The Adjusted Price Index and Monthly Adjusted Consumer Expenditure Basket Weights

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Contents

Background: COVID-19 Pandemic and Price Indexes Based on Current Consumer Spending[1]

[1] Much of this text and analysis was first published in "The Adjusted Price Index and Monthly Adjusted Consumer Expenditure Basket Weights," Gerry O'Donnell and Clément Yélou, Statistics Canada, Catalogue no. 62F0014M, November 10 2021.

The COVID-19 outbreak, declared a pandemic on March 11, 2020, led to economic disruptions that continue to affect financial and labour markets across the globe. At the onset of the pandemic, prices shifted significantly as Canadians entered a sustained period of physical distancing and business closures. As Canadians adapted to staying home and travelling less, demand for a number of consumer goods and services changed, contributing to the first year-over-year decline in the Consumer Price Index (CPI) since 2009. In April and May of 2020, consumer prices were 0.2% and 0.4% lower, respectively, compared with the same months of 2019. While prices for many products have since surpassed pre-pandemic levels, the impact of COVID-19 and various measures to contain its spread continue to impact the CPI.

The COVID-19 pandemic created an unprecedented situation during which the behaviours of Canadians were significantly altered over a very short period of time, affecting consumption patterns which, by design, are not accounted for in the official CPI fixed basket weights. Shifts in household purchasing patterns have implications for the basket weights used in the calculation of the CPI. Typically, expenditure patterns evolve slowly and in a sustained manner over time in response to shifts in relative prices, changes in the level or distribution of household incomes, changing demographics, evolving habits and the availability of new technology. A fixed-basket price index, such as the Canadian CPI, can only reflect these changes when the basket weights are updated. Under normal economic circumstances, any over- or under-estimation of the importance of a given good or service in the CPI is minimized by scheduling basket updates at regular intervals.[2]

In May 2020, Alberto Cavallo from the Harvard Business School published "Inflation with Covid Consumption Baskets"[3]. Cavallo used publicly available data from the Opportunity Insights project, which was in turn based on credit card transaction data, to show that spending on food at home in the United States had surged in March 2020, while spending on many other categories had plummeted. Cavallo constructed a "Covid CPI" using updated consumer expenditure shares, and estimated that inflation was higher using this alternate measure compared to the Bureau of Labor Statistics' All-items US city-average CPI.

In order to assess the impact of COVID-19 on Canadian household expenditures, Statistics Canada, in partnership with the Bank of Canada, explored more current sources of expenditure data to estimate basket weights that reflected shifting consumption patterns during the pandemic. These data were supplemented with transaction and survey data as well as subject matter expertise to derive an alternate set of expenditure weights, and then used to calculate an Adjusted price index series starting March 2020.

On July 13, 2020 Statistics Canada published *Consumer expenditures during COVID-19: An exploratory analysis of the effects of changing consumption patterns on consumer price indexes* [4], the agency's first publication measuring consumer price trends using basket weights updated to reflect the latest monthly consumer spending patterns. This alternative and experimental price index showed a slightly higher rate of price inflation than the official Consumer Price Index (CPI) based on 2017 expenditure patterns when,

[2] The Canadian CPI maintains the fixed basket concept in accordance with best practices established by international price experts and other national statistical agencies, in part because there are no current statistical survey data to inform the magnitude of any change in consumption at the level required for CPI calculation. The CPI basket weights are based primarily on expenditure data from Statistics Canada's Household Final Consumption Expenditures, and are normally updated every two years. The most recent basket update took place with the release of the June 2021 CPI using 2020 expenditure data.

[3] Alberto Cavallo, "Inflation with Covid Consumption Baskets", Harvard Business School, May 28 2020.

[4] Statistics Canada, Catalogue no. 62F0014M.

in the early months of the pandemic, Canadian consumers reduced consumption of goods and services with falling prices, such as traveller accommodation and clothing, and increased their consumption of products with above average price increases, such as food and household cleaning products.

As the pandemic evolved, Statistics Canada updated the study with new methods and results, publishing the Adjusted price index and monthly adjusted consumer expenditure basket weights in The Daily, and data tables 18-10-0263 and 18-10-0264 on October 8 2020, and again on January 12 2021 and April 12 2021.

The monthly adjusted consumer expenditure basket weights made extensive use of aggregate High Frequency Expenditure Network (HFEN) data provided by the Bank of Canada to estimate changes in spending for the majority of products in the 2017 CPI basket[5]. This data was supplemented by a number of other sources to estimate monthly expenditures for more than 500 detailed product classes in the CPI.

The Adjusted price index was derived from these monthly expenditures and used a monthly-chained Laspeyres index, a formulation which used estimates of the previous month's expenditures to aggregate current month price changes emanating from the CPI.

The CPI basket weights were updated with the release of the June 2021 CPI[6]. The new basket weight reference period is 2020, based on data from the national Household Final Consumption Expenditures (HFCE) series, in addition to data from the Survey of Household Spending and the provincial HFCE series. Alternative data for 2020 was used to account for pandemic-related shifts at more detailed levels of CPIs and geographies.

At the same time, Statistics Canada worked to redevelop the methods and data sources for the Adjusted price index and monthly adjusted consumer expenditure basket weights. In addition to the use of a broader range of data sources, a new price index formula was applied to aggregate monthly price changes into an All-items Adjusted price index to address important limitations observed with the monthly-chained Laspeyres index.

This paper outlines the methods used in the initial version and the redeveloped Adjusted price index and monthly adjusted consumer expenditure basket weights. It also highlights important results of this work, and suggests possible avenues for future work.

Monthly Adjusted Consumer Expenditure Basket Weights

Data Sources

In the first months of the pandemic, Statistics Canada faced the challenge of estimating monthly consumer expenditures for perhaps the first time. The agency disseminates a Household Final Consumption Expenditure Implicit price index, but unlike the US Bureau of Economic Analysis' monthly

[5] The major component shelter and the sub-component purchase and leasing of passenger vehicles were not covered by the available expenditure data.

[6] See Statistics Canada, "An Analysis of the 2021 Consumer Price Index Basket Update, Based on 2020 Expenditures", July 21 2021.

Personal Consumer Expenditure Price Index, it is a quarterly estimate published 2 months after the end of a quarter.

In May 2020 the Bank of Canada provided Statistics Canada with aggregate High Frequency Expenditure Network (HFEN) data to help estimate year-over-year changes in spending per month for the majority of products in the 2017 CPI basket with a lag of only three weeks.

In the first iteration, covering reference periods 202002 through 202101, monthly weight estimation started with 2017 CPI basket expenditures based primarily on the 2017 Survey of Household Spending, and then projected these to later months using a variety of data sources and techniques.

During the 2020 CPI basket update, work began on a second version of the Adjusted price index and monthly adjusted consumer expenditure basket weights. Beginning with reference period 202105, monthly weight estimation started with 2020 CPI basket expenditures, which for the first time were based primarily on System of National Accounts Household Final Consumption Expenditures for that year.

Each of the CPI's 500-plus elementary product classes was escalated at the Canada-level using one or more of the sources listed in Table 1. For the SNA-based second version, the sources used were similar to those used to update the CPI basket weights.

Table 1: Data sources used to estimate monthly adjusted consumer expenditure basket weights

Supplier type	Data source	Type of variable used	Periodicity of data	Used in version 1	Used in version 2	Basket share ($p_{2020}q_{2020}$ / $\sum p_{2020}q_{2020}$) of elementary products adjusted using this source in version 2, percent[7]
Data from Statistics Canada programs	Household Final Consumption Expenditures	expenditure	quarterly	✓	✓	83.48
	Retail Commodity Survey	revenue	monthly	×	✓	32.66
	Monthly Retail Trade Survey	revenue	monthly	✓	✓	8.79
	New Motor Vehicle Sales	revenue	monthly	✓		6.31
	Population estimates, quarterly	number of people	quarterly	✓	✓	19.16
	Monthly Survey of Food Services and Drinking Places	revenue	monthly	✓	✓	5.12
	Domestic and international Itinerant aircraft movements	volume	weekly	✓	✓	0.26
	New Housing Price Index data	price index	monthly	✓	✓	7.01
	Passenger bus and urban transit statistics	revenue	monthly	✓	✓	0.20
	Electric power generation statistics	volume	monthly	✓	✓	2.66
	Canadian monthly natural gas distribution statistics	revenue	monthly	✓	✓	0.70
	Consumer Price Index	price index	monthly	✓	✓	11.14
	Labour Force Survey rent data	average price	monthly	✓	✓	6.59
Data supplied to Statistics	Bank of Canada High Frequency Expenditure Network data	year-over-year growth in revenue	monthly	✓	✓	60.56

[7] The rows in this column do not sum to 100. Due to the timeliness of source data, multiple data inputs were used to adjust most products to the most recent reference period of the Adjusted price index.

Canada from external provider	Canada Revenue Agency Goods and Services Tax revenue data	revenue	monthly	✓	✓	58.97
	Grocery retailer scanner data	sales	daily or weekly	✓	✓	15.29
	Airline statistics	revenue	monthly	✗	✓	0.26
	Office of the Superintendent of Financial Institutions mortgage data	mortgage interest outstanding	monthly	✓	✓	3.85
Other data	Electricity volume data	volume	hourly	✓	✓	2.66
	Various news reports	various	various	✓	✗	-

Source: Statistics Canada, internal estimates

In some cases, the individual series from the data source was very similar in coverage to the CPI elementary products. For example, the Monthly Retail Trade Survey's monthly sales estimates for full-service restaurants were used to adjust basket weights for the CPI class food purchased from table-service restaurants. In other cases, the individual proxy series was mapped to a higher-level product class and its monthly expenditure estimates were applied to all lowest product classes. And in a few cases with limited data availability, a proxy series with a different scope was used to escalate a CPI elementary product. One such example was for the shelter utility, water, which was escalated using electric power generation survey data on the assumption that electricity and water consumption would be similarly impacted by the increased demand from working from home and on-line schooling.

Methods

Annual CPI basket values were projected forward using data sources listed in Table 1 by applying the proxy's growth rate between the basket weight reference period and the reference period of the Adjusted price index. For most elementary products, the proxy series was measured in the dollar value of revenues, whereas for some products, the projection of expenditures used a combination of changes in quantities and changes in prices.

In version 1, an attempt was made to preserve an annual concept in the monthly adjusted weights. Where possible, seasonally-adjusted proxy data were used to project 2017 CPI basket expenditures forward. When these were not available, compound monthly growth rates versus the same month in 2017 were calculated from the proxy series, and then projected forward from mid-2017 to January through December 2019 using the compound monthly growth rate. Year-over-year growth rates from the monthly proxy series were then used to project from 2019 to January 2020 and later, and estimates were then constrained to be consistent with the 12-month change in High Frequency Expenditure Network estimates.

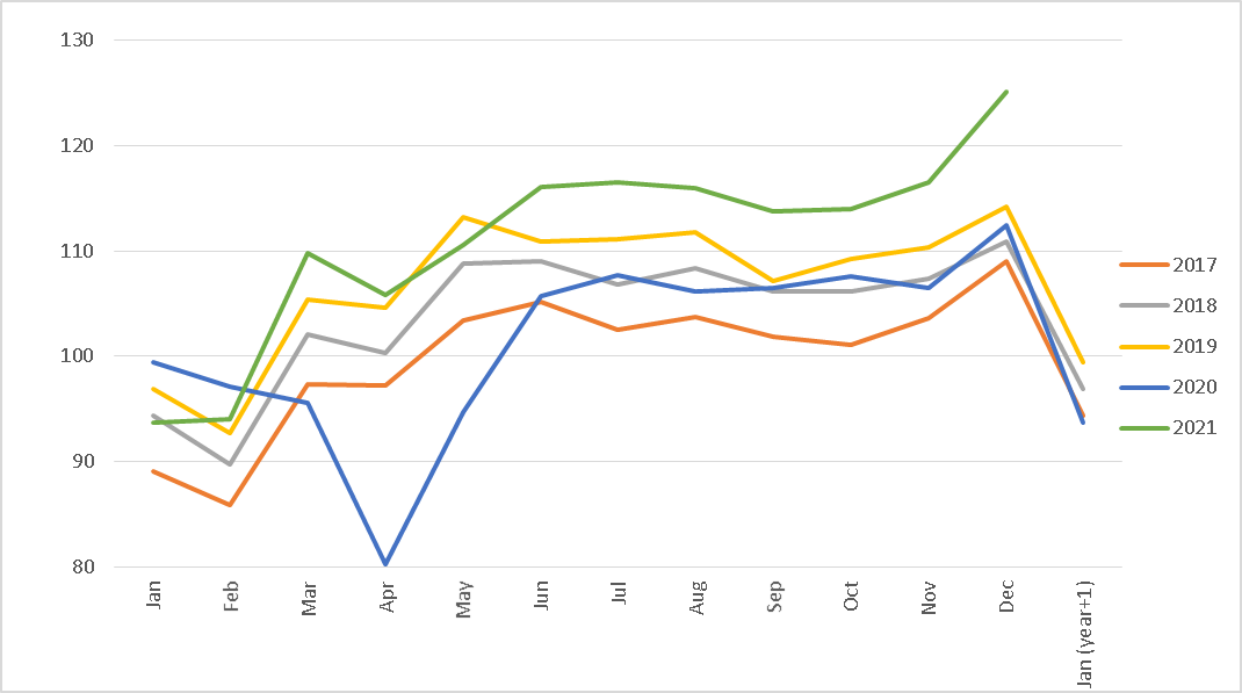
In the second version, raw, not-seasonally-adjusted data were used to project 2020 values forward. Estimates were then constrained to be consistent with the quarterly growth rate in Household Final Consumption Expenditures and the 12-month change in High Frequency Expenditure Network estimates. Appendix 3 provides further details on how monthly adjusted consumer expenditure basket weights were derived for version 2.

Results

After years of trending upward, monthly adjusted consumer expenditures on all items dropped below previous-year levels in March 2020, and then plummeted in April 2020 during the first full month of restrictions on Canadian spending (Chart 1). Afterwards, total spending only surpassed 2019 levels in

February 2021. Price increases for many goods and services contributed to the acceleration in spending in late 2021.

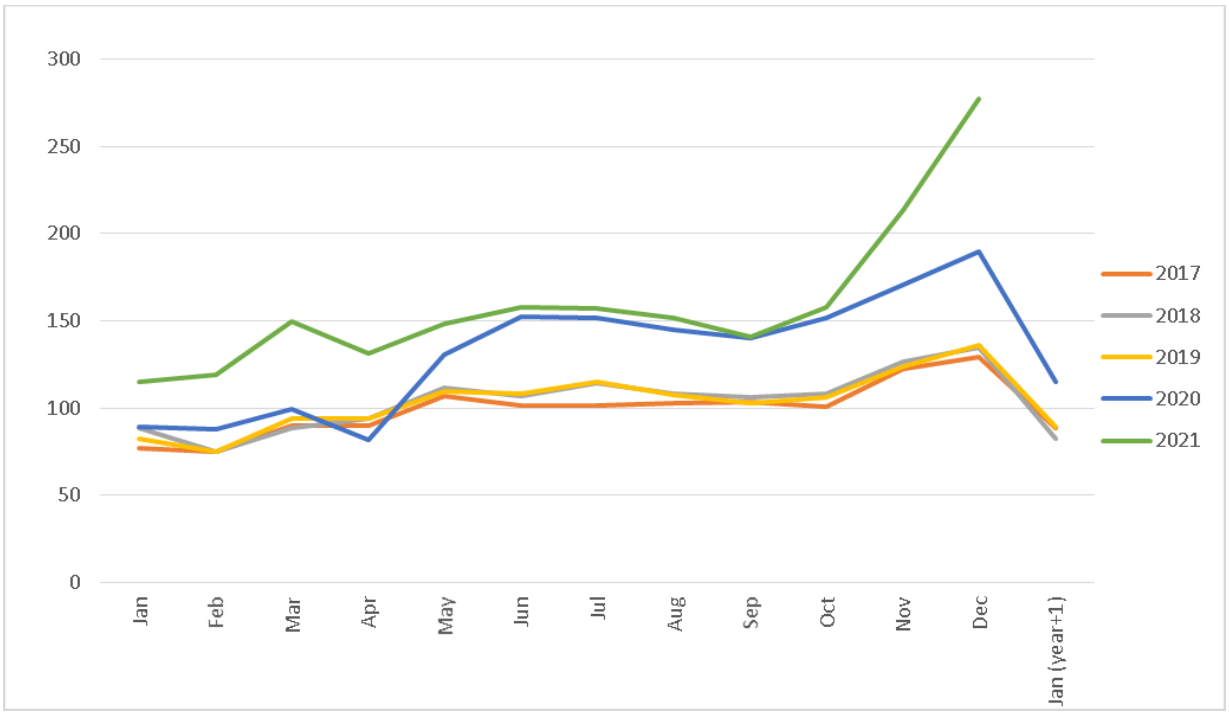
Chart 1: Monthly adjusted consumer expenditure basket weights, All items, January 2017 to December 2021, 2017=100



Source: Statistics Canada, internal estimates

The onset of the pandemic led to an increase in expenditures for some product groups. Many Canadians worked and schooled from home for the first time in spring 2020, and soon increased their spending on items such as household appliances to help improve the experience of prolonged periods at home (Chart 2).

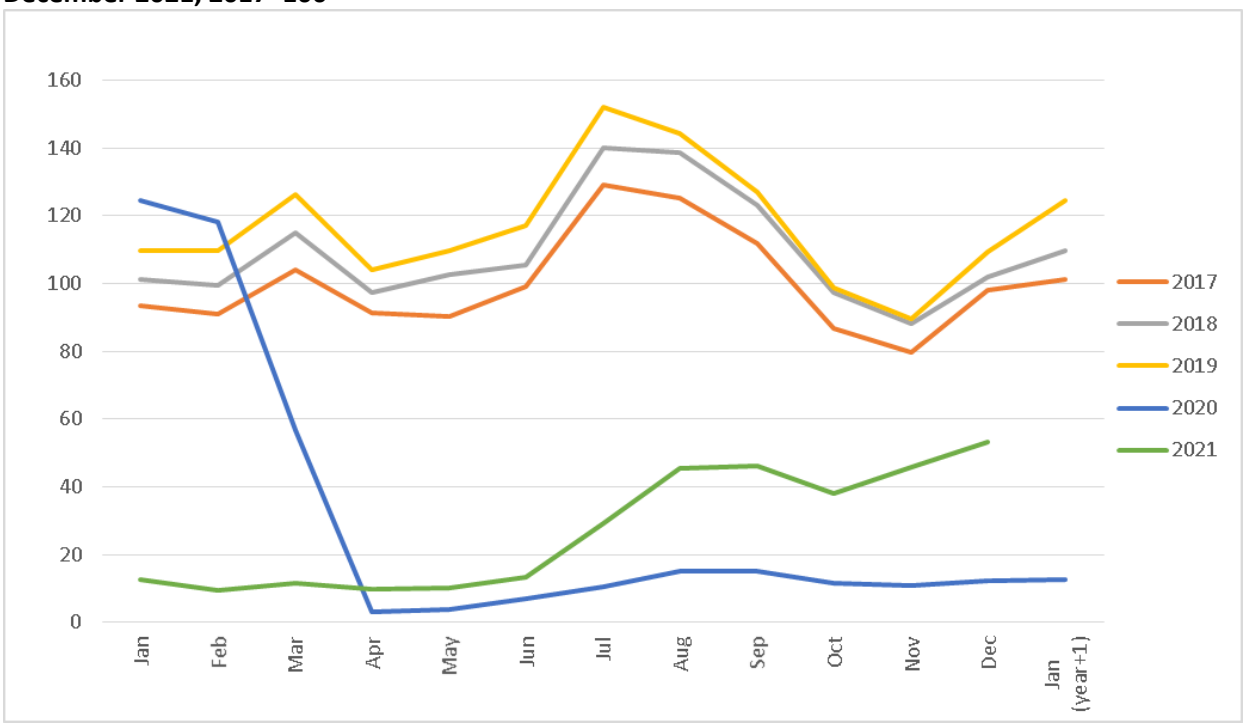
Chart 2: Monthly adjusted consumer expenditure basket weights, Household appliances, January 2017 to December 2021, 2017=100



Source: Statistics Canada, internal estimates

For other product groups, spending was severely restricted at the onset of the pandemic and stayed low for many months. Canadian expenditures on air transportation fell to nearly nil when flights were grounded in April 2020, and remained low until summer 2021 as restrictions slowly eased (Chart 3).

Chart 3: Monthly adjusted consumer expenditure basket weights, Air transportation, January 2017 to December 2021, 2017=100



Source: Statistics Canada, internal estimates

A full set of monthly adjusted consumer expenditure basket weights expressed as shares are available for 8 major aggregates and 110-plus analytical series in Statistics Canada data table 18-10-0264-01 Monthly Adjusted Consumer Expenditure Basket Weights.

Adjusted Price Index

Methods

Statistics Canada's All-items CPI is calculated with a Laspeyres-type formula at the upper level of price aggregation. The Laspeyres formula expresses the change between period 0 and period t in the cost of buying a fixed basket of goods and services, and is calculated by aggregating the prices of the products in the basket using quantities consumed from the price reference period 0 as weights.[8]

The Adjusted price index series from March 2020 to February 2021 was produced using the same geographic and product aggregation structure as the official CPI. However, unlike the official CPI, a monthly-chained Laspeyres index was calculated at the upper level, providing adjusted relatives for the March 2020 to February 2021 Adjusted price index, which was based on estimated previous month weights in order to reflect COVID-19 consumption patterns.

One of the limitations of a Laspeyres price index is that it uses quantities from an earlier period to aggregate prices. A Paasche price index uses quantities from the current period and often reflects substitutions made by consumers in response to price change. A Fisher price index is the geometric average of the Laspeyres and Paasche price indexes, and makes equal use of weights from the earlier period and current period to aggregate prices.

Following redevelopment in 2021, version 2 of the Adjusted price index uses a Fisher price index formula. Appendix 1 provides further details on the Laspeyres, Paasche and Fisher price index formulae.

Another limitation of a monthly-chained Laspeyres index is that the index is subject to chain drift. Chain drift can occur in a chained Laspeyres price index when consumers respond to price increases by reducing quantities consumed, or the reverse, leading to a gap between the chained Laspeyres and fixed-base Laspeyres price index. Appendix 1 provides an example of chain drift.

To overcome the chain drift issue, version 2 of the Adjusted price index uses a Similarity-linked Fisher price index, which is regarded as the most appropriate approach by leading price index experts.[9] In short, the Similarity-linked Fisher is calculated between two periods *t* and *r* such that *r* is prior to *t* and has the least dissimilar (or most similar) set of prices or quantities to period *t*. In our example in Appendix 1, time period 0 has identical prices and quantities to those of period 2, and so period 0 is less dissimilar than period 1. The Fisher price index at *t*=2 would be based on the Fisher price index between period 0 and period 2—in our case a relative of 1, meaning no price change between period 0 and period 2. Appendix 2 provides details on how the Similarity-linked Fisher price index was calculated up to September 2021.

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[8] From Statistics Canada, "Canadian Consumer Price Index Reference Paper," Catalogue no. 62-553-X, February 27 2019, Section 6.24.

[9] See "The Chain Drift Problem and Multilateral Indices" in IMF et.al., *Consumer Price Index Theory*, 2020 (hiips://www.imf.org/ -/media/Files/Data/CPI/companion-publication/chapter-6-chain-drift-problem-and-multilateral-indices.ashx)

The Adjusted price index versions 1 and 2 use Canada-level price changes from the 500-plus elementary products in the CPI.

Results

In the first months of the pandemic, the Consumer Price Index fell substantially, registering a monthly decline of 0.6% in March 2020 and 0.7% in April 2020. There were soon concerns that Canadians had altered their spending so dramatically that the CPI's 2017 basket weights were no longer relevant and the CPI was overstating the price declines experienced by Canadians.

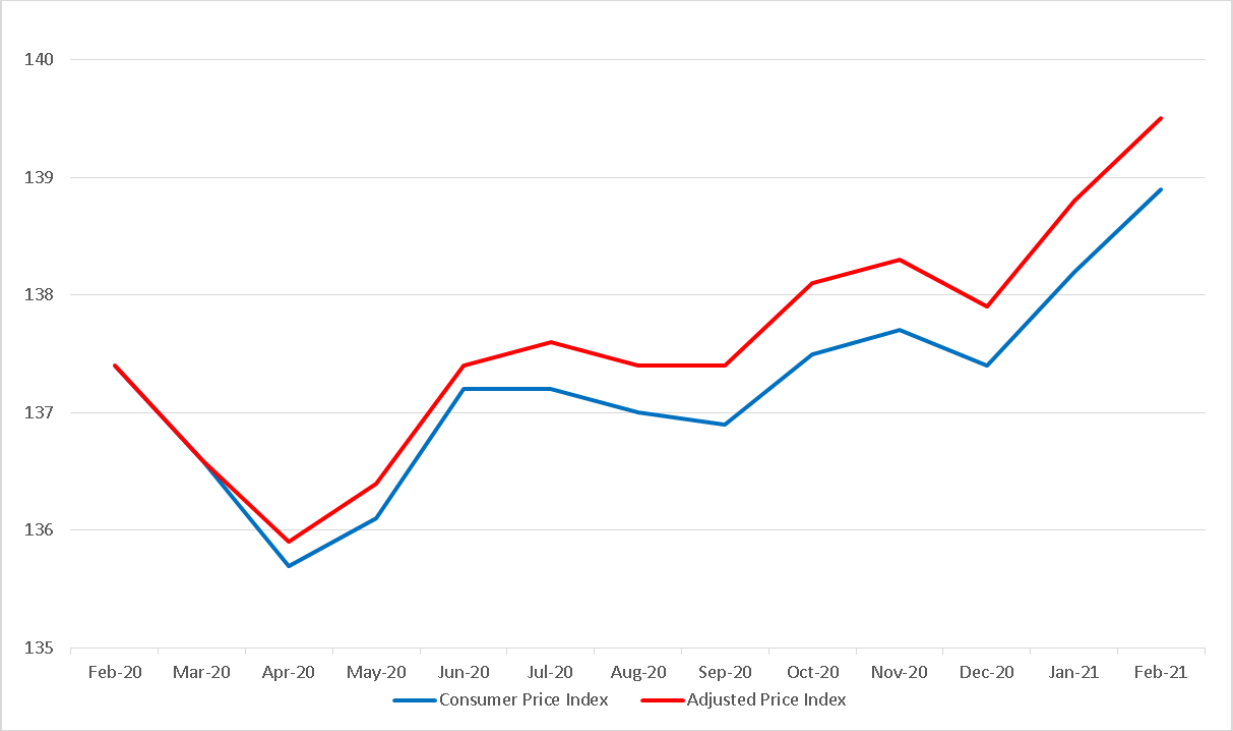
As a result, Statistics Canada's Consumer Price Index program developed and published the Adjusted price in July 2020.

Version 1 of the Adjusted price index, a monthly-chained Laspeyres index set to equal the CPI in February 2020, did indeed trend above the official CPI starting in April 2020. The lockdowns starting in mid-March 2020 prompted Canadians to reduce expenditures on items such as clothing and footwear, leading to a 3.33% basket share for this category in March 2020 in the Adjusted price index versus a 5.40% price-updated basket share in the official CPI. As a result, the March to April 2020 5.9% price decrease for clothing and footwear was much more muted in the Adjusted price index than in the official CPI.

Over the same period, many restaurants were closed and Canadians stocked up on food staples.[10] Spending on food from stores made up 16.67% of all consumer spending in March 2020 in the Adjusted price index compared to a price-updated basket share of 11.49% in the official CPI. The March to April 2020 0.9% price increase for food from stores took on much greater importance in the Adjusted price index than in the official CPI.

Chart 4: Consumer Price Index and Adjusted price index (version 1), February 2020 to February 2021

[10] See Statistics Canada, "Canadian Consumers Prepare for COVID-19", Catalogue no. 62F0014M, April 8 2020 and Statistics Canada, "Canadian Consumers Adapt to COVID-19: A Look at Canadian Grocery Sales up to April 11," Catalogue no. 62F0014M May 11 2020.



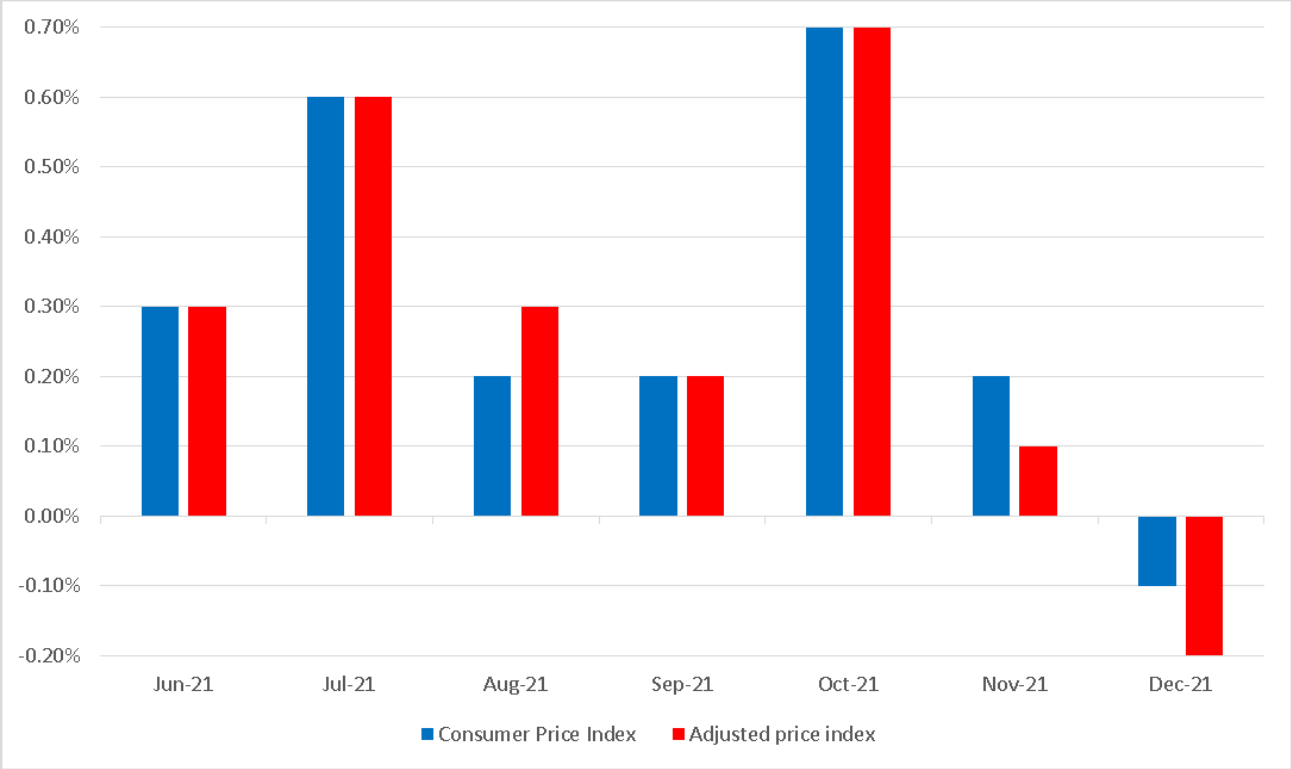
Source: Statistics Canada data tables 18-10-0263-01 Monthly adjusted price index, provisional and 18-10-0004-01 Consumer Price Index, monthly, not seasonally adjusted

In subsequent months, the Adjusted price index continued to exceed the Consumer Price Index (Chart 4), indicating a more elevated rate of price change than the official measure.

The Monthly-chained Laspeyres used for version 1 of the Adjusted price index has several attractive properties: it is simple to calculate, it can be arithmetically decomposed, and it can be produced without weight data for the current reference period. However, it is subject to excessive chain drift.

Upon development of the Adjusted price index version 2, a different pattern emerged (Chart 5). In the seven months to December 2021, the monthly percentage change in the Adjusted price index exceeded that of the CPI in only one month (August), while it was equal to or below the monthly CPI change in all other months.

Chart 5: Consumer Price Index and Adjusted price index (version 2), 1-month percent change, June 2021 to December 2021



Source: Statistics Canada data tables 18-10-0271-01 Adjusted price index, monthly percentage change and 18-10-0004-01 Consumer Price Index, monthly, not seasonally adjusted

In December 2021, the monthly change in the redeveloped Adjusted price index fell below that of the CPI for the fifth straight December[11]. Seasonal expenditures and prices on items such as video equipment in the days before and after Christmas contributed to this gap. Prices for video equipment fell by 3.4% in December 2021, while the monthly adjusted weight for video equipment rose to 0.81% in November and 1.02% the following month. Meanwhile the price-updated basket weight used in the December 2021 CPI was only 0.65%. The video equipment price decrease took on greater importance in the Adjusted price index than in the CPI, contributing to a lower monthly estimate.

Version 1 Adjusted price index levels are available at Statistics Canada data table **18-10-0263-01 Monthly adjusted price index, provisional** **.** Version 2 Adjusted price index estimates expressed as monthly percentage change are available at Statistics Canada data table **18-10-0271-01 Adjusted price index, monthly percentage change.**

Interaction between Weights, Quantities and Prices

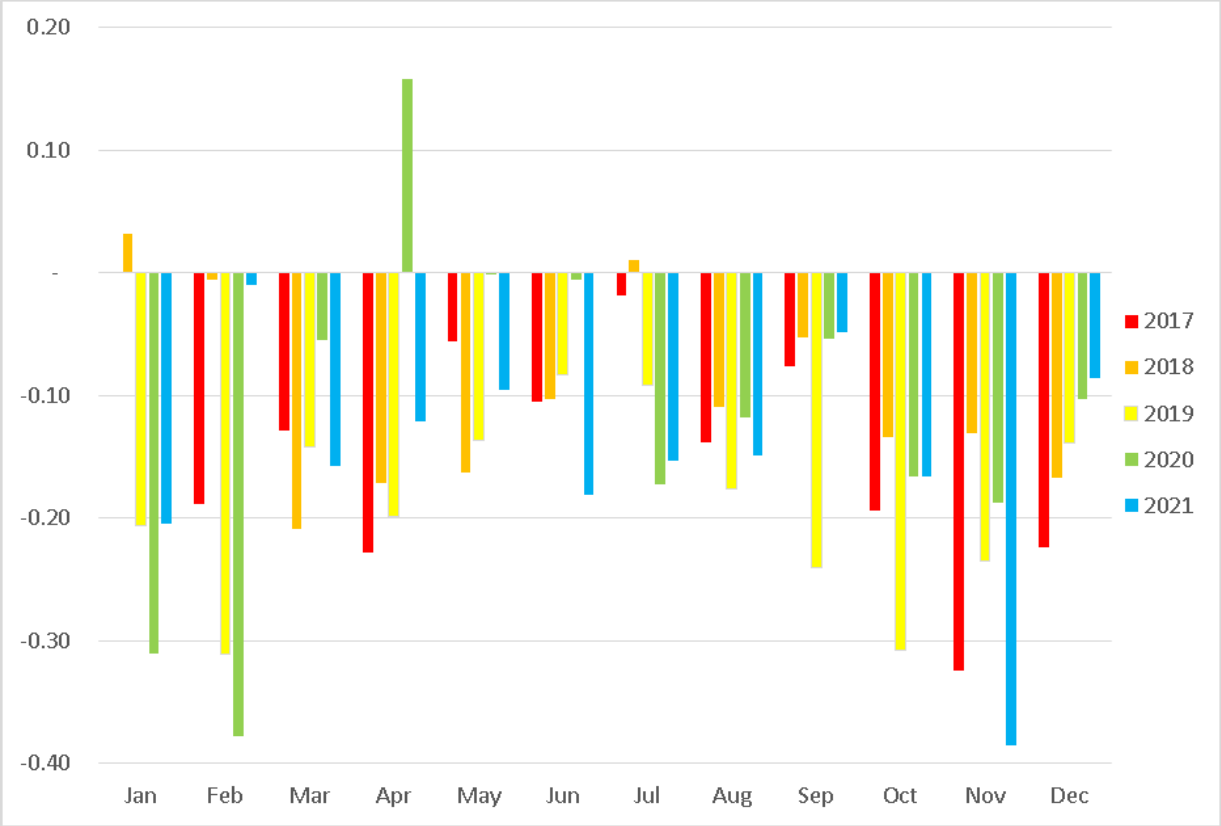
Using data calculated for the Adjusted price index, we can examine the historical relationship between changes in Canadian consumer spending patterns and changes in prices.

Starting from monthly price relatives for the 500-plus elementary products in the Consumer Price Index and estimates of monthly consumer expenditures, we can derive the monthly relative in quantities

[11] Statistics Canada, internal estimates.

consumed[12]. If we then calculate the correlation coefficient between quantity and price relatives by month[13], we can measure the extent to which consumers adjust their demand following price changes and retailers adjust prices resulting from changes in demand.

Chart 7: Pearson correlation coefficient between quantity and price relatives, February 2017 to December 2021



Source: Statistics Canada, internal estimates

In most months from 2017 through 2021, the correlation coefficient between quantity and price relatives is negative, reflecting the tendency for consumers to shift their demand towards goods and services whose prices decline, at least relative to others (Chart 7). The negative relationship tends to strengthen in fall and winter, and moderate in spring and summer.

April 2020, the first full month of lockdown during the COVID-19 pandemic, stands out as an historical anomaly when aggregate price change and quantity change were very positively correlated. Derived monthly quantities for gasoline fell 43.1% in that month as consumers took many fewer trips to work and school, while monthly prices at the pump also decreased (-15.2%) in April. Quantities and prices also

[12] $q_{n,t} / q_{n,t-1} = e_{n,t} / e_{n,t-1} * p_{n,t} / p_{n,t-1}$
where
q_t are quantities of elementary product *n* consumed in period *t*
e_t are expenditures on elementary product *n* in period *t*
p_t are prices for elementary product *n* in period *t*
[13] Weighted by $(e_{n,t-1} + e_{n,t}) / 2$

declined for traveller accommodation (-52.4%, -7.2%) and clothing and footwear (-44.1%, -5.9%), contributing to the positive correlation in April 2020.

Next Steps

Both the official CPI and the Adjusted price index have played key roles in measuring Canada's highly fluid economy and supporting the trajectory of Canada's post-pandemic economic recovery. The Adjusted price index has provided Canadians with data and insights they need on Canada's shifting consumer prices and expenditures as Canada recovers from the COVID-19 pandemic.

On May 9, 2022, Statistics Canada will publish the Adjusted price index and monthly adjusted consumer expenditure estimates to March 2022, and starting with the May 2022 CPI, the official measure will adopt a new basket to reflect 2021 consumer expenditure patterns.

The future of the Adjusted price index and monthly adjusted consumer expenditure basket weights is being discussed with users and stakeholders. Under consideration is whether publication of the index should continue and the use of the index as an analytical series outside of the official CPI, among other potential uses.

With ongoing access to more detailed expenditure data, Statistics Canada would be able to pursue the development of other sought-after indicators, such as measures of inflation for different groups, household types and geography.

Appendix 1: Common Price Index Formulae

Table A1 presents the formulae and example data for the calculation of commonly-used price indexes[14].

Table A1: Common price index formulae, with example

index name	index formula		t = 0				t = 1							t = 2									
		item	q_0	p_0	p_0q_0	P_0	q_1	p_1	p_1q_1	p_1 / p_0	$p_1q_0 = p_0q_0 * p_1 / p_0$	$p_0q_1 = p_1q_1 / (p_1/ p_0)$	P_1	q_2	p_2	p_2q_2	p_2 / p_1	$p_2 / p_0 = p_1 / p_0 * p_2 / p_1$	$p_2q_1 = p_1q_1 * p_2 / p_1$	$p_1q_2 = p_2q_2 / (p_2 / p_1)$	$p_2q_0 = p_1q_0 * p_2 / p_1$	$p_0q_2 = p_2q_2 / (p_2 / p_0)$	P_2
		beef	1	10	10		2	10	20	1.00	10	20		1	10	10	1.00	1.00	20	10	10	10	
		pork	2	20	40		1	25	25	1.25	50	20		2	20	40	0.80	1.00	20	50	40	40	
		sum			50				45		60	40				50			40	60	50	50	
Fixed-base Laspeyres Price Index	$P_{L(F)} = 100 * \Sigma p_t q_0 / \Sigma p_0 q_0$					100.0							120.0										100.0
Fixed-base Paasche Price Index	$P_{P(F)} = 100 * \Sigma p_t q_t / \Sigma p_0 q_t$					100.0							112.5										100.0
Fixed-base Fisher Price Index	$P_{F(F)} = 100 * ((\Sigma p_t q_0 / \Sigma p_0 q_0) * (\Sigma p_t q_t / \Sigma p_0 q_t))^{1/2}$					100.0							116.2										100.0
Monthly Chained Laspeyres Price Index	when t=0, $P_{L(MCh)} = 100$ when t>0, $P_{L(MCh)} = P_{L(MCh),t-1} * \Sigma p_t q_{t-1} / \Sigma p_{t-1} q_{t-1}$					100.0							120.0										106.7
Monthly Chained Paasche Price Index	when t=0, $P_{P(MCh)} = 100$ when t>0, $P_{P(MCh)} = P_{P(MCh),t-1} * \Sigma p_t q_t / \Sigma p_{t-1} q_t$					100.0							112.5										93.8
Monthly Chained Fisher Price Index	when t=0, $P_{F(MCh)} = 100$ when t>0, $P_{F(MCh)} = P_{F(MCh),t-1} * ((\Sigma p_t q_{t-1} / \Sigma p_{t-1} q_{t-1}) * (\Sigma p_t q_t / \Sigma p_{t-1} q_t))^{1/2}$					100.0							116.2										100.0

Note that in time period 1, the result P_1 for the Laspeyres—either the fixed-base or monthly chained—is higher than the Paasche. The Laspeyres uses the earlier period quantities to weight the prices, whereas the Paasche uses the current period's quantities after consumers have substituted some beef for pork, and so the Laspeyres is higher. This occurs in markets where consumers respond to price change by shifting quantities consumed in the opposite direction.

[14] See also Statistics Canada, *The Canadian Consumer Price Index Reference Paper*, Catalogue no. 62-553-X, February 27, 2019, and International Labour Organization/International Monetary Fund/Organisation for Economic Co-operation and Development/Statistical Office of the European Communities/United Nations/The International Bank for Reconstruction and Development/The World Bank, *Consumer Price Index Manual: Theory and Practice*, 2004

The Fisher is the geometric average of the Laspeyres and the Paasche price indexes, either fixed-base or monthly-chained, and its level will always bisect the Laspeyres and Paasche. The Fisher is in the class of "superlative" price indexes which make equal use of weights from both periods whose prices are being compared. Superlative indexes remove the effects of substitution, and can be used to measure its effects when compared to the Laspeyres or Paasche price indexes.

Note also that in time period 2 in the example, the prices and quantities consumed have returned to time period 0 levels, but the monthly-chained Laspeyres price index does not return to their period 0 level, and the monthly-chained Laspeyres price index diverges from the fixed-base Laspeyres price index.

This can be explained using the following:

$P_{L(Ch),1:2} = \sum_n^N p_{n,2}q_{n,1} / \sum_n^N p_{n,1}q_{n,1} = \sum_n^N (p_{n,1}q_{n,1} / \sum_n^N p_{n,1}q_{n,1} * p_{n,2} / p_{n,1})$
$P_{L(F),1:2} = \sum_n^N p_{n,2}q_{n,0} / \sum_n^N p_{n,1}q_{n,0} = \sum_n^N (p_{n,1}q_{n,0} / \sum_n^N p_{n,1}q_{n,0} * p_{n,2} / p_{n,1})$

$P_{L(Ch),1:2} - P_{L(F),1:2} = \sum_n^N (p_{n,1}q_{n,1} / \sum_n^N p_{n,1}q_{n,1} * p_{n,2} / p_{n,1}) - \sum_n^N (p_{n,1}q_{n,0} / \sum_n^N p_{n,1}q_{n,0} * p_{n,2} / p_{n,1})$

where
n is an elementary product
N is the total number of elementary products
0, 1 and 2 are periods
$p_{n,t}$ is the price for elementary product *n* in period *t*
$p_{n,u}q_{n,v}$ is the expenditure on elementary product *n* with period *u* prices and period *v* quantities

The monthly-chained Laspeyres uses period 1 quantities to aggregate period 1 to period 2 price change, whereas the fixed-base Laspeyres uses period 0 quantities.

In our example, consumers have reduced quantities of pork from period 0 to period 1 as the price increased. The relative importance of pork in period 1 used in the monthly-chained Laspeyres ($p_{n,1}q_{n,1} / \sum_n^N p_{n,1}q_{n,1} = 25 / 45 = 56\%$) is less than the period 0 weight of pork used in the fixed-base Laspeyres ($p_{n,1}q_{n,0} / \sum_n^N p_{n,1}q_{n,0} = 40 / 50 = 80\%$). As a result, in period 2, the price drop for pork from period 1 to period 2 will have less impact in the monthly-chained Laspeyres index than in the fixed-base Laspeyres index.

When prices and quantities interact in this way, the monthly-chained Laspeyres price index will exceed the fixed-base Laspeyres price index. This divergence is often referred to as chain drift.

Appendix 2: The Similarity-linked Fisher using predicted share measure of relative price dissimilarity and predicted share measure of relative quantity dissimilarity

The following method was used to derive the Similarity-linked Fisher price index used in the Adjusted price index. Starting with period 1, for each value of *t*, and for all prior periods *r* = 0:*t*-1, compute a Predicted Share measure of relative price dissimilarity:

$$\Delta_{SP}(p^r,p^t,q^r,q^t) = \sum_{n=1}^{N} (p_{n,t}q_{n,t}/\sum p_{n,t}q_{n,t} - (p_{n,r}q_{n,t}/\sum p_{n,r}q_{n,t}))^2 + \sum_{n=1}^{N} (p_{n,r}q_{n,r}/\sum p_{n,r}q_{n,r} - (p_{n,t}q_{n,r}/\sum p_{n,t}q_{n,r}))^2$$

and a Predicted Share measure of relative quantity dissimilarity:

$$\Delta_{SQ}(p^r,p^t,q^r,q^t) = \sum_{n=1}^{N} (p_{n,t}q_{n,t}/\sum p_{n,t}q_{n,t} - (p_{n,t}q_{n,r}/\sum p_{n,t}q_{n,r}))^2 + \sum_{n=1}^{N} (p_{n,r}q_{n,r}/\sum p_{n,r}q_{n,r} - (p_{n,r}q_{n,t}/\sum p_{n,r}q_{n,t}))^2$$

where
$\Delta_{SP}(p^r,p^t,q^r,q^t)$ is the Predicted Share measure of relative price dissimilarity
$\Delta_{SQ}(p^r,p^t,q^r,q^t)$ is the Predicted Share measure of relative quantity dissimilarity
n is an elementary product
N is the total number of elementary products (N = 515)
t is the later period
r is a prior period
$p_{n,t}q_{n,t}$ is the expenditure on elementary product *n* in period *t*
$p_{n,r}q_{n,r}$ is the expenditure on elementary product *n* in period *r*
$p_{n,r}q_{n,t}$ is the expenditure on elementary product *n* in period *t*, multiplied by the change in price on elementary product *n* from period *t:r*
$p_{n,t}q_{n,r}$ is the expenditure on elementary product *n* in period *r*, multiplied by the change in price on elementary product *n* from period *r:t*.

Find the minimum of $\Delta_{SP}(p^r,p^t,q^r,q^t)$ and $\Delta_{SQ}(p^r,p^t,q^r,q^t)$, denoted as min ($\Delta_{SP}(p^r,p^t,q^r,q^t)$, $\Delta_{SQ}(p^r,p^t,q^r,q^t)$). Then find the period *r* with the lowest min ($\Delta_{SP}(p^r,p^t,q^r,q^t)$, $\Delta_{SQ}(p^r,p^t,q^r,q^t)$). Finally, calculate the Fisher price index between *r* and *t* using:

$$P_{F(SPQ),r:t} = (\sum_{n=1}^{N} p_{n,t}q_{n,t} / \sum_{n=1}^{N} p_{n,r}q_{n,t} * \sum_{n=1}^{N} p_{n,t}q_{n,r} / \sum_{n=1}^{N} p_{n,r}q_{n,r})^{1/2}$$

where
$P_{F(SPQ),r:t}$ is the Similarity-linked Fisher price index between periods *r* and *t* using the Predicted Share measure of relative price dissimilarity and the Predicted Share measure of relative quantity dissimilarity
n is an elementary product
N is the total number of elementary products
t is the later period
r is a prior period
$p_{n,t}q_{n,t}$ is the expenditure on elementary product *n* in period *t*
$p_{n,r}q_{n,t}$ is the expenditure on elementary product *n* in period *t*, multiplied by the elementary price index for product *n* from period *t:r*
$p_{n,r}q_{n,r}$ is the expenditure on elementary product *n* in period *r*

$p_{n,t}q_{n,r}$ is the expenditure on elementary product *n* in period *r*, multiplied by the elementary price index for product *n* from period *r:t*

Table A2 illustrates the similarity in prices and quantities between each period from January 2020 to September 2021. Grey shading is applied to the cell for period *r* with the minimum value of $\Delta_{SP}(p^r,p^t,q^r,q^t)$ or $\Delta_{SQ}(p^r,p^t,q^r,q^t)$ for each *t*.

Table A2 also presents the resulting Fisher price index between each period from January 2020 to September 2021. Grey shading is used for the period *r* which satisfies the minimum of $\Delta_{SP}(p^r,p^t,q^r,q^t)$, $\Delta_{SQ}(p^r,p^t,q^r,q^t)$ for each period *t*.

Table A2: Predicted share measure of relative price and quantity dissimilarity and Fisher price index between each period from May 2021 and September 2021

			period *r*			
			202105	202106	202107	202108
period *t*	Predicted Share measure of relative price dissimilarity	202106	0.000005			
		202107	0.000011	0.000004		
		202108	0.000019	0.000010	0.000006	
		202109	0.000021	0.000010	0.000006	0.000002
	Predicted Share measure of relative quantity dissimilarity	202106	0.000492			
		202107	0.001092	0.000396		
		202108	0.001429	0.000700	0.000107	
		202109	0.000880	0.000625	0.000323	0.000292
	Bilateral Fisher Price Index between period r and t	202106	1.003			
		202107	1.009	1.006		
		202108	1.011	1.008	1.002	
		202109	1.013	1.010	1.004	1.002

Appendix 3: Derivation of Monthly Adjusted Consumer Expenditure Basket Weights, Version 2

There were a number of techniques used to estimate monthly consumer expenditures for the 515 elementary products in the Canadian CPI.

The first, used for 79.88% of products by 2020 basket weight, applied the growth rates in revenues or expenditures between 2020 and the reference month of the Adjusted price index for a similar good or service to estimate consumer expenditures on a product in that month.

Starting with the following definitions:
y is a year >= 2020
m is a month from 1 to 12
n is an elementary CPI product class, n = 1,2, …, N
N = 515 elementary CPI product classes
$\sum_{m=1}^{12} E_{2020,m,n}$ is the sum of expenditures from months 1 through 12 in year 2020 on elementary product *n*, as used in the CPI basket weight update
$e_{y,m,n}$ is the estimated expenditure in year *y*, month *m* on elementary product *n*
j is a proxy series for elementary CPI product class n
$v_{y,m,j}$ is the revenue or expenditure in year *y*, month *m* on proxy series *j*

, then method 1 derives monthly elementary product expenditures using:

(1) $e_{y,m,n} = (\sum_{m=1}^{12} E_{2020,m,n} / 12) * v_{y,m,j} / (\sum_{m=1}^{12} v_{2020,m,j} / 12)$

When there were no reasonable sales or expenditure proxy series to escalate a CPI product class, a second method was used. This method, used for 8.48% of products by weight, escalated the 2020 elementary product basket weight by the change in quantities from a proxy series relative to the proxy's 2020 average quantity and the change in the elementary product's CPI versus its 2020 average CPI level.

If we add the following definitions:
$Q_{y,m,j}$ is the quantity or volume level for proxy series *j* in year *y* and month *m*
$p_{y,m,n}$ is the price index for elementary product *n* in year *y* and month *m*

, then method 2a derives monthly elementary product expenditures using:

(2a) $e_{y,m,n} = (\sum_{m=1}^{12} E_{2020,m,n} / 12) * (Q_{y,m,j} / (\sum_{m=1}^{12} Q_{2020,m,j} / 12)) * (p_{y,m,n} / (\sum_{m=1}^{12} p_{2020,m,n} / 12))$

When the price growth from a proxy series was more suitable to use than the price growth from elementary product *n*, a different methods was used. By weight, 13.60%[15] of products were adjusted by escalating the 2020 elementary product basket weight by the change in quantities from a proxy series relative to the proxy's 2020 average level and the change in the levels of a price or price index from a second proxy series versus its 2020 average price or price index level.

[15] Weights do not add to 100%. Multiple methods were used for some products due to limited availability of proxy series for all months.

If we add the following definitions:
k is another proxy series
$P_{y,m,k}$ is the price level or price index for proxy series *k* in year *y* and month *m*

, then method 2b derives monthly elementary product expenditures using:

(2b) $e_{y,m,n} = (\sum_{m=1}^{12} E_{2020,m,n} / 12) * (Q_{y,m,j} / (\sum_{m=1}^{12} Q_{2020,m,j} / 12)) * (P_{y,m,k} / (\sum_{m=1}^{12} P_{2020,m,k} / 12))$

In some cases, due to the timeliness of data, multiple sources were used for the same elementary product class: one proxy source was used for an earlier period, while another was used to bring the series forward to the current reference period. For these cases, the monthly growth rate from the more recent periods were used to escalate the derived expenditures from the earlier periods.

In addition, by weight, 83.48% of elementary products underwent a quarterly adjustment. For those elementary classes whose primary data source in the 2020 basket was national Household Final Consumption Expenditures (HFCE), the quarterly HFCE growth rate was used to constrain the growth rate of all lower-level product groups in all months covered by HFCE quarterly data. This will make the growth rates in the monthly expenditure estimates consistent with the outputs of Statistics Canada's efforts to reconcile estimates from a variety of statistical programs. It should also make the monthly adjusted consumer expenditure weights more consistent with subsequent CPI basket weight updates, which will be largely based on the same data sources.

If we add the following definitions:
A is a set of elementary product classes
n_A is the number of elementary product classes in A
h is an HFCE class similar in scope to aggregate product class *A*
$v_{2020,h}$ is the Household Final Consumption Expenditure on class *h* in year 2020
$v_{2021,Q2,h}$ is the Household Final Consumption Expenditure on class *h* in year 2021, quarter 2
$e'_{y,m,n}$ is the constrained estimated expenditure for elementary product *n* in year *y* and month *m*

, then method 3 constrains the estimated monthly elementary product expenditures. The following illustrates the adjustment in year 2021 month 6 using HCE data for year 2020 and year 2021 quarter 2, and estimated expenditures for year 2020, year 2021 month 6, and year 2021 months 4 through 6 (quarter 2):

(3) $e'_{2021,6,n} = e_{2021,6,n} *((v_{2021,Q2,h} / 3) / (v_{2020,h} / 12)) / ((\sum_{m=4}^{6}\sum_{n=1}^{n\_A} e_{2021,m,n} / 3) / (\sum_{n=1}^{n\_A} E_{2020,n} / 12))$

In the periods following the most recent quarterly adjustment, the monthly growth rates were reapplied to the most recent monthly expenditure value from the constraint in method 3. For year 2021, month 7, the adjustment would be:

(4) $e'_{2021,7,n} = e'_{2021,7,n} * e_{2021,7,n} / e_{2021,6,n}$

Finally, for those products mapped to HFEN expenditure categories, in months following the most recent HFCE quarter, the 12-month growth rate in spending reported by HFEN data was used to adjust the expenditure estimates derived from previous steps. This makes the growth rate for the adjusted

products, in aggregate, consistent with this robust data set. The HFEN adjustment method was applied to 60.56% of products by weight. For year 2021, month 7, the HFEN adjustment would be:

(5) $e''_{2021,7,n} = e'_{2021,7,n} / \sum_{n=1}^{n\_A} e'_{2021,7,n} * (v_{2021,7,b} / v_{2020,7,b}) * \sum_{n=1}^{n\_A} e'_{2020,7,n}$

where:
b is an HFEN class equal in scope to aggregate product class *A*
$e''_{2021,7,n}$ is the HFEN-constrained monthly expenditure for elementary product *n* in year 2021, month 7
$v_{2021,7,b} / v_{2020,7,b}$ is the 12-month growth rate in HFEN expenditures for HFEN class *b* from period 202007 to 202107.