with the problems arising from demolished and vacant rental units and newly constructed (or renovated) units, it is necessary to make more assumptions. The problem is: how can the quality of a new rental property relative to existing rental properties be determined in the period when the new property appears? Similarly in order to construct an estimate of the change in real rental services over the two periods under consideration, it is necessary to know what is the quality or utility of a rental unit which has disappeared relative to rental properties that continue to exist. In order to address these questions, the model of quality adjustment that is explained in sections 3 and 4 of Chapter 8 will be applied.¹²⁸

First, consider the case where there are only three rental properties in scope for periods 0 and 1. Property 1 is present in both periods, property 2 is present in period 0 but not in period 1 (a disappearing property) and property 3 is not present in period 0 but is present in period 1 (a new property).¹²⁹ Denote the real quantity of these three rental properties by q_C , q_D and q_N respectively using definitions (115) for the three properties.¹³⁰ We assume that renters value the relative

¹²⁸ See Diewert (2021).

¹²⁹ The “new” property 3 may not be a truly new property; it may be the case that property 3 was temporarily vacant in period 1. Similarly, property 2 may not permanently disappear in period 1; it may reappear in a subsequent period.

¹³⁰ Thus q_{0C} is set equal to $(1-\Delta)^{A(0,C)}$; $q_{1C} \equiv (1-\Delta)^{A(0,C)+1}$; $q_{0D} \equiv (1-\Delta)^{A(0,D)}$ and $q_{1N} \equiv 1$. Thus an estimate of the age of the rental properties is required along with an estimate for the geometric property depreciation

usefulness or utility of the various properties in scope by using the following *linear valuation* (or utility) *function*:

$$(121) f(q_C, q_D, q_N) \equiv \alpha_C q_C + \alpha_D q_D + \alpha_N q_N$$

where α_C , α_D and α_N are positive constants that reflect the relative value to renters of the 3 properties in scope in periods 0 and 1 and q_C , q_D and q_N are the real quantities for the 3 properties.

In period 0, suppose renters collectively maximize the utility function¹³¹ $f(q_C, q_D, q_N)$ defined by (121) with respect to q_C , q_D and q_N subject to the budget constraint $p_{0C}q_C + p_{0D}q_D = p_{0C}q_{0C} + p_{0D}q_{0D}$ and the non-availability constraint $q_N = 0$ where q_{0C} and q_{0D} are the property depreciation adjusted quantities for the two properties that are available for rent in period 0. The first order conditions for the observed (q_{0C}, q_{0D}) to solve this constrained utility maximization problem are the following conditions:

$$(122) \partial f(q_{0C}, q_{0D}, 0) / \partial q_C = \alpha_C = \lambda^0 p_{0C} ;$$

$$(123) \partial f(q_{0C}, q_{0D}, 0) / \partial q_D = \alpha_D = \lambda^0 p_{0D}$$

where $\lambda^0 > 0$ is the optimal Lagrange multiplier. It can be shown that $\lambda^0 = 1/P^0$ where P^0 can be interpreted as *the period 0 aggregate price level* for the active renters in period 0.¹³² Equations (122) and (123) can be rewritten as follows:

$$(124) p_{0C} = P^0 \alpha_C ;$$

$$(125) p_{0D} = P^0 \alpha_D .$$

In period 1, suppose renters again collectively maximize the utility function $f(q_C, q_D, q_N)$ defined by (121) with respect to q_C , q_D and q_N subject to the period 1 budget constraint $p_{1C}q_C + p_{1N}q_N = p_{1C}q_{1C} + p_{1N}q_{1N}$ and the non-availability constraint $q_D = 0$ where q_{1C} and q_{1N} are the property depreciation adjusted quantities for the two properties that are available for rent in period 1. The first order conditions for the observed (q_{1C}, q_{1N}) to solve this constrained utility maximization problem are the following conditions:

$$(126) \partial f(q_{1C}, 0, q_N) / \partial q_C = \alpha_C = \lambda^1 p_{1C} ;$$

$$(127) \partial f(q_{1C}, 0, q_N) / \partial q_N = \alpha_N = \lambda^1 p_{1N}$$

where $\lambda^1 > 0$ is the optimal period 1 Lagrange multiplier. Again, it can be shown that $\lambda^1 = 1/P^1$ where P^1 can be interpreted as *the period 1 aggregate price level* for the active renters in period 1. Equations (126) and (127) can be rewritten as follows:

$$(128) p_{1C} = P^1 \alpha_C ;$$

$$(129) p_{1N} = P^1 \alpha_N .$$

Note that equations (124), (125), (128) and (129) are a special case of Court's (1939; 109-111) hedonic quality adjustment suggestion number two. He transformed these underlying equations by taking

rate Δ . The corresponding prices are defined as $p_{0C} \equiv R_{0C}/q_{0C}$, $p_{0D} \equiv R_{0D}/q_{0D}$, $p_{1C} \equiv R_{1C}/q_{1C}$ and $p_{1N} \equiv R_{1N}/q_{1N} = R_{1N}$.

¹³¹ Alternatively, assume that each renter has the same linear preferences over alternative rental properties. It turns out that equations (124) and (125) will still be satisfied.

¹³² See Diewert (2021; 8-10).

logarithms of both sides of these equations in order to obtain the *classic time product dummy hedonic regression model*.¹³³

Looking at equations (124), (125), (128) and (129), it can be seen that we have 4 equations in 5 unknowns: the price levels P^0 and P^1 and the three relative quality parameters, α_C , α_D and α_N . Note each α_n measures the relative usefulness of an additional unit of product $n = C, D$ or N to purchasers of the 3 products. It can be seen that the P^1 and the α_n cannot all be identified using observable data; i.e., if P^0 , P^1 , α_C , α_D and α_N satisfy equations (124), (125), (128) and (129) and λ is any positive number, then λP^0 , λP^1 , $\lambda^{-1}\alpha_C$, $\lambda^{-1}\alpha_D$ and $\lambda^{-1}\alpha_N$ will also satisfy these equations. Thus it is necessary to place a normalization (like $P^0 = 1$ or $\alpha_C = 1$) on the 5 parameters which appear in these equations in order to obtain a unique solution. In the index number context, it is natural to set the price level for period 0 equal to unity and so we impose the following normalization on the 5 unknown parameters which appear in equations (124), (125), (128) and (129):

$$(130) P^0 = 1.$$

The unique solution to equations (124), (125), (128), (129) and (130) is:

$$(131) P^0 = 1; P^1 = p_{1C}/p_{0C}; \alpha_C = p_{0C}; \alpha_D = p_{0D}; \alpha_N = p_{1N}/(p_{1C}/p_{0C}) = p_{1N}/P^1.$$

Note that the resulting *price index*, P^1/P^0 , is equal to p_{1C}/p_{0C} , the price ratio for the commodity that is present in both periods. Thus the price index for this very simple model turns out to be a *maximum overlap price index*.¹³⁴

We now consider how companion quantity levels, Q^0 and Q^1 , for the price levels, P^0 and P^1 , can be determined. Define the *aggregate value of rents paid in period t* as V^t for $t = 0, 1$. Making use of the fact that $R_{0N} = 0$ and $R_{1D} = 0$, we have the following expressions for V^0 and V^1 :

$$(132) V^0 \equiv R_{0C} + R_{0D} = p_{0C}q_{0C} + p_{0D}q_{0D};$$

$$(133) V^1 \equiv R_{1C} + R_{1N} = p_{1C}q_{1C} + p_{1N}q_{1N}.$$

The quantity level Q^t for period t can be determined *directly* by evaluating the linear utility function defined by (121) at the period t quantity data or *indirectly* by deflating the period t aggregate value of rents V^t by the period t estimated price level, P^t :

$$(134) Q^0 \equiv \alpha_C q_{0C} + \alpha_D q_{0D} = p_{0C}q_{0C} + p_{0D}q_{0D} = [p_{0C}q_{0C} + p_{0D}q_{0D}]/P^0 = V^0/P^0 = V^0;$$

$$(135) Q^1 \equiv \alpha_C q_{1C} + \alpha_N q_{1N} = p_{0C}q_{1C} + [p_{1N}/P^1]q_{1N} = [p_{1C}/P^1]q_{1C} + [p_{1N}/P^1]q_{1N} = V^1/P^1 = V^1/[P^1/P^0]$$

where the various equalities in (134) and (135) follow by substituting equations (131)-(133) into the direct definitions for Q^0 and Q^1 . Thus real rents in period 0, Q^0 , are set equal to the aggregate

¹³³ For more accessible sources on the log price time product dummy hedonic regression model, see Griliches (1971) and Aizcorbe (2014). Summers (1973) proposed the same model in the international comparisons context where it is known as the *country product dummy model*. This model can also be viewed as a *repeat rent* model that is analogous to the *repeat sales* model that dates back to Bailey, Muth and Nourse (1963).

¹³⁴ Keynes (1930; 94) was an early author who advocated this method for dealing with new goods by restricting attention to the goods that were present in both periods being compared. He called his suggested method the *highest common factor method*. Marshall (1887; 373) implicitly endorsed this method. Triplett (2004; 18) called it the *overlapping link method*.

value of rents in period 0, V^0 , and real rents in period 1, Q^1 , are set equal to the aggregate value of rents in period 1, V^1 , deflated by the maximum overlap rent price index, P^1/P^0 , which in this case where there is only one rental unit in scope that is occupied in both periods, is equal to the following expression:

$$(136) P^1/P^0 = [R_{1C}/R_{0C}]/(1-\Delta).$$

An interesting aspect of this rent model is that the aggregate price and quantity levels, P^0 , P^1 , Q^0 , Q^1 , and the price index, P^1/P^0 , can all be determined by the national statistician using only information on collected rents (the R_{it}) and an estimate for the appropriate monthly geometric property depreciation rate, Δ . Thus detailed information on the characteristics of the rental dwelling units is not required in order to implement this very simple approach which is basically a modified repeat rents index.

It is useful to look at the quantity index, Q^1/Q^0 , that is implied by this simple model.¹³⁵ Using the final expressions in (134) and (135) and definitions (132) and (133), we have:¹³⁶

$$\begin{aligned} (137) Q^1/Q^0 &= [V^1/V^0]/[P^1/P^0] \\ &= [(R_{1C} + R_{1N})/(R_{0C} + R_{0D})]/[P^1/P^0] \\ &= (1-\Delta)[(R_{1C} + R_{1N})/(R_{0C} + R_{0D})]/[R_{1C}/R_{0C}] && \text{using (136)} \\ &= (1-\Delta)[1 + (R_{1N}/R_{1C})]/[1 + (R_{0D}/R_{0C})]. \end{aligned}$$

Thus there are three *growth factors* which determine the overall growth of real rentals :

- $(1-\Delta)$ which is 1 minus the rental property geometric depreciation rate; this factor will reduce the overall growth of real rentals.
- $1 + (R_{1N}/R_{1C})$ which is one plus the ratio of new rental value to continuing rental value in period 1; this growth factor will increase the overall growth of real rentals.
- $1 + (R_{0D}/R_{0C})$ which is one plus the ratio of disappearing rental value to continuing rental value in period 0; this growth factor is in the denominator and hence will decrease the overall growth of real rentals.

In a growing economy with new rental units being added to the marketplace, we would expect the ratio R_{1N}/R_{1C} to exceed the ratio R_{0D}/R_{0C} ; i.e., the availability of new rental¹³⁷ units should normally offset the loss of existing rental units due to demolition and temporary vacancies.

¹³⁵ It is important to construct companion aggregate quantity levels Q^t to complement the aggregate price levels P^t because the methodology outlined here will be applied to a local area or to a specific class of rental properties. These subindexes will have to be aggregated into a national index and in order to do that, it is necessary to have information on expenditure or quantity weights for the various sub-national indexes.

¹³⁶ Note that the decomposition given by (137) does not require a knowledge of A_{it} or any other rental housing characteristic. But the assumption of a common property depreciation rate implicitly implies that the rental properties in scope should have similar characteristics in order to justify the assumption of a common depreciation rate.

¹³⁷ A “new” rental unit includes a rental unit which was available in prior periods but vacant in period 0. Landlords sometimes circumvent local rent controls by renovating their properties so it may be prudent to use the above suggested quality adjustment procedure to capture such renovations rather than attempting to link the “new” rental unit to a prior period.

A problem with this simple model is that there is only one product that is present in both periods. However, it is possible to generalize the present model to allow for multiple overlapping products and for many new and disappearing rental units; see the Appendix for this generalization.

In the period following period 1, the same methodology can be applied to a new bilateral data set where the set of common rental properties in periods 1 and 2 will in general be different. New chain link price and quantity indexes can be calculated and linked up to previous price and quantity levels. Chain drift should not be a problem due to the fact that so many properties will be in the maximum overlap category and price and quantity changes will not be large as we move from period to period.

Thus this very simple rents model can in principle deal with the three big difficulties associated with the pure repeat rents model.

However, there are two main problems with this modified repeat rents model:

- *The model requires an appropriate geometric property depreciation rate.*
- *The model ignores other important characteristics of rental housing that may not remain constant over time, like renovations to the structure and changes in local amenities which affect the utility of the rental property.*

Note that the geometric depreciation rate is applied to the entire property rent which has to cover the user cost of both the structure and the land. Thus properties that have very different mixes of structure and land value will have different overall property depreciation rates. If the land structure mix were to remain constant over time, the assumption of a *property* depreciation rate may be adequate. But of course, the structure part of a property changes its *real value* due to depreciation whereas land does not depreciate. Moreover, the structure to land *nominal value* ratio is likely to change over time.¹³⁸

Thus we turn to a hedonic regression model to address these difficulties.

15. Price Indexes for Rental Housing: Hedonic Regression Approaches

The hedonic regression model that was explained in section 13 above can be applied to property rentals rather than property sales. Thus we now assume that, in addition to the age of the structure on rental property n in period t , $A_{tn} = A(t,n)$, information on the land area and the floor space area of property n in period t , L_{tn} and S_{tn} , is also available. *Quality adjusted structure floor space* for property n in period t , S_{tn}^* , is defined as follows:

$$(138) S_{tn}^* \equiv S_{tn}(1-\delta)^{A(t,n)}; \quad t = 0,1,\dots,T; n \in S(t)$$

where δ is the one period geometric depreciation rate for all structures for the rental properties in scope. The *utility* or *real quantity* of rental property n in period t , q_{tn} , is set equal to the following function of L_{tn} and S_{tn}^* :

$$(139) \begin{aligned} q_{tn} &\equiv L_{tn}^\alpha S_{tn}^{*\beta} && t = 0,1,\dots,T; n \in S(t) \\ &= L_{tn}^\alpha [S_{tn}(1-\delta)^{A(t,n)}]^\beta && \text{using definitions (138)} \\ &= L_{tn}^\alpha S_{tn}^\beta (1-\delta)^{\beta A(t,n)} \end{aligned}$$

¹³⁸ For information on the increasing share of land in housing prices for many economies over the period 1870-2012, see the important paper by Knoll, Schularick and Steger (2017).

$$= L_{tn}^{\alpha} S_{tn}^{\beta} \phi^{A(t,n)}$$

where α and β are positive parameters (which do not necessarily sum to one)¹³⁹ and the constant ϕ is defined as follows:

$$(140) \phi \equiv (1-\delta)^{\beta}.$$

The *constant quality price* of rental property n in period t , p_{tn} , is defined as rents paid, R_{tn} , divided by q_{tn} . The next assumption is that these constant quality prices move in a proportional manner (approximately). Thus we have the following assumptions:

$$(141) p_{tn} \equiv R_{tn}/q_{tn} \approx P^t ; \quad t = 0, 1, \dots, T; n \in S(t).$$

Thus the constant quality rental prices p_{tn} move in an *approximately proportional manner over time*, with the period t factor of proportionality equal to the scalar P^t . Thus P^t can be interpreted as the *price level* for rents in period t . The approximate equalities in equations (141) can be rewritten as the equalities $R_{tn} = P^t q_{tn} e_{tn}$ where e_{tn} is a positive error term with mean equal to 1. Taking logarithms of both sides of these equations leads to the following *time dummy hedonic regression*:

$$(142) \begin{aligned} \ln R_{tn} &= \ln P^t + \ln q_{tn} + \varepsilon_{tn} ; & t = 0, 1, \dots, T; n \in S(t) \\ &= \ln P^t + \alpha \ln L_{tn} + \beta \ln S_{tn} + (\ln \phi) A_{tn} + \ln e_{tn} \\ &= \rho^t + \alpha \ln L_{tn} + \beta \ln S_{tn} + \gamma A_{tn} + \ln e_{tn} \end{aligned}$$

where $\rho^t \equiv \ln P^t$ for $t = 0, 1, \dots, T$ and $\gamma \equiv \ln \phi = \beta \ln(1-\delta)$. The unknown parameters in (142) are the constant quality log rental price levels, $\rho^0, \rho^1, \dots, \rho^T$, and the taste parameters α , β and γ . Once these parameters have been determined, the geometric depreciation rate δ which appears in equation (139) can be recovered from the regression parameter estimates (β^* and γ^*) as follows:

$$(143) \delta^* \equiv 1 - e^{\gamma^*/\beta^*}.$$

An estimate for the property geometric depreciation rate Δ which appeared in equation (115) in the previous section can be obtained using the estimated structure depreciation rate δ^* defined by (143); i.e., solve the equation $(1-\Delta)$ equal to $(1-\delta^*)^{\beta^*}$ for Δ . The solution is:

$$(144) \Delta^* = 1 - (1-\delta^*)^{\beta^*}.$$

If $\beta^* = 1$, then $\Delta^* = \delta^*$. Typically $0 < \beta^* < 1$ in which case, the property depreciation rate Δ^* will be less than the structure depreciation rate δ^* . Thus it can be seen that the hedonic regression model approach to the construction of rental property indexes is not a totally different approach to the earlier matched model approach. The weakness of the matched model approach is that it requires an estimate for the property depreciation rate. It can be seen that the hedonic regression approach can generate an estimate for the property depreciation rate. Thus running an occasional hedonic regression of the form given by (142) will generate an estimate for the property

¹³⁹ Thus the utility function is a Cobb-Douglas function. The analysis in this section follows that of McMillen (2003; 289-290), Shimizu, Nishimura and Watanabe (2010; 795) and Diewert, Huang and Burnett-Isaacs (2017). McMillen assumed that $\alpha + \beta = 1$. The above authors applied their models to the sales of properties but the same model can be applied to property rents. We follow Shimizu, Nishimura and Watanabe in allowing α and β to be unrestricted.

depreciation rate Δ which played a prominent role in the modified repeat rents model outline in the previous section.

The estimated aggregate rental price levels for each period t , P^{t*} , generated by the hedonic regression defined by (142) are defined as the exponentials of the estimated ρ^{t*} :

$$(145) P^{t*} \equiv \exp[\rho^{t*}] ; \quad t = 0, 1, \dots, T.$$

The corresponding aggregate quantity levels Q^{t*} are defined as follows:

$$(146) Q^{t*} \equiv \sum_{n \in S(t)} R_{tn} / P^{t*} ; \quad t = 0, 1, \dots, T.$$

The corresponding *rental price indexes* for periods $t = 0, 1, \dots, T$ are defined as P^{t*} / P^{0*} .

If there were only one stratum and one hedonic regression, then it would not be necessary to calculate the aggregate quantity index Q^{t*} defined by (146). But there will be many strata (classified by location, type of structure and other characteristics) and so to form an aggregate Laspeyres, Paasche or Fisher index of rental prices, it will be necessary to calculate the P^{t*} and Q^{t*} by stratum and then use two stage aggregation to construct regional or national rental price indexes. Since the Laspeyres and Paasche indexes have an equal justification (and are the indexes that use the most representative weights for the two periods being compared), the Fisher index is recommended. It is a symmetric average of the Paasche and Laspeyres that satisfies the time reversal test.

To explain in more detail how the time product dummy model works, exponentiate both sides of equations (142) and drop the error terms. Then for each rental property $n \in S(t)$ in scope for period t , we have the following expression for the rent for property n in period t , R_{tn} :

$$(147) R_{tn} = P^t L_{tn}^\alpha S_{tn}^\beta (1-\delta)^{\beta A(t,n)} ; \quad n \in S(t) \\ = P^t q_{tn}$$

where $q_{tn} \equiv L_{tn}^\alpha S_{tn}^\beta (1-\delta)^{\beta A(t,n)}$ is the *real quantity* or *utility of rental property n in period t* , $L_{tn} = L_n$ is the land area of property n , $S_{tn} = S_n$ is the floor space area of property n , δ is the common geometric structure depreciation rate and $A(t,n) = A_{tn}$ is the age of the structure property n in period t . Thus the model (without error terms) assigns the same price, P^t , to each rental property in scope in period t . *Hence individual rental prices in this model will vary in a proportional manner over time.* Thus any reasonable matched model index number formula for the period t index relative to period 0 will be equal to P^t / P^0 . If $T = 1$, then it can be verified that the hedonic regression price index for period 1, P^1 / P^0 , will be equal to P_{MOL} defined by (117), P_{MOP} defined by (118), P_{MOF} defined by (119) and P_{MOUV} defined by (120).

It can be seen that the hedonic regression model approach to the construction of rental property indexes is not a totally different approach to the earlier matched model approach. The weakness of the matched model approach is that it required an estimate for the property depreciation rate. It can be seen that the hedonic regression approach can generate an estimate for the property depreciation rate.

The problems associated with the hedonic regression approach are twofold:

- Information on the characteristics of the rental properties is required.

- The hedonic regression model may not fit the data very well in which case we can conclude that the somewhat restrictive assumptions of the model do not provide an adequate approximation to reality.

If the sample of rental properties in scope is large enough, then set $T = 1$ and in this case, the hedonic regression model defined by (142) becomes a standard adjacent period time dummy hedonic regression. P^1/P^0 can be used to update the period 0 index level. If T is greater than 1, then we have a rolling window hedonic regression.¹⁴⁰ In this case, there is a problem in determining exactly how to link the results of the new regression in say period $T+1$ to the results of the previous regression for period T . A variety of linking methods have been suggested in the literature.¹⁴¹ In the present context, it is likely that the choice of method will not make a material difference.

We indicate how the above very simple log price hedonic regression model can be generalized to include additional (discrete) characteristics of the properties. Suppose that properties have been classified to 6 postal zones. If property n in period t belongs to postal zone j , then define the dummy variable $d_{j,tn}$ for observation n in period t to equal 1 and if property n in period t does not belong to postal zone j , then define the dummy variable $d_{j,tn}$ for observation n in period t to equal 0. Next, suppose that properties have been classified according to the number of bathrooms m in the structure where 6 is the maximum number of bathrooms. If property n in period t has i bathrooms, then define the dummy variable $d_{i,tn}$ for observation n in period t to equal 1 and if property n in period t does not have i bathrooms, then define the dummy variable $d_{i,tn}$ for observation n in period t to equal 0. Finally, suppose that properties have been classified according to the number of bedrooms k in the structure where the number of bedrooms ranges from 3 to 7. If property n in period t has k bedrooms, then define the dummy variable $d_{k,tn}$ for observation n in period t to equal 1 and if property n in period t does not have k bedrooms, then define the dummy variable $d_{k,tn}$ for observation n in period t to equal 0. Now consider the following generalization of the hedonic regression model defined by (142):

$$(148) \ln R_{tn} = \rho_t + \alpha \ln L_{tn} + \beta \ln S_{tn} + \gamma A_{tn} + \sum_{j=1}^6 \omega_j d_{j,tn} + \sum_{i=1}^6 \eta_i d_{i,tn} + \sum_{k=3}^7 \theta_k d_{k,tn} + \varepsilon_{tn} ; \quad t = 0, 1, \dots, T; n \in S(t).$$

The ω_j parameters affect the quality of the land component of property value while the last two sets of dummy variables affect the quality of the structure component of property value. Not all of the parameters ρ_t , ω_j , η_i and θ_k can be identified; i.e., there is exact multicollinearity associated with the dummy variables associated with these parameters. Thus to identify all of the remaining parameters, we make the following normalizations:

$$(149) \omega_4 \equiv 0; \eta_3 \equiv 0; \theta_5 = 0.$$

¹⁴⁰ See Shimizu, Nishimura and Watanabe (2010) for a worked example of this type of regression model applied to sales of properties rather than to rentals.

¹⁴¹ In the context of rolling window multilateral methods, Ivancic, Diewert and Fox (2011) (IDF) suggested that the movement of the indexes for the last two periods in the new window be linked to the last index value generated by the previous window. Krsinich (2016) suggested that the movement of the indexes generated by the new window over the entire new window period be linked to the previous window index value for the second period in the previous window. Krsinich called this a *window splice* as opposed to the IDF *movement splice*. De Haan (2015; 27) suggested that perhaps the linking period should be in the middle of the old window which the Australian Bureau of Statistics (2016; 12) termed a *half splice*. Finally IDF and Diewert and Fox (2020) suggested taking the geometric mean of all possible ways of linking the results of the new window to the results of the previous window. Diewert and Fox called this the *mean splice* and they thought that this would be the “safest” method of linking.

The model defined by (148) was applied to sales of properties in a suburb of Vancouver Canada and it gave reasonable results for the implied structure depreciation rate; see Diewert, Huang and Burnett-Isaacs (2017). Variants of this model should also work well for rentals of properties.

The results of the present section and the previous section can be summarized as follows:

- The repeat rents model studied in the previous section can be applied provided that some adjustment for the aging of the rental structure is made.
- Hedonic regressions which regress the logarithms of rents on the characteristics of the rental properties plus a time dummy variable can be run for various segments of the rental market. The estimated time dummy coefficients can be converted into period by period price levels which in turn can be converted into rental property price indexes.

The underlying economic structure of the hedonic regression approach can be explained as follows. All renters in the segment of the rental market in scope have the same utility function, $u = f(S, L, A, X, Y, Z)$, where S , L and A are the floor space area, land area and age of the rental property and X , Y and Z are other characteristics of the property. In period t , renters compete with each other to equalize the observed rent to utility ratio for each property. Thus we have for each rental property n in period t the following approximate equalities:

$$(150) R_{tn}/f(S_{tn}, L_{tn}, A_{tn}, X_{tn}, Y_{tn}, Z_{tn}) \approx P_t ; \quad t = 0, 1, \dots, T; n \in S(t)$$

where P_t is the common period t rent to utility ratio across the rental properties in scope. As was seen above, P_t can be interpreted as the period t price level for the properties in scope. It can be seen that equations (150) can be turned into the following (possibly nonlinear) regression model:

$$(151) \ln R_{tn} = \ln P_t + \ln f(S_{tn}, L_{tn}, A_{tn}, X_{tn}, Y_{tn}, Z_{tn}) + e_{tn} ; \quad t = 0, 1, \dots, T; n \in S(t)$$

where the e_{tn} are error terms.

In the following section, we turn our attention to the problems associated with valuing the services of Owner Occupied Housing.

16. Owner Occupied Housing: The User Cost Perspective

Owner Occupied Housing (OOH) is a consumer durable good so the opportunity cost approach to the valuation of the services of a consumer durables that was explained in section 5 could be applied to this valuation problem. Recall that the opportunity cost valuation of an owned consumer durable is simply the *maximum* of the foregone rental or leasing price for the services of the durable during a period of time and the corresponding user cost for the durable. In the previous two sections, the focus was on generating price indexes for rental dwelling units. One approach to the valuation of the services of an owned dwelling unit is to *impute* a rent to it using the rent of a comparable rented unit. This is the *rental equivalence approach* to the valuation of the services of an owned dwelling unit. A second approach to this valuation problem is to construct *user costs* for owned dwelling units. The second approach will be explored in this section.

There are a number of difficulties in applying the usual durables user cost theory to housing:

- Each owned dwelling unit is a unique good due to its unique location and the fact that the structure depreciates over time (and renovations may be undertaken over time).
- Each owned dwelling unit does not trade in each time period. Thus precise period by period market opportunity costs are not readily available.
- An owned dwelling unit is a composite commodity made up of separate land and structure components. In general the price trends in these two components will be different.

In order to deal with the above difficulties, typically, some form of econometric modeling will be required. Thus suppose that some form of hedonic regression on sales of owned dwelling units in scope has been undertaken such as the various builder's models explained in sections 11 and 12 above. Suppose that we have information on a sample of owned properties in scope for periods t and $t+1$ and there are N properties in the sample. We assume asset prices, P_{Ltn} and P_{Stn} ¹⁴² can be assigned to the land and structure areas, L_{tn} and S_{tn} , that can be imputed for rental dwelling n in period t . The aggregate user cost U_{tn} is approximated by the sum of the (end of period) user cost components for land and structures, u_{Ltn} and u_{Stn} respectively. The geometric model of depreciation for structures is used and the one period depreciation rate is $0 < \delta < 1$. The depreciation rate for land is 0. The age of the structure for rental unit n in period t is $A(t,n)$ periods. Setting the *overall user cost value* of unit n in period t and $t+1$ to the sum of the corresponding land and structure user costs leads to the following equations:

$$(152) U_{tn} = u_{Ltn}L_{tn} + u_{Stn}(1-\delta)^{A(t,n)}S_{tn}; \quad n = 1, \dots, N$$

$$= [r_t - i_{Lt}]P_{Ltn}L_{tn} + [r_t - i_{St} + (1+i_{St})\delta]P_{Stn}(1-\delta)^{A(t,n)}S_{tn};$$

$$(153) U_{t+1n} = u_{Lt+1n}L_{tn} + u_{St+1n}(1-\delta)^{A(t,n)+1}S_{tn}; \quad n = 1, \dots, N$$

$$= [r_{t+1} - i_{Lt+1}]P_{Lt+1n}L_{tn} + [r_{t+1} - i_{St+1} + (1+i_{St+1})\delta]P_{St+1n}(1-\delta)^{A(t,n)+1}S_{tn}$$

where r_t is the opportunity cost of capital for the owners of the owned properties in period t and i_{Lt} and i_{St} are the land and structure price inflation rates that owners expect at the beginning of period t . Note that the land and structure areas for unit n , L_{tn} and S_{tn} , typically do not change over time. It is well known in the housing literature that user costs for dwelling units are much more volatile than the corresponding rents for the same units.¹⁴³ Thus in order for the user costs U_{tn} and U_{t+1n} to approximate their market rents (if they were rented), it is necessary to use a nominal smoothed values for the nominal interest rates r_t and particularly for the expected asset inflation rates, i_{Lt} and i_{St} .¹⁴⁴ Note that the quantity of constant quality structure for property n in periods t and $t+1$ are $S_{tn}^* \equiv (1-\delta)^{A(t,n)}S_{tn}$ and $S_{t+1n}^* \equiv (1-\delta)^{A(t,n)+1}S_{tn}$; i.e., the imputed constant quality amount of structure constant quality quantity declines as time increases. The corresponding constant quality amount of land rent, L_{tn} , remains constant over all periods. To form a constant quality overall price index for user costs, calculate Laspeyres, Paasche or Fisher indexes where the price data for periods t and $t+1$ are the vectors $[u_{L1t}, \dots, u_{LNt}; u_{S1t}, \dots, u_{SNt}]$ and $[u_{L1t+1}, \dots, u_{LNt+1}; u_{S1t+1}, \dots, u_{SNt+1}]$ and the quantity data for periods t and $t+1$ are the vectors $[L_{1t}, \dots, L_{Nt}; (1-\delta)^{A(t,1)}S_{1t}, \dots, (1-\delta)^{A(t,N)}S_{Nt}]$ and

¹⁴² P_{Stn} is the price of a square meter of *new* structure of the type used by owned unit n at the beginning of period t .

¹⁴³ On this point, see Genesove (2003), Verbrugge (2008), Shimizu, Nishimura and Watanabe (2010b), Diewert and Nakamura (2011), Garner and Verbrugge (2011) and Suzuki, Asami and Shimizu (2018).

¹⁴⁴ The expected land inflation rate i_{Lt} should be an average of land price inflation over the past 15 to 25 years to reflect the long holding periods that investors have for rental properties and the high transactions costs of buying and selling properties. Diewert and Fox (2018) used a rolling window annualized 25 year inflation rate for land for the 25 years prior to period t to generate very smooth estimates for the expected land inflation rate in their user costs for land in the US.

$[L_{t1}, \dots, L_{tN}; (1-\delta)^{A(t,1)+1} S_{t1}, \dots, (1-\delta)^{A(t,N)+1} S_{tN}]$. Adjustments for new housing and demolitions can be made as well.

It can be seen that it is not a simple matter to implement the user cost approach to valuing the services of OOH. However, at the national level, it may be possible to use national balance sheet estimates for the value of Owner Occupied Housing and for the value of OOH structures. Thus the value of OOH land can be obtained by subtracting the value of OOH structures from the total OOH property value. A rough approximation to the price of OOH land can be obtained as the OOH value of land since the quantity of land in use for housing purposes will not change much from period to period.¹⁴⁵ Aggregate price and quantity indexes for structures used by home owners may be available from the national accounts of the country if the country has a system of Total Factor Productivity accounts.¹⁴⁶ However, this information may only be available on a quarterly or annual basis and on a delayed basis, which limits the usefulness of this information for the construction of a monthly CPI.

However, monthly information on housing sales is often collected by private companies (such as real estate associations). This information usually includes information on housing characteristics. Thus it becomes possible to implement hedonic regression models along the lines explained in sections 11 and 12 above and the information from these regressions can be used in order to implement simplified user cost approaches. It should be noted that Iceland has used a simplified user cost approach to value the services of OOH in its CPI for many years without encountering opposition to the use of user costs.¹⁴⁷

17. Valuing the Services of OOH: User Costs versus Rental Equivalence

In this section, the various factors that cause the user cost of an owned dwelling unit to differ from a rental price for a comparable property will be examined.¹⁴⁸ In addition, other factors that affect user costs for house in general will be discussed.¹⁴⁹

- Utilities such as electricity, water and natural gas may be included in the rent for a dwelling unit that is similar to an owned unit. The user cost of an owned unit should exclude these costs since these expenditures are covered in other categories of a Consumer Price Index.
- When calculating the user cost of the owner of a dwelling unit of renting the unit, there is the problem of determining what is the correct market rental opportunity cost. It turns out

¹⁴⁵ For an example of this methodological approach to obtaining housing land price indexes, see Knoll, Schularick and Steger (2017).

¹⁴⁶ The use of user costs to measure capital input in production accounts can be traced back to Dale Jorgenson and his coauthors; see Hall and Jorgenson (1967), Christensen and Jorgenson (1969) and Jorgenson (1989).

¹⁴⁷ See Gudnason and Jonsdottir (2011). Simplified user costs are also discussed in Diewert (2005a), Verbrugge (2008) and Hill, Steurer and Walzl (2020).

¹⁴⁸ Our discussion here is similar to that of Hill, Steurer and Walzl (2020) who note that the services a household obtains from renting a dwelling are not necessarily the same as the services obtained by an owner-occupier. One difference between our analysis and their analysis is that their user cost formula is a single user cost formula that applies to the entire property. However, depreciation affects only the structure part of rents and if one attempts to adjust a market rent for this aging factor, it is necessary to apply the depreciation adjustment only to the structure part of rents.

¹⁴⁹ There are many papers that compare user costs with equivalent rents. For U.S. studies see Verbrugge (2008) (2012), Garner and Verbrugge (2009) (2011) and Adams and Verbrugge (2021). For comparisons for Belgium, see Goeyvaerts and Buyst (2019) and for Ireland, see Coffey, McQuinn and O'Toole (2020).

that all rents paid in say period t for comparable units to an owned unit can be classified into 3 categories: (i) the rental agreement is not being renegotiated during this period; (ii) the rental agreement is renegotiated during this period with the same tenants and (iii) the rental agreement is a new one with new tenants. Typically, there are no escalations of rents for continuing tenants during the leasehold period and often, renegotiated rents with continuing tenants are also sticky; i.e., there is not much change in these renegotiated rents.¹⁵⁰ For purposes of measuring the user cost of an owner of renting an owned unit, category (iii) rents should be used as the appropriate comparable market rent.¹⁵¹

- Property taxes will be included in market rents and they should also be included in an owner's user cost.
- Normal maintenance expenditures on the structure will be part of market rents. These expenditures should not be included in an owner's user cost for a dwelling unit which is being used by the owner since these expenditures by home owners should already be included in other expenditure categories in the CPI. Landlords may also have considerable overhead expenses that are associated with the management of rental properties. These expenses can perhaps be grouped together with maintenance expenditures.
- The structure depreciation rate for rented dwelling units may be higher than the rate for comparable owned dwelling units, since owners are likely to take better care of their property and will avoid property damage. This expected difference in the value of depreciation should be deducted from the market rent that is applied to a comparable owned home.
- The owners of rental properties need to charge a small premium to the rents that they receive from rented units in order to cover the loss of rental income due to vacancies. This vacancy premium does not apply to the user cost of an owned unit and thus the comparable market rent for an owned unit should be adjusted downward to account for this vacancy factor.
- Insurance payments are included in market rents. However, in the CPI, insurance payments made by owner occupiers of their dwelling units will typically be included in another category so in this case, the imputed insurance premiums should be deducted from the market rent that is applied to a comparable owned home.
- The opportunity cost of capital for a landlord and for an owner living in a dwelling unit may be different. A landlord who rents properties to tenants may include a risk premium in his or her cost of capital to account for possible downturns in the rental market.
- It is likely that there is an *owner's premium* to owning rather than renting. A poor person may not qualify for a mortgage loan to purchase a dwelling unit so he or she is forced to rent rather than purchase. A richer person has the choice between renting or owning a dwelling unit of the same quality. If the richer person is risk averse, he or she will probably prefer to own the same quality dwelling unit rather than renting to eliminate the transactions costs of moving if evicted. The risks of unforeseen increases in rents demanded by the landlord are also eliminated by owning rather than renting. This factor

¹⁵⁰ On the stickiness of rents, see Shimizu, Nishimura and Watanabe (2010b), Lewis and Restieaux (2015; 72-75), Gallin and Verbrugge (2019), Coffey, McQuinn and O'Toole and Suzuki, Asami and Shimizu (2021). Lewis and Restieaux label their three categories as (i) Occupied Let, (ii) Renewal and (iii) New Let. Their category (i) is a stock measure that includes all occupied rental units while their categories (ii) and (iii) match up with categories (ii) and (iii) in the text above. Rents in categories (ii) and (iii) may be subject to rent controls which means that rents in these categories do not reflect current opportunity costs. The problems caused by rent controls are discussed by Diaz and Luengo-Prado (2008) and Coffey, McQuinn and O'Toole (2020).

¹⁵¹ However, when constructing a rental price index for renters, rents for all 3 categories should be used.

may help to explain why property investors do not purchase high end properties for rental purposes: there is a lack of demand to rent expensive properties and thus user costs for the landlord cannot be covered by market rents for high end properties.

Recall that the *total user cost* of dwelling unit n in period t was U_{tn} defined by (152).¹⁵² Define period t property *value* of the same property n , V_{tn} , as the sum of its land value and structure value:

$$(154) V_{tn} \equiv P_{Ltn}L_{tn} + P_{Stn}(1-\delta)^{A(t,n)}S_{tn} ; \quad n = 1, \dots, N.$$

where P_{Ltn} is the price per meter squared of a unit of land and P_{Stn} is the price per meter squared of a unit of new structure of the type on property n for period t . Define the period t , property n *land and structure shares of total property value* as:

$$(155) s_{Ltn} \equiv P_{Ltn}L_{tn}/V_{tn} ; s_{Stn} \equiv P_{Stn} (1-\delta)^{A(t,n)}S_{tn}/V_{tn} ; \quad n = 1, \dots, N.$$

Then using (152) and the above definitions, the ratio of total user cost to property value for property n in period t can be written as follows:

$$(156) U_{tn}/V_{tn} = [r_t - i_{Lt}]s_{Ltn} + [r_t - i_{St} + (1+i_{St})\delta]s_{Stn} ; \quad n = 1, \dots, N.$$

Recall that r_t is a smoothed longer term opportunity cost of capital for period t , i_{Lt} is the long term expected land price inflation rate, i_{St} is a long term expected structure price inflation rate and δ is the geometric structure depreciation rate. The *rent to capital value ratio* or *capitalization rate*¹⁵³ defined by (156) does not take into account the complications that were discussed above; i.e., the *user cost* U_{tn} that would apply to an owner occupier of dwelling unit n in period t is not equal to the *rent* R_{tn} that a landlord would charge to a tenant for the same dwelling unit. Thus it is necessary to modify (156) to take into account these complications. Define v_t as the period t rate of expected loss of rental income due to vacancies (as a fraction of period t capital value), define m_{tn} as expected period t maintenance and overhead expenditures for property n divided by the corresponding period t structure value,¹⁵⁴ define the land tax rate τ_{Ltn} as the ratio of land taxes paid by the owners of property n in period t to the imputed land value $P_{Ltn}L_{tn}$ and the structure tax rate τ_{Stn} as the ratio of structure property taxes paid in period t for property n to imputed structure value, $P_{Stn} (1-\delta)^{A(t,n)-1}S_{tn}$. Finally, define π_{tn} as the ratio of insurance payments made in period t by property n to imputed structure value, $P_{Stn} (1-\delta)^{A(t,n)-1}S_{tn}$. Using the above discussion on complications to the standard user cost model, it can be seen that a more meaningful rent to value ratio decomposition for property n in period t is given by the following modification of (156) for $n = 1, \dots, N$:

$$(157) R_{tn}/V_{tn} = [r_t - i_{Lt} + v_t + \tau_{Ltn}]s_{Ltn} + [r_t - i_{St} + (1+i_{St})\delta + v_t + \tau_{Stn} + m_{tn} + \pi_{tn}]s_{Stn} .$$

¹⁵² For convenience, we repeat this formula: $U_{tn} = [r_t - i_{Lt}]P_{Ltn}L_{tn} + [r_t - i_{St} + (1+i_{St})\delta]P_{Stn} (1-\delta)^{A(t,n)}S_{tn}$.

¹⁵³ Crone, Nakamura and Voith (2000) used hedonic techniques to estimate both a rent index and a selling price index for housing in the U.S. They also suggested that *capitalization rates* (i.e., the ratio of the market rent of a housing property to its selling price) can be applied to an index of housing selling prices in order to obtain an imputed rent index for OOH. As will be shown below, capitalization rates are functions of many variables, some of which can change considerably over time. Also, it will be seen that capitalization rates for rented houses are not exactly appropriate as estimators for capitalization rates for owned houses.

¹⁵⁴ Older structures will probably have higher m_{tn} ratios.

If property tax payments are not a separate category in the CPI, then the appropriate user cost for an owner of property n in period t , U_{tn} , as a fraction of property value, V_{tn} , is equal to the following expression:

$$(158) U_{tn}/V_{tn} = [r_t - i_{Lt} + \tau_{Ltn}]S_{Ltn} + [r_t - i_{St} + (1+i_{St})\delta + \tau_{Stn}]S_{Stn}$$

Note that the terms v_t , m_{tn} and π_{tn} have been dropped from (158). Thus the differences between (157) and (158) are equal to the following expressions for $n = 1, \dots, N$:

$$(159) R_{tn}/V_{tn} - U_{tn}/V_{tn} = v_t + [m_{tn} + \pi_{tn}]S_{Stn}.$$

It can be seen that simply applying the rent of a comparable rented dwelling unit to an owned unit will overstate the appropriate user cost that should be applied to the owned unit. The above computations did not take into account the possibility that the depreciation rate for a rental property is greater than the corresponding depreciation rate for a similar owned property.

The user cost formulae defined by (157)-(159) look rather complicated and they require information that may not be available to the statistician. Thus additional assumptions may have to be made which allow approximate user costs for owned dwelling units to be calculated. In situations where equivalent rental prices are not available, this may be the only feasible method to value the services of OOH. For example, the European Union issued the following regulation in 2005 that gives guidance in forming estimates of the services of OOH when equivalent rental prices are not available:

“Under the user-cost method, the output of dwelling services is the sum of intermediate consumption, consumption of fixed capital (CFC), other taxes less subsidies on production and net operating surplus (NOS). For owner occupied dwellings, no labour input is recorded for work done by the owners (1). Experience suggests that CFC and NOS are the two largest items, each representing 30 to 40 % of output.

CFC should be calculated based on a perpetual inventory model (PIM) or other approved methods. A separate estimate for the owner-occupied residential buildings should be available. The net operating surplus should be measured by applying a constant real annual rate of return of 2.5% to the net value of the stock of owner-occupied dwellings at current prices (replacement costs). The real rate of return of 2.5% is applied to the value of the stock at current prices since the increase in current value of dwellings is already taken account of in the PIM. The same rate of return should be applied to the value of the land at current prices on which the owner-occupied dwellings are located.

The value of land at current prices may be difficult to observe annually. Ratios of land value to the value of buildings in different strata may be derived from an analysis of the composition of the costs of new houses and associated land.” Eurostat (2005).

To value the services of OOH in Iceland, the highly simplified user cost formula $U_t = (r_t^* + \delta)P_t$ was used, where U_t is the period t property user cost, r_t^* is a real interest rate (varied between 3.6% and 4.3%), δ is an annual property depreciation rate (set equal to 1.25%) and P_t is a period t constant quality property price index.¹⁵⁵

¹⁵⁵ See Gudnason and Jonsdottir (2011; 148). Note that as in the case of Iceland, the depreciation rate is applied to total property value and not to just the structure value. This may be an acceptable approximation if the shares of land and structure in total property value remain roughly constant over time. However, the empirical results of Knoll, Schularick and Steger (2017) on house price inflation in 14 advanced economies indicate that the share of land has increased substantially in recent years.

The Office for National Statistics in the UK used the user cost formula $U_t = (r + m + \delta - i)P_t$ to value the services of OOH, where r is a rate of return which includes a risk premium, δ is a depreciation rate, m is the maintenance rate, i is the expected capital appreciation rate of the unit and P_t is a period t property price index.¹⁵⁶ For other simplified user cost formulae, see Verbrugge (2008) and Garner and Verbrugge (2009). When they set i equal to expected CPI inflation, reported rents approximated the corresponding user costs fairly well.

Returning back to the user cost formulae defined by (157) and (158), there is another factor which will tend to make the user cost valuation of the services of an owned dwelling unit much bigger than the corresponding actual rental price: households who rent tend to be poorer than households who own. *Thus renters simply cannot afford to rent high end housing units.* High end dwelling units that do rent will tend to rent for prices that are much less than their long run user costs.¹⁵⁷ In advanced countries, the rent to property value ratio for the more expensive properties tends to be about one half the rent to property value ratio for the least expensive properties.¹⁵⁸ Thus it is likely that the widespread use of the rental equivalence approach to the valuation of the services of owner occupied housing results in a measures of the value of housing services which are much lower than valuations based on long run user costs.

There is one additional troublesome issue that has not been discussed thus far and that is the issue of what to do with transfer costs. Transfer costs are the costs associated with the purchase of a dwelling unit. These costs include transactions taxes, legal fees and real estate agent fees. These costs can be substantial. Thus when a household purchases a dwelling unit, the final cost of the purchase should include all of the associated transfer costs. According to user cost theory, the appropriate valuation of the property at the end of the period should be the value of the sale of the house after transfer costs. This viewpoint suggests that the transactions costs of the purchaser should be immediately expensed in the period of purchase. However, from the viewpoint of a landlord who has just purchased a dwelling unit for rental purposes, it would not be sensible to charge the tenant the full cost of these transaction fees in the first month of rent. The landlord would tend to capitalize these costs and recover them gradually over the time period that the landlord expects to own the property. Thus take the capitalized transfer costs that are charged to property n in period t and divide by total property value V_{tn} to obtain the imputed property transfer cost ratio, λ_{tn} . The new rental cost formula for rented unit n in period t , the counterpart to (157), becomes the following formula:

$$(160) R_{tn} = [r_t - i_{Lt} + v_t + \tau_{Ltn} + \lambda_{tn}]P_{Ltn}L_{tn} + [r_t - i_{St} + (1 + i_{St})\delta + v_t + \tau_{Stn} + m_{tn} + \pi_{tn} + \lambda_{tn}]P_{Stn}(1 - \delta)^{A(t,n) - 1}S_{tn}.$$

From the viewpoint of an owner of a newly purchased dwelling unit, the owner does not actually sell the unit in the next period; the owner holds on to the dwelling unit for periods that range from 10 to 20 years on average. Thus it is probably best to regard the transfer costs as a fixed cost that should be amortized over the expected holding period before the dwelling unit is sold again. If this amortisation is appropriate, then the new user cost formula that is the counterpart to (158) is the following formula which should be used to value the services of the owned unit if it is not rented out to tenants:

¹⁵⁶ See Lewis and Restieaux (2015; 156). We have changed their notation to match up with our notation.

¹⁵⁷ Often high end houses that are not being used by their owners are rented out at prices that are far below their user costs just so someone will be in the house to maintain it and deter theft and vandalism. This is the “caretaker” explanation for falling ratios of rents to property value as property values increase.

¹⁵⁸ See Heston and Nakamura (2009) (2011). Aten (2018) found similar results for the US. Shimizu, Diewert, Nishimura and Watanabe (2012) found that user cost valuations for OOH in Tokyo were about 1.7 times as big as the equivalent rent estimates.

$$(161) U_{tn} = [r_t - i_{Lt} + \tau_{Ltn} + \lambda_{tn}] P_{Ltn} L_{tn} + [r_t - i_{St} + (1 + i_{St})\delta + \tau_{Stn} + \lambda_{tn}] P_{Stn} (1 - \delta)^{A(t,n)-1} S_{tn} .$$

The above discussion indicates that it is not a straightforward matter to determine the conceptually correct rental equivalent price to value the services of an owned dwelling unit.¹⁵⁹

18. The Payments Approach and the Household Costs Index

A fifth possible approach to the treatment of Owner Occupied Housing in a CPI, the *payments approach*, was described by Goodhart as follows:

“The second main approach is the payments approach, measuring actual cash outflows, on down payments, mortgage repayments and mortgage interest, or some subset of the above. ... Despite its problems, such a cash payment approach was used in the United Kingdom until 1994 and still is in Ireland.” Charles Goodhart (2001; F350-F351).

Thus the *payments approach* to owner occupied housing is a modified *cash flow approach* to the *costs* of operating an owner occupied dwelling.¹⁶⁰ It consists mainly of mortgage interest and principal payments along with property taxes. Imputations for capital gains, for the cost of capital tied up in house equity and depreciation are ignored in this approach. This leads to the following objections to this approach; i.e., it ignores the opportunity costs of holding the equity in the owner occupied dwelling, it ignores depreciation and it uses nominal interest rates without any offset for anticipated changes in the price of land and the structure over the accounting period. In general, due to its omission of depreciation, the payments approach will tend to lead to smaller monthly expenditures on owner occupied housing than the rental equivalence, user cost and opportunity cost approaches, except during periods of high inflation, when the nominal mortgage rate term may become very large without any offsetting item for possible house price inflation.¹⁶¹ This feature of the payments approach makes it unsuitable for measuring the services of OOH in a cost of living index.

The payments approach (like the acquisitions approach) is not a suitable approach if the goal of consumer price measurement is to measure the *flow* of consumption services. The rental equivalence, user cost and opportunity cost approaches are useful for measuring the flow of consumption services. The acquisitions approach is useful for central bank monitoring of marketplace consumer price inflation due to its avoidance of imputations (except imputations for quality change are allowed).

¹⁵⁹ For a more comprehensive decomposition of the user cost formula for an owned dwelling unit with a mortgage on the unit, see Diaz and Luengo-Prada (2008), Diewert, Nakamura and Nakamura (2009), Diewert and Nakamura (2011) and Goeyvaerts and Buyst (2019).

¹⁶⁰ It is not a true cash flow approach because it omits the outlays for the purchase of a dwelling unit and it omits the potential benefits from the eventual sale of the unit. The Office for National Statistics (ONS) in the United Kingdom correctly labels this class of index as a *Household Costs Index* (HCI). The ONS describes this type of index as follows: “More specifically, they will aim to measure how much the nominal disposable income of different household groups would need to change, in response to changes in costs, to enable households to purchase the same quantity of goods and services of the same quality. Put simply, the broad approach of the HCI is to measure the outgoings of households.” ONS (2017; 2).

¹⁶¹ See the comparison of alternative OOH price indexes for the United Kingdom using the rental equivalence approach and the payments approach made by the ONS (2017; 10) (2018; 3). The latter publication also implements the acquisitions approach and compares the three indexes for the UK. The payments approach index is much more volatile than the other two indexes.

The current corona virus pandemic has created an important use for the payments approach, which as indicated above, is essentially a cash flow approach; i.e., how much money is required to allow a home owner to cover the out of pocket costs associated with home ownership. For households who own their own home and lose their sources of income due to government mandated lockdowns of sectors of the economy, it would be useful for the government to have estimates of the cash costs of keeping pandemic affected home owners in their dwelling units. However, note that what is required to meet this purpose are estimates of actual household costs rather than an index of their costs.

Another rationale for the payments approach has been developed by Astin and Leyland and we outline it below.

Astin and Leyland (2015; 1) labelled their index version of the payments approach as a *Household Inflation Index* (HII) and they described it as a measure of “inflation as perceived and experienced by households in their role as consumers”. Thus broadly speaking, they wanted to produce a consumer price index which would more closely reflect consumer *experience* and *perceptions* of the inflation that they are experiencing. On page 3 of their paper, they outlined more specifically how their HII would differ from say the European Union’s Harmonized Index of Consumer Prices (HICP) which Astin was instrumental in setting up:

- The HII would be a democratic index rather than a plutocratic index;¹⁶²
- Interest paid on car loans, student loans and credit cards are household expenditures which would be in scope for their index;
- The HII would include domestic household tourist expenditures abroad and exclude the consumption expenditures of foreign tourists in the home country;¹⁶³
- The HII would include gross insurance premiums paid by households for cars, travel and health.¹⁶⁴

¹⁶² This terminology dates back to Prais (1959). In practical terms, what the authors suggested is that national statistical agencies should construct separate consumer price indexes for different groups of households that are demographically homogeneous. This is sensible advice. The demographic groups should be further classified into at least two subgroups depending on whether the households are renters or owners of dwelling units. The owners of dwelling units could be further decomposed into groups depending on the size of their mortgage debt. Owners of houses with no outstanding mortgages do not require the same compensation to maintain their level of housing service consumption as renters. As cash transactions become obsolete, banks and other financial institutions that issue household credit and debit cards will have information on household purchases at the individual household level. Thus in the future, it will become easier to construct consumer price indexes for groups of households classified by their demographic characteristics and location.

¹⁶³ Including expenditures made by foreign visitors in a CPI is called the *domestic treatment* of household transactions and excluding foreign visitor expenditures while including national expenditures made by national residents abroad is called the *national treatment*. Thus Astin and Leyland argued for the national treatment of tourist expenditures in their CPI concept. On the other hand, Astin (1999, 6-7) argued for the domestic treatment of tourist expenditures for the HICP, which is satisfactory if one wants an inflation index which is suitable for central bank monitoring of inflation. Diewert (2002; 595-596) argued that the domestic perspective was appropriate if one wanted a measure of consumer price inflation from a domestic producer perspective but the national perspective was preferred for a measure of consumer inflation faced by residents in the country under consideration.

¹⁶⁴ The gross premiums approach simply uses the total premium amount as the value of a property insurance policy held by a household. The net premiums approach subtracts either actual claims or the expected value of payments for claims on the policies in force for the period under consideration. From a national accounts perspective, the net claims approach can be justified. But the gross claims approach can be justified on a consumer theory basis; see Diewert (1993; 415-423). However, in either case, the

Astin and Leyland (2015) suggest that if the main purpose of a CPI is for the national indexation of pensions and only one CPI is available for this purpose, then a democratic CPI is better for this purpose than the usual plutocratic CPI.¹⁶⁵ Note that interest paid on car loans would be explicitly included in a user cost approach to household vehicle services and interest on capital tied up would be implicitly included in the monthly or annual fee for a leased car. Thus interest payments made explicitly or implicitly by households appear in the non payment approaches to the treatment of durables.

Astin and Leyland (2015; 3, 22) also made the following specific suggestions on how expenditures on OOH should be treated in their proposed HII; their proposed HII should include the following categories of household expenditure:

- Total mortgage payments (interest and principal) for the dwelling;
- The transactions costs associated with the purchase of a house (transactions taxes; legal fees; real estate agent fees);
- State and local property taxes;
- Insurance;
- Spending on renovations and extensions;
- Minor repairs and maintenance.

Typically, the payments approach applied to owner occupied housing would not include the principal component of mortgage payments but Astin and Leyland properly note that these payments are *experienced* by households and hence they advocated including *total* mortgage payments in their Household Inflation Index.

The transactions costs associated with the purchase of a house should be in scope for an acquisitions CPI as well as in a CPI that was based on the user cost approach.¹⁶⁶ If the OOH component of the CPI were based on the rental equivalence approach, these transactions costs may be partially included in the imputed rent applied to the owned dwelling unit.¹⁶⁷

State and local property taxes paid by home owners on a continuing basis are definitely part of the costs of the services of owned housing and should be included in the user cost approach to housing. These costs are implicitly included in the rental equivalence approach.

separation of the net or gross premium payments into price and quantity components is a complex matter where standard practice has not yet emerged. For example, suppose the risk associated with a claim increases over time. Should the price of the policy be quality adjusted downward which would be consistent with insurance services as a payment per unit risk?

¹⁶⁵ A plutocratic CPI implicitly gives a higher expenditure weight to the consumer price index of a well off household. In theory, a democratic CPI should give an equal weight to all households when forming the aggregate CPI. However, rather than producing a democratic CPI, if enough information on the spending habits of different groups is available, then it may be preferable to apply a separate CPI that reflected the spending habits of the particular group under consideration; i.e., *it may be preferable to publish CPIs for different demographic groups.*

¹⁶⁶ Conceptually, these transactions costs should be amortized over the expected holding period for a house purchase if one uses the user cost approach.

¹⁶⁷ However, the transactions costs of purchasing a rental property could have a longer amortization period if the rental property were held by the landlord for a longer time period than the average holding period for an owner of a property using the property to provide personal housing services.

Property insurance costs are imbedded in rents and so these costs are included in market rents. Thus using the rental equivalence approach to OOH, housing insurance payments should not be added to the equivalent rent. However, if the user cost approach is used for valuing the services of OOH, then housing insurance payments should be included in the user cost formula (along with property taxes). If insurance payments are a separate elementary category in the CPI, housing insurance payments could be included in the insurance subindex; i.e., it is necessary to avoid double counting of household expenditures in constructing a CPI.

Household expenditures on renovations and extensions of an owned dwelling unit should be taken into account in a CPI. If a user cost approach is being used, then these expenditures should be applied to the structure component of the overall property user cost; i.e., these expenditures should be deflated and added to the owned structure stock for the following period. Thus a renovation to an owned property should lead to an increase in the real quantity of the structure on the property but it may be difficult to capture this quality improvement using the rental equivalence approach. Depending on the details of how the rental equivalence approach to OOH is being implemented, it may be necessary to treat household expenditures on renovations of an owned dwelling unit as a separate category in the CPI. These expenditures should be amortized but it may be acceptable to simply treat these expenditures as current expenditures instead of recognizing that the benefits of these renovation expenditures extend over time. Minor repairs and maintenance also have benefits that extend over time but the time horizon of these benefits will tend to be relatively short and so immediate expensing of these expenditures is an acceptable approximation.

The above discussion of the Astin and Leyland proposal shows that many aspects of their suggested index are reasonable and not entirely inconsistent with the other approaches to the treatment of durables that we have considered in this paper.¹⁶⁸ However, while their proposed Household Inflation Index is a reasonable index that can reflect household experience and perceptions of inflation, it is not an index that can measure household consumption of the services of durable goods because it focuses on the immediate costs associated with the purchase of durable goods and ignores possible future benefits of these purchases. Thus the payments approach does not lead to indexes which are suitable for indexation purposes.

The Office for National Statistics (ONS) in the United Kingdom has basically implemented much of the Astin and Leland proposed Household Inflation Index on an ongoing basis¹⁶⁹ and compared their new index with traditional acquisition and rental equivalence type CPIs; see the ONS (2018). However, the ONS (properly) recognized that the HII is focused on *costs* and so they renamed the index as a Household Costs Index (HCI). The ONS describes their HCI in a methodology paper as follows:

“The Household Costs Indices (HCIs) are a set of experimental measures, currently in development 1, that aim to more closely reflect UK households’ experience of changing prices and costs. More specifically, they will aim to measure how much the nominal disposable income of different household groups would need to change, in response to changes in costs, to enable households to purchase the same quantity of goods and services of the same quality. Put simply, the broad approach of the HCIs is to measure the outgoings of households.” Office for National Statistics (2017; 2).

¹⁶⁸ For a more complete discussion of the Astin and Leyland proposals, see the ONS (2017).

¹⁶⁹ See the Office for National Statistics (2018).

The ONS (2017; 2) noted that its HCI differs from a traditional consumer price index¹⁷⁰ that uses the rental equivalence approach to the treatment of OOH in the following four ways:

- The use of democratic weighting;
- The use of a payments approach for measuring owner occupiers' housing costs (OOH);
- The inclusion of a measure of interest costs on credit card debt;
- The use of gross expenditure to calculate the weight for insurance premiums.

The above dot points show that the ONS Household Costs Index is very similar to the Astin and Leyland Household Inflation Index. Both indexes are versions of the payments approach. One major difference is that the ONS treatment of the payments approach includes mortgage interest on owned dwellings but excludes repayment of principal (whereas the HHI includes repayment of principal).¹⁷¹

The ONS cautions users that there are problems with the use of the payments approach:

“Using a payments-based approach is commonly considered to be the best construct for assessing changes in net money incomes over time. This is in line with the stated aims of the HCIs, as briefly set out in section 1 of this article. However, the inclusion of nominal interest payments on mortgage debt is not without its problems conceptually. Its inclusion has been criticised as the treatment of interest flows is not consistent across persons (or households). For example, Charles Goodhart (2001) describes that if a borrower is worse off in some way when interest rates rise, then equivalently a lender owning an interest bearing asset is better off, and it may be analytically unsound to include one but not the other.” Office for National Statistics (2017; 10).

The Goodhart objection to the payments approach is similar to our major objection: the approach measures the *costs* facing households but does not always recognize possible offsetting *benefits* that may accrue to households. However, a payments type index can be useful as an index of household outlays and hence *perceptions of inflation*, which was the reason why Astin and Leyland introduced their version of the payments approach to the measurement of household inflation.

The ONS compares its versions of the rental equivalence, acquisitions and payments approaches to the measurement of the services of owner occupied dwellings on a regular ongoing basis; see ONS (2018; 3) for a chart of the three types of index for the UK over the years 2005-2018 on a quarterly basis. This chart shows the volatility of the payments based index as compared to the other two indexes. The rental equivalence index shows a steady upward growth with the net acquisitions index being slightly more volatile and finishing above the rental equivalence index. The payments index finished up far below the other two indexes. This work by the ONS shows that the choice of methodology for the treatment of OOH in a CPI matters.

The ONS has provided a number of publications that explain in some detail both the rationale for the four main approaches to the treatment of OOH as well as data sources and methods; see ONS (2016) (2017) (2018). These publications should be useful for statistical agencies who are

¹⁷⁰ The traditional CPI that the ONS uses for comparison purposes (which they call the CPIH) is identical to Eurostat's Harmonized Index of Consumer Prices (HICP) except that the services of OOH are measured by the rental equivalence approach plus local property taxes (Council Taxes); see the ONS (2016; 3). The HICP simply omits the services of OOH.

¹⁷¹ See the ONS (2017; 8-9). The ONS payments approach to OOH is compared to the rental equivalence approach for the UK over the years 2006-2016. In the future, the ONS intends to produce HCIs with and without principal payments.

planning to offer alternative analytical indexes for the treatment of Owner Occupied Housing in a consumer price index. However, some comments on how the ONS constructs its rental equivalence and acquisitions indexes for OOH may be useful.

The ONS (2016; 33) explains that it constructs its *net acquisitions approach index for OOH* as follows: prices are based on a price index for new house sales but the weights for these prices are set equal to the value of residential construction during the time period under consideration. The underlying price concept which the ONS would like to implement for its net acquisitions index is the price of the structure component of new dwelling unit sales to owners of houses who live in them. In other words, the land component of the selling price is to be stripped out of the sale price. The ONS recognizes that its empirical measures of price and expenditure are flawed for this treatment of OOH: the prices collected are sales of new dwelling units to *all* purchasers (purchasers who intend to live in the dwelling unit and hence are in scope and purchasers who plan to rent the dwelling unit to tenants and hence are not in scope for OOH) and more importantly, the selling prices of new dwelling units include a land component which is supposed to be excluded. The residential investment weights are also flawed because the investment includes investments in new rental units which should be excluded. The reason for the above desired treatment of the acquisitions approach applied to new dwelling units is that Eurostat would like to implement this net approach¹⁷² to new house sales for its Harmonized Index of Consumer Prices.¹⁷³ A possible better solution to implementing this pricing concept is to simply use the deflator for residential building investments which is already constructed by countries as part of their national accounts. This deflator could be improved if the residential building price index could be decomposed into two strata: one stratum for sales intended for purchasers who plan to live in the new residential structure and another stratum for investments in rental properties. But even if this latter decomposition of the residential construction price index were not made, using an overall residential construction price index along with estimates for the value of new rental buildings and for total residential construction¹⁷⁴ would lead to a price index which should be much closer to the desired (by Eurostat) price index for OOH. The above limitations of the ONS acquisitions price index for OOH should be kept in mind when looking at their chart for the acquisitions, rental equivalence and payments indexes for OOH in the UK; see the ONS (2018; 3).¹⁷⁵

There are also problems with the ONS (2018; 3) rental equivalence price index series. In ONS (2016; 21-23), the ONS explained how it constructed its rental equivalence index. A sample of rental prices is collected across the UK and then the prices are stratified according to: (i) type of dwelling unit,¹⁷⁶ (ii) postal code; (iii) number of bedrooms and (iv) furnished or unfurnished.

¹⁷² It is a net approach because the gross purchase price of a new dwelling unit is to be net of the land price component of the selling price. It is also a net approach because it excludes intra-household sales of residential housing units.

¹⁷³ There is already an EU regulation that requires member countries to produce such a monthly acquisitions type index for OOH but since not all EU countries are yet able to comply with the regulation, the current HICP still ignores OOH.

¹⁷⁴ The OOH expenditure weight could be obtained by subtracting the value of rental residential investment from total residential investment value. A possible reason for not implementing this version of the net acquisitions approach to OOH is that national statistical agencies are not in a position to produce a monthly construction cost index in a timely manner.

¹⁷⁵ It is likely that the ONS (2018; 3) acquisitions index has an upward bias relative to the Eurostat target net acquisitions index because the ONS price index has a substantial land price component in it which will reflect rapidly increasing land prices in the UK over the sample period.

¹⁷⁶ The four categories are: (i) detached house; (ii) semi-detached house; (iii) terraced house and (iv) flat or maisonette.

Given our earlier discussion of the application of hedonic regression models to the construction of house price indexes and rental indexes, it can be seen that the list of stratifying characteristics is not ideal. The number of bedrooms can act as a proxy for floor space area but there is no information on land plot area and no information on the age of the structure. The latter omission is particularly important. The evidence from hedonic regressions for both selling prices and rental prices indicates that the aging of the structure leads to a quality decline in structure service of about 1% per year for a residential property. Thus if the land and structure components of property value are equal, the neglect of structure depreciation could lead to a downward bias of about 0.5% per year in a rental price index that does not take into account the quality decline due to aging of the property. This is a substantial bias. The ONS should stratify rental properties according to the age of the structure in order to take this bias into account (or move to a hedonic regression framework with the age of the structure as an explanatory variable).

There is another potential bias in the ONS rental equivalence index for OOH. The rental equivalence approach to valuing the services of OOH is an *opportunity cost approach*. The choice to live in an owned dwelling unit rather than rent it out means that the owner of the structure is giving up *the current market rent* that the owner of the unit could get if the unit were rented. This is the appropriate opportunity cost from the viewpoint of the rental equivalence approach to valuing the services of an owned dwelling unit. Thus the appropriate opportunity cost is the *current* rent for a property that is similar to the owned property to a new tenant but the opportunity cost that the ONS (2016) uses is the average of all *existing* rental prices for similar properties.¹⁷⁷ The latter average will tend to be lower than new rents if there is rental price inflation and higher if there is rental price deflation.¹⁷⁸ Thus the ONS procedures undervalue the rental opportunity costs of living in an owned dwelling unit under conditions of general inflation.¹⁷⁹

Recall the discussion in the previous section that compared the rental equivalence approach to the opportunity cost approach to the valuation of owned housing services. The opportunity cost approach sets the true opportunity cost of living in an owned dwelling unit as the maximum of its market rental price and its user cost. In many countries, the ratio of house rent to property value approximately doubles as we move from less expensive to more expensive properties.¹⁸⁰ This means that, in general, the rental equivalence approach to the valuation of OOH will give a much smaller expenditure *weight* to the services of OOH as compared to the user cost and opportunity cost approaches.

The above limitations of the ONS rental equivalence price index for OOH should be kept in mind when looking at the ONS charts for the acquisitions, rental equivalence and payments indexes for OOH in the UK; see the charts in ONS (2018; 3).

¹⁷⁷ Existing (contractual) rental prices are appropriate for valuing rental properties in a CPI. But they are not appropriate for use in the rental equivalence approach (except as an approximation): the rental equivalence approach requires the use of current opportunity costs, not historical costs.

¹⁷⁸ The ONS is well aware of this difference: “There is an important difference between newly let properties and existing tenants; price rises are highest when properties are newly let compared with existing tenants renewing a lease.” Office for National Statistics (2016; 50).

¹⁷⁹ The use of all contract rents instead of renewal contract rents to value the services of a house will lead to a lower *weight* in the CPI (under conditions of general inflation) but it may not affect the corresponding rate of *change* in the price index.

¹⁸⁰ See footnotes 157 and 158 in the previous section.

We conclude this section by reviewing some issues concerning the timing of payments made by households for the consumption of durable goods. Consider the following quotation from the ONS:

“Consumption expenditure can be measured in three ways which it is important to distinguish. These ways are:

Acquisition means that the total value of all goods and services delivered during a given period is taken into account, whether or not they were wholly paid for during the period.

Use means that the total value of all goods and services consumed during a given period is taken into account.

Payment means that the total payments made for goods and services during a given period is taken into account, whether or not they were delivered.

For practical purposes, these three concepts cannot be distinguished in the case of non-durable items bought for cash, and they do not need to be distinguished for many durable items bought for cash. The distinction is, however, important for purchases financed by some form of credit, notably major durable goods, which are acquired at a certain point of time, used over a considerable number of years, and paid for, at least partly, some time after they were acquired, possibly in a series of instalments. Housing costs paid by owner-occupiers are an obvious example.” Office for National Statistics (2010; 6).

In what follows, we will look at the problems associated with the three methods of valuation in a number of specific cases.¹⁸¹

Case 1: The payment period coincides with the acquisition period. Let P_1 be the acquisition price for such a unit of a durable good in period 1. Then the acquisition price in period 1 is obviously P_1 , the payments price is also P_1 and the period 1 user cost price is p_1 and its exact form depends on the model of depreciation that is applicable for this particular durable good. In other words, there are no problems in sorting out the three methods of valuation in this case.

Case 2: The initial payment period coincides with the acquisition period but payments for the purchase of the durable continue on for subsequent periods. Suppose that payments must be made for T periods and the sequence of monetary payments is $\pi_1, \pi_2, \dots, \pi_T$. Suppose also that the sequence of expected one period financial opportunity costs of capital for the purchasing household is r_1, r_2, \dots, r_{T-1} . Then the discounted stream of payments, P_1 , is the period 1 (expected) cost of purchasing the good where P_1 is defined as follows:

$$(162) P_1 \equiv \pi_1 + (1+r_1)^{-1}\pi_2 + (1+r_1)^{-1}(1+r_2)^{-1}\pi_3 + \dots + (1+r_1)^{-1}(1+r_2)^{-1} \dots (1+r_{T-1})^{-1}\pi_T.$$

In this case, the acquisitions price for the durable good in period 1 is defined to be P_1 , the payments price is π_1 and the user cost will be determined using the appropriate depreciation model, where P_1 is taken to be the beginning of the period price for the durable good. In a subsequent period $t \leq T$, the acquisitions price for the used durable good will be 0, the payments price will be π_t and the period t user cost value v_t will be determined using the appropriate depreciation model for this type of durable good. *If* the useful life of the durable good happens to equal T and *if* the period t payment is equal to the corresponding period t user cost valuation v_t for

¹⁸¹ We will address the problems from the viewpoint of the approach to intertemporal consumption theory that dates back to Hicks (1946).

$t = 1, 2, \dots, T$, then obviously, the period t user cost valuation v_t will be equal to the observable period t payment π_t .¹⁸²

There are problems associated with the computation of the P_1 defined by (162); i.e., in order to compute P_1 when the durable good is purchased during period 1, the sequence of future payments π_t has to be known and guesses will have to be made on the magnitudes of the sequence of expected nominal interest rates r_t . However, the important point to be made here is that P_1 defined by (162) will be *less* than the simple sum of the π_t , $\sum_{t=1}^T \pi_t$, provided that the nominal interest rates r_t are positive.

Case 3: The full payment for the good (or service) is made in period 1 but the services of the commodity are not delivered until period t . Let the period 1 payment be π_1 as usual. Thus the sequence of payments associated with the purchase of the commodity under consideration is π_1 for period 1 and 0 for all subsequent periods. The acquisition of the commodity does not take place until period t but the appropriate acquisition price P_t is not the period 1 payment, π_1 , but the following *escalated period 1 price*:

$$(163) P_t \equiv (1+r_1)(1+r_2) \dots (1+r_{t-1})\pi_1.$$

The logic behind this valuation is the following one. During period 1 when the product was paid for, the payment could have been used to pay down debt (at the interest rate r_1) or the payment could have been used to invest in an asset that earned the rate of return r_1 . Thus after one period, the opportunity cost of the investment in the pre-purchased product has grown to $\pi_1(1+r_1)$, after two periods, the opportunity cost has grown to $\pi_1(1+r_1)(1+r_2)$, ..., and by period t when the good or service is acquired, the opportunity cost has grown to $\pi_1(1+r_1)(1+r_2) \dots (1+r_{t-1})$, which is (163). The important point to be made here is that P_t will be *greater* than the period 1 prepayment, π_1 , provided that the nominal interest rates r_t are positive. Since the product has not been acquired by the household for periods $1, 2, \dots, t-1$, the corresponding user cost valuations, v_1, v_2, \dots, v_{t-1} , should be set equal to 0. However, when period t is reached, "normal" user costs can be calculated for durable goods using the P_t defined by (163) as the beginning of period t price of the durable, assuming that the form of depreciation is known.

Prepayment for services or durable goods is widespread; e.g., trip and hotel reservations made in advance and paid for in advance are service examples and prepayment for condominium units that are under construction is a durable good example.

Case 4: The good or service is acquired in period 1 but is not paid for until period 2. In this case, the sequence of payments is $0, \pi_2, 0, \dots, 0$. The commodity is acquired in period 1 and the appropriate period 1 acquisition price is P_1 defined as follows:

$$(164) P_1 \equiv (1+r_1)^{-1}\pi_2.$$

The justification for this acquisition price runs as follows: The purchasing household lays aside the amount of money P_1 to buy the product in period 1. This money is invested and earns the one period rate of return r_1 . Thus when period 2 comes along, the household has $P_1(1+r_1) = \pi_2$ which

¹⁸² The period t user cost valuation v_t for a unit of the durable good that is t periods old can be converted into an equivalent amount of a new unit of a durable good if the geometric or one hoss shay model of depreciation is applicable for the durable good under consideration. Otherwise, units of the durable good of different ages at the same point in time need to be aggregated using an index number formula.

is just enough money to complete the purchase in period 2. Thus P_1 is an appropriate period 1 acquisitions price. If the commodity is a durable good, then assuming that the form of depreciation is known, P_1 defined by (164) can be used as the beginning of period 1 price for the period 1 user cost and the entire sequence of user costs can be calculated.

This form of pricing is used as a way of offering lower prices for a wide variety of products. A particular application of this model to a service is the use of credit cards to purchase consumption items. A household that pays its balance owed on time can avoid interest charges and thus can postpone payment for its household purchases for up to one month in many cases.¹⁸³

If interest rates are very low, then statistical agencies may well find it is not worth taking into account the above refinements. However, if nominal interest rates are high, it may be necessary to make some of the above adjustments.¹⁸⁴

It can be seen that the durability of housing creates a host of measurement problems that statistical agencies are not well equipped to handle.

19. The Treatment of Household Monetary Balances in a CPI

The treatment of financial services in a Consumer Price Index is a controversial topic. The academic literature has not come to a general consensus on how to model many financial services provided to households. However, given the importance of financial services in all economies, it may be useful to outline some of the issues surrounding this topic.

We will concentrate on household banking services in this section.¹⁸⁵ It is clear that many services that banks provide to households are reasonably simple to model; i.e., it is straightforward to collect prices on the costs of using the services of a safety deposit box. It is not so straightforward to measure the services of bank household deposit services or bank loans to households. However, it is possible to adapt the basic user cost theory explained in section 4 above to model the services of household transferable deposits¹⁸⁶ and time or savings deposits held in banks or other financial institutions.

Recall from section 4 that r^0 was the household's opportunity cost of financial capital at the beginning of period 0. In the national accounts banking literature, r^0 is called the *household reference rate of return on safe assets* for the period under consideration. We assume that the bank providing household deposit services pays the deposit holder an interest rate of r_D^0 on its holdings of bank deposits of the type under consideration at the end of the accounting period. For a checking account, r_D^0 will typically be equal to zero. For a savings or time deposit account, r_D^0 will typically be a number that is less than r^0 .¹⁸⁷ Then the *beginning of the period user cost* u_D^0 of holding a dollar of deposits (on average) throughout period 0 is:¹⁸⁸

¹⁸³ However, a household that does not pay off its balance owed in a timely fashion will find itself in Case 3 above.

¹⁸⁴ We note that the above adjustments for the timing of payments have implications for the system of national accounts that have not been fully worked out.

¹⁸⁵ There are also important controversies surrounding the treatment of insurances services in a CPI.

¹⁸⁶ Before internet banking became popular, these deposits were called checking deposits.

¹⁸⁷ Under current conditions, for some countries, r_D could be a small negative number. For most countries that exhibit low inflation, r_D will be a small positive number.

¹⁸⁸ This user cost of money dates back to Diewert (1974), who did not include the deposit interest rate term, r_D^0 . This extra term was introduced by Donovan (1978) and Barnett (1978) (1980).

$$(165) u_D^0 \equiv 1 - (1+r_D^0)/(1+r^0) = (r^0 - r_D^0)/(1+r^0).$$

The above user cost looks at the opportunity cost of holding a dollar of bank deposits at the beginning of the accounting period (as opposed to investing the dollar at the rate of return of r^0 or to paying off outstanding debts at the interest rate of r^0) but at the end of the accounting period, the deposit holder gets the dollar back plus interest r_D^0 earned in tying up that dollar for the period but this amount, equal to $1+r_D^0$, needs to be discounted by one plus the opportunity cost of capital, $1+r^0$.

As usual, instead of discounting costs and benefits to the beginning of the accounting period, the costs and benefits can be anti-discounted to the end of the accounting period, which leads to the following *end of the period user cost* u_D^{0*} of holding a dollar of deposits throughout the period:

$$(166) u_D^{0*} \equiv (1+r^0)u_D^0 = (r^0 - r_D^0).$$

Define the household's nominal *asset value* of bank deposits held at the beginning of period 0 as V_D^0 and define the corresponding nominal value of *deposit services* for period 0 as v_D^0 . Given the end of period user cost for a bank deposit, p_D^0 , and the (asset) value of household bank deposits at the beginning of period 0, V_D^0 , the *imputed (nominal) value of bank deposit services from the household perspective*, v_D^0 , is defined as the product of p_D^0 and V_D^0 :

$$(167) v_D^0 \equiv u_D^{0*}V_D^0 = (r^0 - r_D^0)V_D^0.$$

The end of period user cost of holding a dollar's worth of bank deposits defined by (166) and the corresponding value of total deposit services defined by (167) are derived using a household opportunity cost perspective.

The question which now arises is: what is the real value of deposit services to the household; i.e., what is the appropriate deflator for the nominal service flow v_D^0 defined by (167)? The answer to this question is not clear cut.

In order to answer the above question, it is necessary to ask what the *purpose* of the deposit holdings is. Feenstra (1986) and others provide an answer to this purpose question: cash balances or their deposit equivalents are held in order to buy consumer goods and services. The idea here is that consumers receive income flows from selling their labour services or from dividend and bond interest payments at regular intervals. These income flows are converted into cash or bank deposits at the beginning of the payment period and then are spent over the course of the payment period in order to purchase consumer goods and services. This is termed a *cash in advance model*. Thus if the household purpose in holding bank deposits is to buy consumer goods and services, then it seems reasonable to deflate V_D^0 by the corresponding period 0 aggregate consumer price level (excluding financial services), P_C^0 say, to obtain the equivalent amount of real consumption that the nominal value of deposit balances, V_D^0 , could purchase; i.e., define the *consumption equivalent* of the household's nominal deposit balances, q_D^0 , as follows:¹⁸⁹

$$(168) q_D^0 \equiv V_D^0/P_C^0.$$

¹⁸⁹ Feenstra (1986) provided a formal model of a cash in advance economy that justifies the deflation of nominal household bank balances by a consumer price index. Alternatively, we can make a simple opportunity cost argument to justify deflating V_D by P_C : by holding deposits, the household gives up current consumption. Note that the conceptually correct CPI to do the deflation should be based on the *acquisition approach* to the construction of a CPI.

Now deflate the value of household deposit *services*, v_D^0 defined by (167), by q_D^0 in order to obtain the *price for bank deposit services from the household perspective* p_D^0 defined as follows:

$$\begin{aligned}
 (169) \quad p_D^0 &\equiv v_D^0/q_D^0 \\
 &= [(r^0 - r_D^0)V_D^0]/[V_D^0/P_C^0] && \text{using (167) and (168)} \\
 &= (r^0 - r_D^0)P_C^0.
 \end{aligned}$$

Note that the price level for deposit services for period 0, p_D^0 , is proportional to the consumer price level for goods and services in period 0, P_C^0 . The corresponding real value of deposit services for period 0, q_D^0 , is set equal to the period 0 nominal household stock of monetary balances, V_D^0 , deflated by the consumer price level for period 0, P_C^0 .¹⁹⁰ We note that the data variables which appear in equations (167)-(169) are all relatively easy to measure, with the exception of the reference rate or opportunity cost of financial capital interest rate, r^0 . There is no easy answer on how exactly to measure this interest rate.¹⁹¹

The cash in advance approach to modeling the demand for monetary services can be applied to the household demand to hold currency and transferable deposits. Since many time deposit bank accounts also allow households to use these deposits to buy goods and services, the above model could also be applied to these accounts. To get a rough idea of the relative size of these two types of monetary accounts and their relationship to total annual purchases of consumer goods and services, the data from the Integrated Macroeconomic Accounts for the US for the year 2019 can be used; see the Bureau of Economic Analysis (2020). For 2019, final consumption expenditures were 14.56 trillion dollars; household holdings of currency and transferable deposits were 1.26 trillion dollars and holdings of time and savings deposits were 10.16 trillion dollars. It can be seen that these holdings of household monetary assets are much larger than the amounts that cash in advance models would predict. Thus households are holding large amounts of bank deposits for reasons other than for the purpose of funding their normal purchases of consumer goods and services.

Monetary theory suggests several additional reasons for consumers to hold currency and bank deposits:

- As a *store of value*; i.e., to save up funds for future major purposes such as buying an automobile or house;
- For *precautionary purposes*; i.e., as a form of self insurance against future income shocks;
- For *portfolio balancing purposes*.

The above purposes reflect the fact that a large fraction of consumer holdings of currency and bank deposits are probably held for investment purposes broadly speaking, rather than as a means of facilitating current period purchases of consumer goods and services. Thus statistical agencies constructing a CPI may want to rule holdings of currencies and deposits as being out of scope. On the other hand, it would be useful for statistical agencies to produce a supplementary CPI which

¹⁹⁰ This user cost approach to modeling the price, quantity and value of household monetary services is due to Donovan (1978), Barnett (1978) (1980), Fixler (2009) and Barnett and Chauvet (2011). For discussions on how the user cost approach to modeling monetary services in both the household and production accounts, see Fixler and Zieschang (1991) (1992) (1999), Diewert, Fixler and Zieschang (2011) (2016), Diewert (2014) and Diewert and Fox (2018) (2019).

¹⁹¹ See the discussion between Fixler (2009), Basu (2009) and Wang, Basu and Fernald (2009).

includes the services of monetary deposits along the lines indicated above because household holdings of monetary deposits have a direct opportunity cost in foregone consumption and including monetary services in a broader measure of consumption would be useful for some analytic purposes.

It should be mentioned that not all economists subscribe to the above user cost approach for modeling the household demand for monetary services. The Basu, Fernald, Inklaar and Wang approach to modeling bank outputs and inputs is critical of the above deflation based user cost approach to modeling the price and quantity of financial services presented in this section.¹⁹² Rather than defining the real quantity of financial services as being proportional to suitably deflated stocks of financial assets held by banks or households, the above authors suggest that *direct measures* of the services rendered by consuming financial services be constructed (such as the number of transactions) and then the nominal service flows would be deflated by these direct measures, yielding an implicit price index for the services, as an alternative to deflating nominal asset holdings by a price index.¹⁹³ We have two responses to this methodology:

- Direct transactions fees are taken into account separately in our suggested user cost approach (although some free services may be omitted in this approach) and
- The transactions fee approach seems to be a cost of production approach which is not necessarily relevant for consumers of the service.

However, economists have not settled on a universally accepted methodology for modeling the household demand to hold bank deposits, so statistical agencies need to keep this fact in mind.

20. Summary and Conclusion

It is clear that constructing constant quality price indexes for consumer durables is not as conceptually simple as constructing price indexes for nondurables and services where the matched model approach can guide index construction. The fundamental problem of accounting arises when constructing a price index for the services of a durable good: *imputations will have to be made in order to decompose the initial purchase cost into period by period service flow components over the life time of the durable good.* The method of imputation will involve assumptions which may not be accepted by all interested parties. In spite of this difficulty, it will be useful for statistical agencies to construct analytical series for the services of long lived consumer durables that can be made available to the public. This will meet the needs of different users.¹⁹⁴

When constructing property price indexes based on sales of properties, there is another factor that reinforces the argument for multiple price indexes: when transactions are sparse, property indexes based on the sparse data can be very volatile. Thus for some purposes, it may be useful to

¹⁹² See Wang (2003), Wang, Basu and Fernald (2009), Basu (2009), Basu, Inklaar and Wang (2011), Inklaar and Wang (2010) and Colangelo and Inklaar (2012).

¹⁹³ See Inklaar and Wang (2010) and Colangelo and Inklaar (2012) for empirical estimates of the differences between the demand side deflation approach and an approach incorporating ‘engineering’ indicators of financial service delivery.

¹⁹⁴ Hill, Steurer and Waltl (2020), using Australian data, found substantial differences using the three main approaches to the valuation of OOH. This emphasizes the need for statistical agencies to produce estimates for all three approaches if possible.

construct a smoothed index (that is revised for a certain number of months) in addition to a volatile real time index.¹⁹⁵

For non-housing consumer durables, at present, statistical agencies produce consumer price indexes based on the *acquisitions approach*. This type of index is useful for measuring consumer price inflation based on market transactions, with minimal imputations (except for possible quality change). In addition to this standard index, statistical agencies should produce supplementary indexes based on the *user cost approach* in order to more accurately measure the flow of services generated by stocks of consumer durables.¹⁹⁶

The valuation of the services of housing is very difficult due to the fact that housing services are unique: the location of each dwelling unit is unique and the location affects the land price component of the property and thus affects rents and user costs. Moreover, the structure component of housing does not remain constant over time due to depreciation of the structure and to renovation expenditures. Various methods that can deal with these difficulties (to some degree at least) were explained in sections 11-17. The details of the methods are too complex to summarize here but the suggested methods based on various hedonic regression models have been applied and offer possible ways forward.

For Owner Occupied Housing, the three main approaches should be implemented by statistical agencies to serve the needs of different users. There are two possible versions for the *acquisitions approach* depending on how the treatment of new dwelling purchases is treated: (i) construct a price index for the purchase of new dwelling units in an inclusive basis, including the price of land or (ii) exclude land cost from the purchase cost. The latter index should be well approximated by a construction cost index (with appropriate margins added for developer margins). The inclusive index will be useful for new house buyers, who have to pay for the land plot as well as the new structure. A *rental equivalence price index* for the services of OOH should also be constructed. For many countries, such an (implicit) index is already available as part of the national accounts valuation for the services of OOH.¹⁹⁷ A *user cost index* for the services of OOH should also be constructed since the user cost valuation for the services of a high end dwelling unit will typically be much greater than the corresponding price that the unit could rent for.¹⁹⁸ If the rental equivalent rent and user cost for an owned unit are constructed and are of the same quality, then applying the *opportunity cost approach* to the valuation of the services of the owned unit is appropriate.

For rented housing, the measurement problems are perhaps not so severe; monthly or weekly rents can be observed for the same rental unit and so it would seem that the usual matched model methodology could be applied in this situation. However, an index based on the matched model methodology and normal index number theory will generally have an upward bias because of the neglect of depreciation or a lowering of quality due to the aging of the structure. In order to deal

¹⁹⁵ See Rambaldi and Fletcher (2014) on various smoothing methods that could be used. Diewert and Shimizu (2020) suggested a very simple method which worked well in their empirical application.

¹⁹⁶ The rental equivalence approach could be used for durables that are rented or leased but typically, most consumer durables are not rented. Depreciation rates will in most cases be based on educated guesses. Durable stock estimates can be made once depreciation rates have been determined. The current value of household stocks of consumer durables should also be constructed and added to household balance sheets.

¹⁹⁷ However, if possible, the equivalent rents should be based on new contract rents in order to provide a current opportunity cost for using the services of an owned dwelling unit; recall the discussion on this point in section 17.

¹⁹⁸ Recall the evidence on this point in Heston and Nakamura (2009) (2011) and others.

with this bias, it will in general require a hedonic regression approach with age as one of the explanatory variables.

We will conclude by noting some specific recommendations that emerge from this Chapter:

- There are three main approaches for the treatment of consumer durables in a CPI: the acquisitions approach, the rental equivalence approach and the user cost approach.
- The acquisitions approach is suitable (for most purposes) for durable goods with a relatively short expected useful life.
- The acquisitions approach is particularly useful for central bankers who want consumer inflation indexes that are largely free from imputations.
- The acquisitions approach provides an index for purchases of a durable good and this index is a required input into the construction of a user cost index.
- The remaining two approaches are useful for measuring the flow of services yielded by consumer durables over their useful lives.
- At present, only the flow of services for OOH is estimated by national statistical agencies (using the rental equivalence or user cost approaches) because this information is required for the international System of National Accounts; i.e., the flow of services for other durable goods is not measured at present.
- The acquisitions approach will substantially understate the value of the service flow from consumer durables that have relatively long lives. Hence at least one of the rental equivalence or user cost approaches should be implemented by statistical agencies for durables with long lives.¹⁹⁹ Examples of long lived durables are automobiles and household furnishings.
- The rental equivalence approach to the valuation of the services provided by consumer durables is the preferred method of valuation (with the exception of OOH) when rental or leasing markets for the class of durables exist, because, in principle, no imputations are required to implement this method.²⁰⁰
- However, when rental markets for the durable good under consideration are thin or do not exist, then the user cost approach should be used to value the services of the durable good.
- The user cost approach requires the construction of a price index for new acquisitions of the durable. It also requires a model of depreciation and assumptions about the opportunity cost of capital and about expected asset inflation rates. Thus the user cost approach necessarily involves imputations.
- In order to avoid unnecessary volatility in the user costs, long run expected asset inflation rates should be used in the user cost formula.²⁰¹
- Rental markets for high end dwelling units are generally nonexistent or very thin and hence, it may not be possible to use the rental equivalence approach for high end OOH. Even if some rental information on high end housing units is available, usually these rents are far below the corresponding user costs.

¹⁹⁹ If the acquisitions approach is used in the headline CPI, the alternative approaches can be published as experimental or supplementary series.

²⁰⁰ However, for housing, the “comparable” rental property may not be exactly the same as the owned unit. Moreover, the observed rents may include insurance services and the services of some utilities and possibly furniture. It will be difficult to extract these costs from the observed rent.

²⁰¹ The long run asset inflation rate over the past 20 or 25 years or the long run rate of inflation in housing rents could be used to predict future asset inflation rates. Many other prediction methods could be used; see for example Verbrugge (2008). However, the focus should be on predicting long run asset inflation rather than period to period inflation.

- The “true” opportunity cost for using the services of a consumer durable is the maximum of its rental price (if it exists) and its user cost. Thus the use of the rental equivalence approach to value the services of a high end housing unit will understate the “true” service flow by a substantial amount.²⁰²
- In order to construct national balance sheets and to measure national multifactor productivity, it is necessary to decompose the selling prices of dwelling units into structure and land components. This can be done for both detached housing and condominium units using hedonic regression techniques; see sections 11 and 12 above. This decomposition is also required in order to construct accurate user costs for housing units since depreciation applies to the structure but not to the land component of the property.
- When constructing price indexes for rental housing, statistical agencies need to make an adjustment to observed rents for the same unit for depreciation of the structure and possible improvements to the structure.
- When using observed rents to measure the service flow for comparable owned properties, statistical agencies should use new contract rents to evaluate the service flow for the owned units since rents for continuing tenants may be sticky and not reflect current opportunity costs.
- When constructing user costs for OOH, statistical agencies need to avoid double counting of some housing related costs that may appear elsewhere in the CPI such as insurance costs. Similar double counting problems may arise with housing rents, which may include the services of some utilities or furniture and of course, the housing rent will include insurance costs. In principle, these associated costs should be deducted from the observed rent and placed in the appropriate classification of the CPI. In practice, this is a difficult imputation problem.
- A variant of the acquisitions approach is sometimes applied to OOH. This variant excludes the land component of the purchase of a new house. As mentioned earlier, this variant reduces to a construction cost index for housing with some allowance made for builders’ profit margins. This variant generates valuations for OOH that may be far below the comparable rental equivalent and user cost valuations. It is difficult to justify the use of this variant in a CPI.²⁰³
- A more comprehensive measure of the flow of consumption services would include estimates for the flow of services from storable goods and for household holdings of currency and transferable deposits.

Which of the three main methods for valuing the purchase of a consumer durable should be used for indexing pensions or indexing salaries for consumer inflation? This is a difficult question to answer. If we start out with the idea that we want a national consumer price index, then if there were no durable goods, a national acquisitions price index would be the target index. But it is not clear that this is the “correct” price index once we recognize the existence of consumer durables: an acquisitions index does not recognize the imputed costs of previously purchased consumer durable goods. Thus in order to deal with this difficulty, we need to move to a rental equivalence index or a user cost index if rental markets are thin. But if a national index based on say the rental

²⁰² Long run user costs and rents will tend to be approximately equal to each other for lower end housing units since this type of housing unit will be built by property developers who provide rental housing and they need to set rents that are approximately equal to their long run user costs. However, short run dynamics can cause user costs and rents to diverge even for lower end housing units.

²⁰³ It is not a “true” acquisitions price that is observed in the marketplace since it involves imputations to subtract the land value from the property sale. The resulting acquisitions price obviously does not reflect the total services provided by the purchase.

equivalence approach were used to determine pension payments for veterans or retired civil servants or for employees in an industry, the resulting payments do not take into account that different households have different holdings of consumer durables (housing in particular) and they do not need to be compensated for their consumption of existing holdings. There are additional complications that need to be addressed:

- If the goal is to maintain the purchasing power of a certain group of households (such as retirees or veterans), then an appropriate index needs to be constructed for the relevant group.
- The relevant group may live in different regions of the country and so, in principle, separate indexes need to be constructed for each region by group.

Appendix: Adjusting Housing Rental Price Indexes for New and Disappearing Units

A problem with the simple repeat rents model that was proposed in section 14 is that the model that extended the modified repeat rents index to deal with new and disappearing units was highly simplified. In this Appendix, this simple model is generalized to allow for multiple overlapping products and for many new and disappearing rental units.²⁰⁴

Suppose that there are M rental properties in scope for the rental price index that are present in periods 0 and 1. Suppose further that for rental property n in period t that has a structure on it of age $A(t,m)$ on it, its utility or *real quantity* q_{tm} as a function of the structure age is defined as follows:

$$(A.1) \quad q_{tm} \equiv (1-\Delta)^{A(t,m)} ; \quad t = 0,1; m = 1,\dots,M$$

where Δ is the assumed common to all rental units *geometric property depreciation rate* that is due to structure depreciation. As in section 14, the observed rent for property m in period t is R_{tm} . The constant quality price for property m in period t , p_{tm} , is defined as the observed rent R_{tm} divided by the corresponding real quantity q_{tm} :

$$(A.2) \quad p_{tm} \equiv R_{tm}/q_{tm} = R_{tm}/(1-\Delta)^{A(t,m)} ; \quad t = 0,1; m = 1,\dots,M.$$

In period 0, there are also J rental properties that disappear in period 1. The observed rents, structure ages and constant quality prices and quantities for period 0 for these disappearing rental units are R_{D0j} , $A^\circ(0,j)$, p_{D0j} and q_{D0j} respectively for $j = 1,\dots,J$. The constant quality prices and quantities for these units satisfy the following relationships: $\Delta^{A^\circ(0,j)}$

$$(A.3) \quad q_{D0j} \equiv (1-\Delta)^{A^\circ(0,j)} ; \quad j = 1,\dots,J;$$

$$(A.4) \quad p_{D0j} \equiv R_{D0j}/q_{D0j} = R_{D0j}/(1-\Delta)^{A^\circ(0,j)} ; \quad j = 1,\dots,J.$$

In period 1, there are also K newly occupied rental properties that appear in period 1. The observed rents, structure ages and constant quality prices and quantities for period 1 for these new rental units are R_{N1k} , $A^*(1,k)$, p_{N1k} and q_{N1k} respectively for $k = 1,\dots,K$. The constant quality prices and quantities for these units satisfy the following relationships:²⁰⁵

²⁰⁴ This more general model is based on section 4 in Diewert (2021).

²⁰⁵ If the new period 1 rental unit has a new structure, then $A^*(1,k)$ is set equal to 0; if the “new” rental unit consists of an old structure that was not rented in period 0, then $A^*(1,k)$ is set equal to the age of the structure in months if the index is a monthly index.

$$(A.5) q_{N1k} \equiv (1-\Delta)^{A^*(1,k)}; \quad k = 1, \dots, K;$$

$$(A.6) p_{N1k} \equiv R_{N1k}/q_{N1k} = R_{N1k}/(1-\Delta)^{A^*(1,k)}; \quad k = 1, \dots, K.$$

Thus for each rental unit m that is rented in periods 0 and 1, the tenant occupying rental unit m experiences a utility decline going from period 0 to 1 that is equal to:

$$(A.7) q_{1m}/q_{0m} = (1-\Delta)^{A(0,m)+1}/(1-\Delta)^{A(0,m)} = 1-\Delta; \quad m = 1, \dots, M.$$

Using definitions (A.2), the corresponding rates of price change are given by:

$$(A.8) p_{1m}/p_{0m} = [R_{1m}/(1-\Delta)^{A(0,m)+1}]/[R_{0m}/(1-\Delta)^{A(1,m)}] = [R_{1m}/R_{0m}]/(1-\Delta); \quad m = 1, \dots, M.$$

The *maximum overlap Laspeyres rent index*, P_{MOL} , is defined as follows :

$$\begin{aligned} (A.9) P_{MOL} &\equiv \frac{\sum_{m=1}^M p_{1m}q_{0m}/\sum_{m=1}^M p_{0m}q_{0m}}{\sum_{m=1}^M [R_{1m}/(1-\Delta)^{A(0,m)+1}]/[\sum_{m=1}^M R_{0m}]} && \text{using (A.1) and (A.2)} \\ &= \frac{\sum_{m=1}^M [R_{1m}/(1-\Delta)]/\sum_{m=1}^M R_{0m}}{[\sum_{m=1}^M R_{1m}/\sum_{m=1}^M R_{0m}]/(1-\Delta)} \\ &= P_{RR}/(1-\Delta) \end{aligned}$$

where P_{RR} is the *repeat rents index* defined as

$$(A.10) P_{RR} \equiv \sum_{m=1}^M R_{1m}/\sum_{m=1}^M R_{0m}.$$

The *maximum overlap Paasche rent index*, P_{MOP} , is defined as follows :

$$\begin{aligned} (A.11) P_{MOP} &\equiv \frac{\sum_{m=1}^M p_{1m}q_{1m}/\sum_{m=1}^M p_{0m}q_{1m}}{\sum_{m=1}^M R_{1m}/\sum_{m=1}^M [R_{0m}/(1-\Delta)^{A(0,m)}]/[(1-\Delta)^{A(0,m)+1}]} && \text{using (A.1) and (A.2)} \\ &= \frac{\sum_{m=1}^M R_{1m}/\sum_{m=1}^M R_{0m}(1-\Delta)}{[\sum_{m=1}^M R_{1m}/\sum_{m=1}^M R_{0m}]/(1-\Delta)} \\ &= P_{RR}/(1-\Delta) && \text{using definition (A.10).} \end{aligned}$$

The *maximum overlap Fisher rent index*, P_{MOF} , is defined as the geometric mean of the maximum overlap Laspeyres and Paasche indexes:

$$\begin{aligned} (A.12) P_{MOF} &\equiv [P_{MOL}P_{MOP}]^{1/2} \\ &= P_{RR}/(1-\Delta) && \text{using (A.9) and (A.11).} \end{aligned}$$

Thus the maximum overlap Laspeyres, Paasche and Fisher rent indexes are all equal to the repeat rents index P_{RR} divided by $(1-\Delta)$ where Δ is the property rental geometric depreciation rate.

The property depreciation rate allows us to adjust the observed rent for each rental unit for quality changes due to the aging of the structure but it does not allow us to compare the utility of each rental unit with an alternative rental unit. In order to form overall price and quantity indexes that take into account the new and disappearing rental units, it is necessary to make some stronger assumptions. Thus we assume that tenants evaluate the relative utility of the various rental units that are available according to the following utility function:

$$(A.13) f(q_1, \dots, q_M; q_{D1}, \dots, q_{Dj}; q_{N1}, \dots, q_{Nk}) \equiv \sum_{m=1}^M \alpha_m q_m + \sum_{j=1}^J \beta_j q_{Dj} + \sum_{k=1}^K \gamma_k q_{Nk}$$

where the α_m , β_j and γ_k are positive parameters which reflect the relative utilities of the various rental properties that are available in any given period. The “observed” quantities, q_{0m} , q_{1m} , q_{D0j} and q_{N1k} , for the various available rental properties in periods 0 and 1 are defined by (A.1), (A.3) and (A.5).

In period t , a tenant occupying rental unit m incurs the rental cost $R_{tm} = p_{tm}q_{tm}$. The utility benefit to the tenant $B_{tm} = \alpha_m q_{tm}$. Since it is assumed that each tenant has the same preferences, the cost benefit ratios, $R_{tm}/B_{tm} = p_{tm}q_{tm}/\alpha_m q_{tm} = p_{tm}/\alpha_m$ should be approximately equal to a constant which we can interpret as an aggregate price level P^t ; i.e., utility maximizing tenants should bid up rents for units m where R_{tm}/B_{tm} is low and avoid rental units where R_{tm}/B_{tm} is relatively high. Thus for period 0, the following approximate equalities should hold:

$$(A.14) R_{0m}/\alpha_m q_{0m} \approx P^0; \quad m = 1, \dots, M;$$

$$(A.15) R_{D0j}/\beta_j q_{D0j} \approx P^0; \quad j = 1, \dots, J.$$

Now use definitions (A.1) and (A.3) to eliminate q_{0m} and q_{D0j} from (A.14) and (A.15). After a bit of rearrangement, we obtain the following approximate equalities:

$$(A.16) R_{0m} \approx P^0 \alpha_m (1-\Delta)^{A(0,m)}; \quad m = 1, \dots, M;$$

$$(A.17) R_{D0j} \approx P^0 \beta_j (1-\Delta)^{A(0,j)}; \quad j = 1, \dots, J.$$

The same logic can be applied to the rental units that are available in period 1. Thus for period 1, the following approximate equalities should hold:

$$(A.18) R_{1m}/\alpha_m q_{1m} \approx P^1; \quad m = 1, \dots, M;$$

$$(A.19) R_{N1k}/\gamma_k q_{N1k} \approx P^1; \quad k = 1, \dots, K.$$

Again use definitions (A.1) and (A.3) to eliminate q_{1m} and q_{N1k} from (A.18) and (A.19) in order to obtain the following approximate equalities:

$$(A.20) R_{1m} \approx P^1 \alpha_m (1-\Delta)^{A(1,m)}; \quad m = 1, \dots, M;$$

$$(A.21) R_{N1k} \approx P^1 \gamma_k (1-\Delta)^{A(1,k)}; \quad k = 1, \dots, K.$$

If we take logarithms of both sides of equations (A.16), (A.17), (A.20), (A.21), define $\phi \equiv 1-\Delta$ and add error terms to the resulting equations, it can be seen that we have an *adjacent period time dummy hedonic regression model* which can be used to obtain estimates for the M unknown α_m , the J unknown β_j , the K unknown γ_k , and the 3 unknown parameters, P^0 , P^1 and Δ . There are $2M + J + K + 3$ degrees of freedom in the regression. However, it can be seen that not all parameters can be identified; it will be necessary to impose a normalization on the parameters such as $P^0 = 1$ or $\alpha_1 = 1$. The age of the structure on each rental property is the only rental property characteristic which is required to run the hedonic regression.²⁰⁶

Suppose the normalization $P^0 = 1$ is used in the hedonic regression. Denote the estimates for P^1 and Δ by P^{1*} and Δ^* . We need to define the resulting aggregate real rental quantities for the two

²⁰⁶ But typically, the properties in scope will have some similar characteristics; e.g., they will be classified by type of property, furnished or unfurnished and by local neighbourhood. The adequacy of the model should be judged by the fit of the regression.

periods under consideration. We first define some subaggregate rental values. Define the value of rents for units that are present in both periods as the continuing aggregate rents, R_C^0 and R_C^1 , for periods 0 and 1 as follows;

$$(A.22) R_C^0 \equiv \sum_{m=1}^M R_{0m} ; R_C^1 \equiv \sum_{m=1}^M R_{1m}.$$

Define the aggregate rents for the units that are present in one period but absent in the other period as follows:

$$(A.23) R_D^0 \equiv \sum_{j=1}^J R_{D0j} ; R_N^1 \equiv \sum_{k=1}^K R_{N1k}.$$

Denote the aggregate price levels for the rental units in scope for periods 0 and 1 by P^0 and P^1 and the corresponding aggregate quantity levels by Q^0 and Q^1 . These aggregates are defined as follows:

$$(A.24) P^0 \equiv 1 ; P^1 \equiv P^{1*} ; Q^0 \equiv R_C^0 + R_D^0 ; Q^1 \equiv (R_C^1 + R_N^1)/P^{1*}.$$

In order to justify the above definitions for the period 0 and 1 aggregates, suppose the approximate equalities (A.14) and (A.15) hold exactly. Then it can be seen that

$$(A.25) \begin{aligned} Q^0 &\equiv R_C^0 + R_D^0 \\ &= \sum_{m=1}^M R_{0m} + \sum_{j=1}^J R_{D0j} && \text{using (A.22) and (A.23)} \\ &= \sum_{m=1}^M P^0 \alpha_m q_{0m} + \sum_{j=1}^J P^0 \beta_j q_{D0j} && \text{using (A.14) and (A.15)} \\ &= \sum_{m=1}^M \alpha_m q_{0m} + \sum_{j=1}^J \beta_j q_{D0j} && \text{using } P^0 \equiv 1. \end{aligned}$$

Thus Q^0 is equal to period 0 aggregate utility, $\sum_{m=1}^M \alpha_m q_{0m} + \sum_{j=1}^J \beta_j q_{D0j}$. Now suppose the approximate equalities (A.18) and (A.19) hold exactly. Using (A.24), we have:

$$(A.26) \begin{aligned} Q^1 &\equiv (R_C^1 + R_N^1)/P^{1*} \\ &= (\sum_{m=1}^M R_{1m} + \sum_{k=1}^K R_{N1k})/P^{1*} && \text{using (A.22) and (A.23)} \\ &= (\sum_{m=1}^M P^{1*} \alpha_m q_{1m} + \sum_{k=1}^K P^{1*} \gamma_k q_{N1k})/P^{1*} && \text{using (A.18) and (A.19)} \\ &= \sum_{m=1}^M \alpha_m q_{1m} + \sum_{k=1}^K \gamma_k q_{N1k}. \end{aligned}$$

Thus Q^1 is equal to period 1 aggregate utility, $\sum_{m=1}^M \alpha_m q_{1m} + \sum_{k=1}^K \gamma_k q_{N1k}$.

It is useful to analyze the factors that influence the growth of real aggregate rents. Using definitions (A.24), we have the following decomposition which is a counterpart to the decomposition (137) for real rents in section 14 of the main text:

$$(A.27) \begin{aligned} Q^1/Q^0 &= [(R_C^1 + R_N^1)/P^{1*}]/[R_C^0 + R_D^0] && \text{using definitions (A.24)} \\ &= [1/P^{1*}][R_C^1/R_C^0][1+(R_N^1/R_C^1)]/[1+(R_D^0/R_C^0)] \\ &= P_{RR}[1/P^{1*}][1+(R_N^1/R_C^1)]/[1+(R_D^0/R_C^0)] \end{aligned}$$

where $P_{RR} \equiv R_C^1/R_C^0$ is the repeat rents price index for the rental properties that are occupied in both periods.

If a reasonable estimate for the rental property depreciation rate Δ^* is available to the statistical agency, then there is an alternative to running the hedonic regression defined by the logarithms of equations (A.16), (A.17), (A.20), (A.21). This alternative approach simply sets P^{1*} , which plays a

crucial role in definitions (A.24), equal to the maximum overlap Fisher index P_{MOF} defined by (A.12). Thus the definitions in (A.24) are replaced by the following definitions:

$$(A.28) P^0 \equiv 1; P^{1*} \equiv P_{\text{MOF}} \equiv P_{\text{RR}}/(1-\Delta^*); Q^0 \equiv R_C^0 + R_D^0; Q^1 \equiv (R_C^1 + R_N^1)/P^{1*}.$$

Under these conditions, the decomposition of Q^1/Q^0 becomes the following one:

$$(A.29) \begin{aligned} Q^1/Q^0 &= P_{\text{RR}}[1/P^{1*}][1+(R_N^1/R_C^1)]/[1+(R_D^0/R_C^0)] && \text{using (A.27)} \\ &= [1-\Delta^*][1+(R_N^1/R_C^1)]/[1+(R_D^0/R_C^0)] && \text{using (A.28)} \end{aligned}$$

which is analogous to (137) in the main text.

References

- Adams, B. and R. Verbrugge (2021), “Location, Location, Structure Type: Rent Divergence within Neighborhoods”, Manuscript, Washington D.C.: Bureau of Labor Statistics.
- Aizcorbe, A. (2014), *A Practical Guide to Price Index and Hedonic Techniques*, Oxford, UK: Oxford University Press.
- Astin, J. (1999), “The European Union Harmonized Indices of Consumer Prices (HICP)”, *Statistical Journal of the United Nations ECE* 16 (1999), 123-135.
- Astin, J. and J. Leyland (2015), “Towards a Household Inflation Index”, unpublished paper prepared for the Royal Statistical Society.
- Aten, B.H. (2018), “Valuing Owner-Occupied Housing: An Empirical Exercise Using the American Community Survey (ACS) Housing Files”, Research Paper, Bureau of Economic Analysis, Washington DC, March.
- Australian Bureau of Statistics (2016), “Making Greater Use of Transactions Data to Compile the Consumer Price Index”, Information Paper 6401.0.60.003, November 29, Canberra: ABS.
- Bailey, M.J., R.F. Muth and H.O. Nourse (1963), “A Regression Method for Real Estate Price Construction”, *Journal of the American Statistical Association* 58, 933-942.
- Bank for International Settlements (2018), “BIS Property Prices”, BIS Statistics Warehouse, Basel: BIS.
- Barnett, W. (1978), “The User Cost of Money”, *Economics Letters* 2, 145-149.
- Barnett, W.A., (1980), “Economic Monetary Aggregates: An Application of Aggregation and Index Number Theory”, *Journal of Econometrics* 14, 11-48.
- Barnett, W.A. and M. Chauvet (2011), “How Better Monetary Statistics Could Have Signalled the Financial Crisis”, *Journal of Econometrics* 161, 6-23.
- Basu, S. (2009), “Incorporating Financial Services in a Consumer Price Index: Comment”, pp. 266-271 in *Price Index Concepts and Measurement*, W.E. Diewert, J. Greenlees and C.

- Hulten (eds.), *Studies in Income and Wealth*, Volume 70, Chicago: University of Chicago Press.
- Basu, S., R. Inklaar and J.C. Wang (2011), “The Value of Risk: Measuring the Services of U.S. Commercial Banks”, *Economic Inquiry* 49(1), 226-245.
- Beidelman, C. (1973), *Valuation of Used Capital Assets*, Sarasota Florida: American Accounting Association.
- Beidelman, C.R. (1976), “Economic Depreciation in a Capital Goods Industry”, *National Tax Journal* 29, 379-390.
- Böhm-Bawerk, E. V. (1891), *The Positive Theory of Capital*, W. Smart (translator of the original German book published in 1888), New York: G.E. Stechert.
- Bostic, R.W., S.D. Longhofer and C.L. Readfearn (2007), “Land Leverage: Decomposing Home Price Dynamics”, *Real Estate Economics* 35:2, 183-2008.
- Bureau of Economic Analysis (2020), “Integrated Macroeconomic Accounts for the United States, Table S.3.a, Households and Nonprofit Institutions Serving Households”, March 30. <https://www.bea.gov/data/special-topics/integrated-macroeconomic-accounts>
- Bureau of Labor Statistics (1983), “Changing the Home Ownership Component of the Consumer Price Index to Rental Equivalence”, *CPI Detailed Report*, Washington DC: BLS.
- Burnett-Issacs, K., N. Huang and W.E. Diewert (2021), “Developing Land and Structure Price Indexes for Ottawa Condominium Apartments”, *Journal of Official Statistics* 36:4, 736-802.
- Cairns, R.D. (2013), “The Fundamental Problem of Accounting”, *Canadian Journal of Economics* 46:2, 634-655.
- Canning, J.B. (1929), *The Economics of Accountancy*, New York: The Ronald Press Co.
- Christensen, L.R. and D.W. Jorgenson (1969), “The Measurement of U.S. Real Capital Input, 1929-1967,” *Review of Income and Wealth* 15(4): 293-320.
- Church, A.H. (1901), “The Proper Distribution of Establishment Charges, Part III”, *The Engineering Magazine* 21, 904-912.
- Clapp, J.M. (1980), “The Elasticity of Substitution for Land: The Effects of Measurement Errors”, *Journal of Urban Economics* 8, 255-263.
- Coffey, C., K. McQuinn and C. O’Toole (2020), “Rental Equivalence, Owner-Occupied Housing and Inflation Measurement: Micro-level Evidence from Ireland”, Economic and Social Research Institute Working Paper 685, Dublin, Ireland.
- Colangelo, A. and R. Inklaar (2012), Bank Output Measurement in the Euro Area—a Modified Approach”, *The Review of Income and Wealth* 58:1, 142-165.

- Court, A. T. (1939), "Hedonic Price Indexes with Automotive Examples", pp. 98-117 in *The Dynamics of Automobile Demand*, New York: General Motors Corporation.
- Crone, T.M., L.I. Nakamura and R. Voith (2000), "Measuring Housing Services Inflation", *Journal of Economic and Social Measurement* 26, 153-171.
- Crone, T.M., L.I. Nakamura and R.P. Voith (2011), "Hedonic Estimates of the Cost of Housing Services: Rental and Owner Occupied Units", pp. 51-68 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.
- Crosby, N.S. Devaney and V. Law (2012), "Rental Depreciation and Capital Expenditure in the UK Commercial Real Estate Market, 1993-2009", *Journal of Property Research* 29:3, 227-246.
- Davis, M.A. and J. Heathcote (2007), "The Price and Quantity of Residential Land in the United States", *Journal of Monetary Economics* 54:8, 2595-2620.
- de Haan, J. (2015), "Rolling Year Time Dummy Indexes and the Choice of Splicing Method", Room Document at the 14th meeting of the Ottawa Group, May 22, Tokyo. <http://www.stat.go.jp/english/info/meetings/og2015/pdf/t1s3room>
- Díaz, A., and M.J. Luengo-Prado (2008), "On the User Cost and Homeownership", *Review of Economic Dynamics* 11:3, 584-613.
- Diewert, W.E. (1974), "Intertemporal Consumer Theory and the Demand for Durables," *Econometrica* 42, 497-516.
- Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 114-145.
- Diewert, W.E. (1977), "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium," pp. 73-126 in *Equilibrium and Disequilibrium in Economic Theory*, E. Schwödiauer (Ed.), Dordrecht, Holland: Reidel Publishing Co.
- Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, Dan Usher (ed.), University of Chicago Press, Chicago.
- Diewert, W.E. (1993), "Symmetric Means and Choice under Uncertainty", pp. 355-433 in *Essays in Index Number Theory*, Volume 1 (W.E. Diewert and A.O. Nakamura editors), Amsterdam: North-Holland.
- Diewert, W.E. (2002), "Harmonized Indexes of Consumer Prices: Their Conceptual Foundations", *Swiss Journal of Economics and Statistics* 138:4, 547-637.
- Diewert, W.E. (2005a), "Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates", pp. 479-542 in *Measuring Capital in the New Economy*, C. Corrado, J. Haltiwanger and D. Sichel (eds.), Chicago: University of Chicago Press.
- Diewert, W.E. (2005b), "Adjacent Period Dummy Variable Hedonic Regressions and Bilateral Index Number Theory", *Annales d'Économie et de Statistique* 79/80, 759-786.

- Diewert, W.E. (2008), "OECD Workshop on Productivity Analysis and Measurement: Conclusions and Future Directions", pp. 11-36 in *Proceedings From the OECD Workshops on Productivity Measurement and Analysis*, Paris: OECD.
- Diewert, W.E. (2010), "Alternative Approaches to Measuring House Price Inflation", Discussion Paper 10-10, Department of Economics, The University of British Columbia, Vancouver, Canada, V6T 1Z1.
- Diewert, W.E. (2014), "The Treatment of Financial Transactions in the SNA: A User Cost Approach", *Eurostat Review of National Accounts and Macroeconomic Indicators* 1, 73-89.
- Diewert, W.E. (2021), "Quality Adjustment Methods", Draft Chapter 8 in Consumer Price Index Theory, Washington D.C.: International Monetary Fund, published online at: <https://www.imf.org/en/Data/Statistics/cpi-manual>.
- Diewert, W.E., J. de Haan and R. Hendriks (2011), "The Decomposition of a House Price Index into Land and Structures Components: A Hedonic Regression Approach", *The Valuation Journal* 6, 58-106.
- Diewert, W.E., J. de Haan and R. Hendriks (2015), "Hedonic Regressions and the Decomposition of a House Price index into Land and Structure Components", *Econometric Reviews* 34, 106-126.
- Diewert, W.E., D. Fixler and K. Zieschang (2011), "The Measurement of Banking Services in the System of National Accounts", Discussion Paper 11-04, Department of Economics, University of British Columbia, Vancouver, Canada.
- Diewert, W.E., D. Fixler and K. Zieschang (2016), "Problems with the Measurement of Banking Services in a National Accounting Framework", pp. 117-176 in *National Accounting and Economic Growth*, John M. Hartwick (ed.), Cheltenham UK: Edward Elgar Publishing.
- Diewert, W.E. and K.J. Fox (2016), "Sunk Costs and the Measurement of Commercial Property Depreciation", *Canadian Journal of Economics* 49:4; 1340-1366.
- Diewert, W.E. and K.J. Fox (2018), "Alternative User Costs, Productivity and Inequality in US Business Sectors", pp. 21-69 in *Productivity and Inequality*, W.H. Greene, L. Khalaf, P. Makkissi, R. Sickles, M. Veall and M.-C. Voia (eds.), New York: Springer.
- Diewert, W.E. and K.J. Fox (2019), "Money and the Measurement of Total Factor Productivity", *Journal of Financial Stability* 42, 84-89.
- Diewert, W.E. and K.J. Fox (2020) "Substitution Bias in Multilateral Methods for CPI Construction Using Scanner Data," *Journal of Business and Economic Statistics*, published online at: <https://doi.org/10.1080/07350015.2020.1816176>
- Diewert, W.E. and R.J. Hill (2010), "Alternative Approaches to Index Number Theory", pp. 263-278 in *Price and Productivity Measurement*, W.E. Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Victoria Canada: Trafford Press.

- Diewert, W.E., N. Huang and K. Burnett-Issacs (2017), “Alternative Approaches for Resale Housing Price Indexes”, Discussion Paper 17-05, Vancouver School of Economics, The University of British Columbia, Vancouver, Canada, V6T 1L4.
- Diewert, W.E. and D.A. Lawrence (2000), “Progress in Measuring the Price and Quantity of Capital”, pp. 273-326 in *Econometrics Volume 2: Econometrics and the cost of Capital: Essays in Honor of Dale W. Jorgenson*, L. J. Lau (ed.), Cambridge, MA: The MIT Press.
- Diewert, W.E. and A.O. Nakamura (2011), Accounting for Housing in a CPI”, pp. 7-32 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.
- Diewert, W.E., A.O. Nakamura and L.I. Nakamura (2009), “The Housing Bubble and a New Approach to Accounting for Housing in a CPI”, *Journal of Housing Economics* 18:3, 156-171.
- Diewert, W.E., K.G. Nishimura, C. Shimizu and T. Watanabe (2020), *Property Price Index: Theory and Practice*, Tokyo: Springer Japan.
- Diewert, W. E. and C. Shimizu (2015), “Residential Property Price Indexes for Tokyo,” *Macroeconomic Dynamics* 19, 1659-1714.
- Diewert, W. E. and C. Shimizu (2016), “Hedonic Regression Models for Tokyo Condominium Sales,” *Regional Science and Urban Economics* 60, 300-315.
- Diewert, W. E. and C. Shimizu (2017), “Alternative Approaches to Commercial Property Price Indexes for Tokyo,” *Review of Income and Wealth* 63:3, 492-519.
- Diewert, W.E. and C. Shimizu (2019), “Residential Property Price Indexes: Spatial Coordinates versus Neighbourhood Dummy Variables”, Discussion Paper 19-09, Vancouver School of Economics, The University of British Columbia, Vancouver, Canada, V6T 1L4.
- Diewert, W.E. and C. Shimizu (2020), “Alternative Land Price Indexes for Commercial Properties in Tokyo”, *Review of Income and Wealth* 66:4, 784-824.
- Diewert, W.E. and H. Wei (2017), “Getting Rental Prices Right for Computers: Reconciling Different Perspectives on Depreciation”, *Review of Income and Wealth*, 63:S1,149-168.
- Donovan, D. (1978), “Modeling the Demand for Liquid Assets: An Application to Canada”, *IMF Staff Papers* 25, 676-704.
- European Central Bank (2018), *Residential Property Price Index Statistics*, Statistical Data Warehouse, Frankfurt: European Central Bank.
- Eurostat, IMF, OECD, UN and World Bank (1993), *System of National Accounts 1993*, United Nations: New York.
- Eurostat (2001), *Handbook on Price and Volume Measures in National Accounts*, Luxembourg: European Commission.

- Eurostat (2005), “On the principles for estimating dwelling services for the purpose of Council Regulation (EC, Euratom) No 1287/2003 on the harmonisation of gross national income at market prices”, *Official Journal of the European Union*, Commission Regulation (EC) No 1722/2005, October 20.
- Eurostat (2013), *Handbook on Residential Property Prices Indices (RPPIs)*, Luxembourg: Publications Office of the European Union.
- Eurostat (2017), *Technical Manual on Owner-Occupied Housing and House Price Indices*, Brussels: European Commission.
- Feenstra, Robert C. (1986), “Functional Equivalence between Liquidity Costs and the Utility of Money”, *Journal of Monetary Economics* 17:2, 271-291.
- Fenwick, D. (2009), “A Statistical System for Residential Property Price Indices”, Eurostat-IAOS-IFC Conference on Residential Property Price Indices, Bank for International Settlements, November.
- Fenwick, D. (2012), “A Family of Price Indices”, The Meeting of Groups of Experts on Consumer Price Indices, UNECE/ILO, United Nations Palais des Nations, Geneva Switzerland, May 30-June 1.
- Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.
- Fixler, D. (2009), “Incorporating Financial Services in a Consumer Price Index”, pp. 239-266 in *Price Index Concepts and Measurement*, W.E. Diewert, J. Greenlees and C. Hulten (eds.), Studies in Income and Wealth, Volume 70, Chicago: University of Chicago Press.
- Fixler, D. and K.D. Zieschang (1991), “Measuring the Nominal Value of Financial Services in the National Income Accounts”, *Economic Inquiry*, 29, 53-68.
- Fixler, D. and K.D. Zieschang (1992), “User Costs, Shadow Prices, and the Real Output of Banks,” pp. 219-243 in Z. Griliches (ed.), *Output Measurement in the Service Sector*, Chicago: University of Chicago Press.
- Fixler, D. and K.D. Zieschang (1999), “The Productivity of the Banking Sector: Integrating Financial and Production Approaches to Measuring Financial Service Output,” *The Canadian Journal of Economics* 32, 547-569.
- Francke, M.K. (2008), “The Hierarchical Trend Model”, pp. 164-180 in *Mass Appraisal Methods: An International Perspective for Property Valuers*, T. Kauko and M. Damato (eds.), Oxford: Wiley-Blackwell.
- Francke, M. K. and G. A. Vos. (2004), “The Hierarchical Trend Model for Property Valuation and Local Price Indices,” *Journal of Real Estate Finance and Economics* 28, 179-208.
- Gallin, J. and R.J. Verbrugge (2019), “A Theory of Sticky Rents: Search and Bargaining with Incomplete Information”, *Journal of Economic Theory* 183, 478-519.
- Garcke, E. and J.M. Fells (1893), *Factory Accounts: Their Principles and Practice*, Fourth Edition, (First Edition 1887), London: Crosby, Lockwood and Son.

- Garner, T.I., and R.J. Verbrugge (2009), “Reconciling User Costs and Rental Equivalence: Evidence from the U.S. Consumer Expenditure Survey”, *Journal of Housing Economics* 18:3, 172-192.
- Garner, T.I. and R. Verbrugge (2011), “The Puzzling Divergence of Rents and User Costs, 1980-2004: Summary and Extensions”, pp. 125-146 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.
- Genesove, D (2003) , “The Nominal Rigidity of Apartment Rents”, *The Review of Economics and Statistics* 85 (4), 844-853.
- Gilman, S. (1939), *Accounting Concepts of Profit*, New York: The Rolland Press Co.
- Goeyvaerts, G. and E. Buyst (2019), “Do Market Rents Reflect User Costs?”, *Journal of Housing Economics* 44:1, 112-130.
- Goodhart, C. (2001), “What Weights should be Given to Asset Prices in the Measurement of Inflation?”, *The Economic Journal* 111 (June), F335-F356.
- Griliches, Z. (1971), “Introduction: Hedonic Price Indexes Revisited”, pp. 3-15 in *Price Indexes and Quality Change*, Z. Griliches (ed.), Cambridge MA: Harvard University Press.
- Gudnason, R. and G. Jonsdottir (2011), “Owner Occupied Housing in the Icelandic CPI,” pp. 147-150 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.
- Hall, R.E. (1971), “The Measurement of Quality Change from Vintage Price Data”, pp. 240-271 in *Price Indexes and Quality Change*, Z. Griliches (ed.), Cambridge MA: Harvard University Press.
- Hall, R.E. and D.W. Jorgenson (1967), “Tax Policy and Investment Behavior”, *American Economic Review* 57, 391-414.
- Harper, M.J., E.R. Berndt and D.O. Wood (1989), “Rates of Return and Capital Aggregation Using Alternative Rental Prices”, pp. 331-372 in *Technology and Capital Formation*, D. W. Jorgenson and R. Landau (eds.), The MIT Press, Cambridge, MA.
- Heston, A. and A.O. Nakamura (2009) “Questions about the Equivalence of Market Rents and User Costs for Owner Occupied Housing,” *Journal of Housing Economics* 18, 273–279.
- Heston, A. and A.O. Nakamura (2011), Reported Prices and Rents of Housing: Reflections of Costs, Amenities or Both?”, pp. 117-124 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.
- Hicks, J. R. (1946), *Value and Capital*, Second Edition, Oxford: Clarendon Press.
- Hill, R.J. (2013), “Hedonic Price Indexes for Residential Housing: A Survey, Evaluation and Taxonomy”, *Journal of Economic Surveys* 27, 879-914.

- Hill, R.J. and M. Scholz (2018), “Can Geospatial Data Improve House Price Indexes? A Hedonic Imputation Approach with Splines”, *Review of Income and Wealth* 64:4, 737-756.
- Hill, R.J., M. Scholz, C. Shimizu and M. Steurer (2018), “An Evaluation of the Methods used by European Countries to compute their Official House Price Indices”, *Economie et Statistique / Economics and Statistics Numbers* 500-502, 221-238.
- Hill, R.J., M. Steurer and S.R. Wautl (2020), “Owner Occupied Housing, Inflation and Monetary Policy”, Graz Economic Papers 2020-18, Department of Public Economics, University of Graz, Graz, Austria.
- Hoffmann, J. and C. Kurz (2002), “Rent Indices for Housing in West Germany: 1985 to 1998”, Discussion Paper 01/02, Economic Research Centre of the Deutsche Bundesbank, Frankfurt.
- Hotelling, H. (1925), “A General Mathematical Theory of Depreciation”, *Journal of the American Statistical Association* 20, 340-353.
- Hulten, C.R. (1990), “The Measurement of Capital”, pp. 119-158 in *Fifty Years of Economic Measurement*, E. R. Berndt and J. E. Triplett (eds.), Chicago: the University of Chicago Press.
- Hulten, C.R. (1996), “Capital and Wealth in the Revised SNA”, pp. 149-181 in *The New System of National Accounts*, J.W. Kendrick (ed.), New York: Kluwer Academic Publishers.
- Hulten, C.R. and F.C. Wykoff (1981a), “The Estimation of Economic Depreciation using Vintage Asset Prices”, *Journal of Econometrics* 15, 367-396.
- Hulten, C.R. and F.C. Wykoff (1981b), “The Measurement of Economic Depreciation”, pp. 81-125 in *Depreciation, Inflation and the Taxation of Income from Capital*, C.R. Hulten (ed.), Washington D.C.: The Urban Institute Press.
- Hulten, C.R. and F.C. Wykoff (1996), “Issues in the Measurement of Economic Depreciation: Introductory Remarks”, *Economic Inquiry* 34, 10-23.
- Inklaar, R. and J.C. Wang (2010), “Real Output of Bank Services: What Counts is What Banks Do, Not What They Own”, paper presented at the 31st General Conference of the International Association for Research in Income and Wealth at St. Gallen, Switzerland, August 22-28.
- ILO, Eurostat, IMF, OECD, UNECE and the World Bank (2004), *Consumer Price Index Manual: Theory and Practice*, Peter Hill (ed.), Geneva: International Labour Office.
- Ivancic, L., W.E. Diewert and K.J. Fox (2011), “Scanner Data, Time Aggregation and the Construction of Price Indexes”, *Journal of Econometrics* 161, 24-35.
- Jorgenson, D.W. (1989), “Capital as a Factor of Production”, pp. 1-35 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge MA: The MIT Press.
- Jorgenson, D.W. (1996), “Empirical Studies of Depreciation”, *Economic Inquiry* 34, 24-42.

- Karlin, S. (1959), *Mathematical Methods and Theory in Games, Programming and Economics*, Volume 1, Reading MA: Addison-Wesley Publishing Co.
- Katz, A.J. (1983), “Valuing the Services of Consumer Durables”, *The Review of Income and Wealth* 29, 405-427.
- Keynes, J.M. (1930), *Treatise on Money*, Vol. 1. London: Macmillan.
- Knoll, K., M. Schularick and T. Steger (2017), “No Price Like Home: Global House Prices, 1870-2012”, *American Economic Review* 107(2), 331-353.
- Koev, E. and J.M.C. Santos Silva (2008), “Hedonic Methods for Decomposing House Price Indices into Land and Structure Components”, unpublished paper, Department of Economics, University of Essex, England, October.
- Konüs, A.A. and S.S. Byushgens (1926), “K Probleme Pokupatelnoi Cili Deneg”, *Voprosi Konyunkturi* 2, 151-172.
- Krsinich, F. (2016), “The FEWS Index: Fixed Effects with a Window Splice”, *Journal of Official Statistics* 32, 375-404.
- Kuhn, H.W. and A.W. Tucker (1951), “Nonlinear Programming”, pp. 481-492 in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley CA: University of California Press.
- Lebow, D.E. and J.B. Rudd (2003), “Measurement Error in the Consumer Price Index: Where do we Stand?”, *Journal of Economic Literature* 41, 159-201.
- Lewis, R. and A. Restieaux (2015), *Improvements to the Measurement of Owner Occupiers’ Housing Costs and Private Housing Rental Prices*, Newport, UK: Office for National Statistics.
- Malpezzi, S., L. Ozanne and T. Thibodeau (1987), “Microeconomic Estimates of Housing Depreciation”, *Land Economics* 63, 372-385.
- Marshall, A. (1887), “Remedies for Fluctuations of General Prices”, *Contemporary Review* 51, 355-375.
- Marshall, A. (1898), *Principles of Economics*, Fourth Edition, London: The Macmillan Co.
- Matheson, E. (1910), *The Depreciation of Factories and their Valuation*, Fourth Edition, London: E. & F.N. Spon.
- McMillen, D.P. (2003), “The Return of Centralization to Chicago: Using Repeat Sales to Identify Changes in House Price Distance Gradients”, *Regional Science and Urban Economics* 33, 287-304.
- OECD (2001), *Measuring Productivity: Measurement of Aggregate and Industry-Level Productivity Growth*, Paris: OECD.

- Office for National Statistics (2010), *Consumer Price Indexes Technical Manual: 2010 Edition*, Newport, U.K.: Office for National Statistics.
- Office for National Statistics (2016), *CPIH Compendium*, Newport, U.K.: Office for National Statistics.
- Office for National Statistics (2017), *Household Costs Indices: Methodology*, Newport, U.K.: Office for National Statistics.
- Office for National Statistics (2018), *Measures of Owners Occupiers' Housing Costs, UK: April to June 2018*, Newport, U.K.: Office for National Statistics.
- Peasnell, K.V. (1981), "On Capital Budgeting and Income Measurement", *Abacus* 17:1, 52-67.
- Prais, S.J. (1959), "Whose Cost of Living?", *The Review of Economic Studies* 26, 126-134.
- Rambaldi, A.N., R.J. McAllister, K. Collins and C.S. Fletcher (2010), "Separating Land from Structure in Property Prices: A Case Study from Brisbane Australia", School of Economics, The University of Queensland, St. Lucia, Queensland 4072, Australia.
- Rambaldi, A.N, and C.S. Fletcher (2014), "Hedonic Imputed Property Price Indexes: The Effects of Econometric Modeling Choices", *Review of Income and Wealth* 60:S2, S423-S448.
- Schreyer, P. (2001), *OECD Productivity Manual: A Guide to the Measurement of Industry-Level and Aggregate Productivity Growth*, Paris: OECD.
- Schreyer, P. (2009), *Measuring Capital*, Statistics Directorate, National Accounts, STD/NAD(2009)1, Paris: OECD.
- Schwann, G.M. (1998), "A Real Estate Price Index for Thin Markets", *Journal of Real Estate Finance and Economics* 16:3, 269-287.
- Shimizu, C., W.E. Diewert, K. Nishimura and T. Watanabe (2012), "The Estimation of Owner Occupied Housing Indexes using the RPPI: The Case of Tokyo", Meeting of the Group of Experts on Consumer Price Indices, Geneva, May 28.
- Shimizu, C., K. Nishimura and T. Watanabe (2010a), "Housing Prices in Tokyo: A Comparison of Hedonic and Repeat Sales Measures", *Journal of Economics and Statistics* 230, 792-813.
- Shimizu, C., K. Nishimura and T. Watanabe (2010b), "Nominal Rigidity of Housing Rent", *Financial Review* 106:1, 52-68.
- Shimizu, C., H. Takatsuji, H. Ono and K. G. Nishimura (2010), "Structural and Temporal Changes in the Housing Market and Hedonic Housing Price Indices," *International Journal of Housing Markets and Analysis*, 3(4), 351-368.
- Shimizu, C. and T. Watanabe (2011), "Nominal Rigidity of Housing Rent", *Financial Review* 106:1, 52-68.

- Silver, M.S. (2018), “How to Measure Hedonic Property Price Indexes Better”, *EURONA* 1/2018, 35-66.
- Solomons, D. (1961), “Economic and Accounting Concepts of Income”, *The Accounting Review* 36, 374-383.
- Statistics Portugal (Instituto Nacional de Estatística) (2009), “Owner-Occupied Housing: Econometric Study and Model to Estimate Land Prices, Final Report”, paper presented to the Eurostat Working Group on the Harmonization of Consumer Price Indices”, March 26-27, Luxembourg: Eurostat.
- Summers, R. (1973), “International Comparisons with Incomplete Data”, *Review of Income and Wealth* 29:1, 1-16.
- Suzuki, M., Y. Asami and C. Shimizu (2021), “Housing Rent Rigidity under Downward Pressure: Unit-level Longitudinal Evidence from Tokyo”, *Journal of Housing Economics* 52, Available online.
- Triplett, J. (2004), *Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes*, Directorate for Science, Technology and Industry, DSTI/DOC(2004)9, Paris: OECD.
- Triplett, J.E. and B.P. Bosworth (2004), *Productivity in the U.S. Services Sector: New Sources of Economic Growth*, Washington D.C.: Brookings Institution Press.
- Ueda, K., K. Watanabe and T. Watanabe (2020), “Consumer Inventory and the Cost of Living Index: Theory and Some Evidence from Japan”, unpublished paper, Waseda University (E-mail: kozo.ueda@waseda.jp).
- Verbrugge, R. (2008), “The Puzzling Divergence of Rents and User Costs, 1980-2004”, *Review of Income and Wealth* 54:4, 671-699.
- Verbrugge, Randal J. (2012). Do the Consumer Price Index’s Utilities Adjustments for Owners’ Equivalent Rent Distort Inflation Measurement? *Journal of Business & Economic Statistics*, 30(1), 143-148.
- Walras, L. (1954), *Elements of Pure Economics*, translated by W. Jaffe (first published in 1874), London: George Allen and Unwin.
- Wang, J.C. (2003), “Loanable Funds, Risk and Bank Service Output”, Federal Reserve Bank of Boston, Working Paper Series No. 03-4.
<http://www.bos.frb.org/economic/wp/wp2003/wp034.htm>
- Wang, J.C., S. Basu and J.G. Fernald (2009), “A General Equilibrium Asset Approach to the Measurement of Nominal and Real Bank Output”, pp. 273-328 in *Price Index Concepts and Measurement*, W.E. Diewert, J. Greenlees and C. Hulten (eds.), Studies in Income and Wealth, Volume 70, Chicago: University of Chicago Press.
- White, K.J. (2004), *Shazam: User’s Reference Manual*, Version 10, Vancouver, Canada: Northwest Econometrics Ltd.

Wold, H. (1944), "A Synthesis of Pure Demand Analysis, Part 3", *Skandinavisk Aktuarietidskrift* 27, 69-120.