Data Envelopment Analysis (DEA) assessment of composite indicators of infrastructure endowment

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Abstract

In Data Envelopment Analysis the distance from the best practice frontier can be interpreted as the economic performance of sample units. In the present paper this distance is used as an efficiency measure to correct the weighted average of non-substitutable sub-indicators of infrastructure endowment.

Keywords: benefit-of-the-doubt indicators, composite indicators, Data Envelopment Analysis, infrastructure endowment. JEL classification: C18; R50; H54.

1. Introduction

Data envelopment analysis (DEA),first introduced by Farrell (1957) and successively developed by Charnes, Cooper e Rhodes (1978), is a linear programming technique; it defines the *best practice frontier* that serves as a benchmark and computes the relative distance between each unit and the frontier. This distance can be interpreted as the economic performance of the units in the sample. Within the context of composite indicators this interpretation has been used to reassess indicators, see for example Mahlberg and Obersteiner (2001); Despotis (2005), and their reassessment of the Human Development Index; also see Somarriba and Pena (2009), and Sharpe and Andrews (2010) for applications within the context of quality of life and economic well being respectively.

In this paper, however, we make use of the distance from the best practice frontier as an efficiency measure to *correct* a composite indicator of endowment. In fact whenever it is reasonable to assume non-substitutability among the sub-indicators, their weighted average should also take into account the combination (or relative proportion) between the sub indicators used in the aggregation function. We therefore suggest the use of DEA to measure the efficiency of the combination of inputs whose weighted aggregation defines the composite indicator of infrastructure endowment; and to correct the composite indicator by taking into account the efficiency of the combination of sub-indicators.

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The views expressed in this paper are solely those of the authors and to not involve the responsibility of Istat.

2. Data envelopment analysis

DEA measures the relative efficiency of decision making units on the basis of multiple inputs and outputs. The efficiency of a unit is defined as the weighted sum of its outputs divided by a weighted sum of its inputs. “The weights for inputs and outputs are estimated by a linear programme so as to maximize the relative efficiency of each unit” (Despotis, 2005).

Farrel (1957) introduced the concept of *best practice frontier* which delineates the technological limits of what a country can achieve with a given level of resources. The distance from the frontier can be used as a performance indicator.

The techniques used to measure the efficiency of a set of firms can be adapted and used also in the context in which a synthetic objective overall index is to be built.

We use an input-oriented DEA, which is a mathematical programming method that could achieve the task (Coelli , 1996)

The computation of the envelope and the performance indicator can be reduced to a linear programme for each individual unit.

The model assumes *N* inputs and *M* outputs for each of the *I* units (for us regions). For the i-th unit the inputs are represented by an array $\underline{x}_i$ and the outputs are represented by an array $\underline{q}_i$. A first problem's formulation is the following: for each unit *i* the ratio of all the outputs over all the inputs is defined as

$$f(\underline{u}, \underline{v}) = \frac{\underline{u}'\underline{q}_i}{\underline{v}'\underline{x}_i}$$

where $\underline{u}$ is an array of output weights and where $\underline{v}$ is an array of input weights . The model seeks to maximize f , which represents the efficiency of the unit *i*, subject to the constraints that all the efficiency measures must be less than or equal to one. Moreover the weights must be positive. This linear programme is solved for each unit assigning to it the most favourable weights. A problem with this formulation is that it has infinite solutions of the form ($\delta\underline{u}$, $\delta\underline{v}$) for $\delta > 0$. To avoid it, the following constraint is introduced

$$\underline{v}'\underline{x}_i = 1$$

Thus the problem can be written as

$$\begin{cases} \max\limits_{\underline{u},\underline{v}} \underline{u}'\underline{q}_i \\ \underline{v}'\underline{x}_i = 1 \\ \underline{u}'\underline{q}_1 - \underline{v}'\underline{x}_1 \leq 0 \\ \vdots \\ \underline{u}'\underline{q}_I - \underline{v}'\underline{x}_I \leq 0 \\ \underline{u}, \underline{v} \geq 0 \end{cases}$$

Lastly we call the model DEA with no input if for the array $\underline{x}_i$ we have $\underline{x}_i=(1,...,1)'$.

The model computes the weights so that the unit under investigation is valued as best as possible. The weights can differ from unit to unit; and range between 0 and 1.

A particular weakness of DEA must be underlined. Any unit supporting the frontier is credited as equally well performing even if it is superior with respect to one indicator but performs poorly with respect to all the others. For such a unit DEA computes a high weight for the indicator on which the unit is superior and a low weight for all the others. In fact as Cherchye et al. (2007) affirm: "*the core idea is that a good relative performance of a country in one particular sub indicator dimension indicates that this country considers the policy dimension concerned as relatively important*". Such a data oriented weighting method is justifiable in the typical composite indicator context of uncertainty about and consensus on an appropriate weighting scheme, and opens the way to the "benefit of the doubt" indicators(Cherchye et al., 2007, 2008).

3. DEA as a measure of the efficiency of a combination of indicators

3.1 DEA with no inputs

Assume we have an infrastructure endowment composite indicator obtained as a weighted average of different sub-indicators. For the sake of simplicity assume equal weighs of the different (possibly normalized) sub-indicators. This means that a situation in which subindicator 1 assumes value 100 and sub indicator 2 assumes value 50 is equivalent to a situation in which sub indicator 1 assumes value 50 and sub indicator 2 100 : the value for the composite infrastructure endowment indicator remains the same. The question is: do we agree that the sub-indicators are substitutable? Can we define and measure the efficiency of their combination and define on this basis an equivalence between different values (and different combinations) of sub-indicators?

DEA provides a useful insight.

Let us take as an example, 8 health care infrastructure endowment indicators for 20 italian regions:

1. public health care expenditure per 10.000 residents
2. National health care staffx 1.000 residents
3. medical specialist x 10.000 residents
4. Primary Care Trusts Number x 1.000.000 residents
5. physicians 10.000 residents
6. emergency medical service x 1.000.000 residents
7. n. inpatient beds x 10.000 residents
8. rehabilitation centres/long-term care centre/nursing x 1.000.000 residents

These indicators are necessary and not substitutable: how could health care be provided without physicians or impatient beds or without a sensible combination of these or any of the other indicators?

First of all we perform an unconstrained DEA without input to obtain the so called Benefit of the Doubt (BoD) composite indicator. These and the next results shed a light on how *doubtful* some composite indicators can be.

As a result of DEA without input we have (in table 1a) that more than 50% of the regions are classified as efficient and thus would obtain the highest value for the composite BOD indicator of infrastructure endowment. It is the precise aim of DEA to value each unit as best as possible allowing for different combinations of inputs.

Table 1a - Efficiency for DEA with no input (scenario A.1)

REGIONS	Efficiency	REGIONS	Efficiency
Piemonte	95,86%	Marche	93,76%
Valle d'Aosta	100,00%	Lazio	100,00%
Lombardia	91,32%	Abruzzo	100,00%
Trentino Alto Adige	100,00%	Molise	100,00%
Veneto	88,97%	Campania	84,84%
Friuli Venezia Giulia	100,00%	Puglia	93,51%
Liguria	100,00%	Basilicata	100,00%
Emilia Romagna	100,00%	Calabria	100,00%
Toscana	100,00%	Sicilia	97,03%
Umbria	100,00%	Sardegna	100,00%

As for the weights, they are reported in table 1b.

Table 1b - Weights for DEA with no input (scenario A.1)

	Piemonte	**Valle d'Aosta**	**Lombardia**	**Trentino Alto Adige**	**Veneto**	**Friuli Venezia Giulia**	**Liguria**
u_1	0,00%	1,45%	0,00%	1,66%	0,00%	1,10%	2,40%
u_2	37,85%	7,90%	0,00%	77,19%	9,55%	34,80%	42,73%
u_3	5,16%	5,23%	50,19%	1,54%	0,00%	5,72%	32,52%
u_4	0,00%	68,85%	0,00%	0,63%	17,86%	7,28%	7,23%
u_5	44,49%	14,53%	0,00%	4,97%	70,48%	50,06%	13,83%
u_6	0,00%	0,03%	0,00%	0,10%	0,00%	0,05%	0,08%
u_7	12,51%	0,35%	49,81%	10,34%	2,11%	0,47%	0,85%
u_8	0,00%	1,65%	0,00%	3,57%	0,00%	0,54%	0,35%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
	Emilia Romagna	**Toscana**	**Umbria**	**Marche**	**Lazio**	**Abruzzo**	**Molise**
u_1	0,59%	2,81%	0,48%	0,00%	2,77%	0,29%	0,00%
u_2	41,21%	67,27%	2,60%	15,10%	2,22%	1,13%	8,22%
u_3	30,44%	6,66%	5,51%	0,00%	77,10%	5,69%	12,85%
u_4	1,98%	0,22%	19,44%	0,00%	7,23%	2,23%	24,29%
u_5	1,66%	17,45%	71,75%	83,11%	8,15%	89,55%	17,28%
u_6	0,04%	0,07%	0,01%	0,00%	0,18%	0,02%	9,89%
u_7	24,02%	0,89%	0,15%	1,79%	1,71%	0,40%	22,14%
u_8	0,06%	4,63%	0,06%	0,00%	0,64%	0,70%	5,33%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

Table 1b continued **- Weights for DEA with no input (scenario A.1)**

	Campania	Puglia	Basilicata	Calabria	Sicilia	Sardegna
u_1	56,16%	0,00%	2,79%	1,43%	0,00%	2,06%
u_2	0,00%	0,00%	12,79%	6,63%	0,00%	11,50%
u_3	11,57%	0,00%	8,72%	71,81%	6,79%	9,84%
u_4	0,00%	0,00%	34,65%	3,48%	0,00%	35,30%
u_5	31,37%	98,06%	29,60%	5,39%	92,46%	26,05%
u_6	0,00%	0,00%	9,30%	10,27%	0,21%	0,05%
u_7	0,00%	1,79%	0,91%	0,85%	0,00%	0,59%
u_8	0,90%	0,15%	1,23%	0,15%	0,54%	14,62%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

It should be noticed that many weights are null, and this, for necessary sub-indicators does not make much sense. We thus performed a DEA with weights constrained to be positive. The number of efficient regions(see table 2a) decreased sensibly.

Table 2a - Efficiency for DEA with no input and non negative constraints (scenario B.1)

REGIONS	Efficiency	REGIONS	Efficiency
Piemonte	87,31%	Marche	86,40%
Valle d'Aosta	100,00%	Lazio	88,92%
Lombardia	77,37%	Abruzzo	93,10%
Trentino Alto Adige	96,97%	Molise	100,00%
Veneto	79,88%	Campania	75,39%
Friuli Venezia Giulia	97,67%	Puglia	84,80%
Liguria	95,50%	Basilicata	100,00%
Emilia Romagna	91,15%	Calabria	98,68%
Toscana	97,66%	Sicilia	89,59%
Umbria	87,77%	Sardegna	100,00%

Looking at the weights (table 2b) it is easy to see that there is very great variability among them. Moreover in some cases a not-too-bad overall performance is due almost exclusively to one single sub indicator; for example Toscana an Lazio (high weights respectively on u_2 - national health care staff- and u_3 number of specialists) but nearly in the same situation also Sicilia, Puglia, Abruzzo and Campania with non negligeable weights only on u_5 - n. of physicians - and u_3 medical specialists (respectively around 83% and 8% in all three regions).

Table 2b - Weights for DEA with no input and non negative constraints (scenario B.1)

	Piemonte	Valle d'Aosta	Lombardia	Trentino Alto Adige	Veneto	Friuli Venezia Giulia	Liguria
u_1	1,39%	2,28%	2,70%	1,82%	1,36%	1,39%	1,55%
u_2	59,42%	8,46%	2,70%	78,59%	60,55%	59,42%	70,00%
u_3	8,62%	5,08%	51,22%	1,82%	1,36%	8,62%	20,69%
u_4	1,39%	67,81%	2,70%	1,82%	1,36%	1,39%	1,55%
u_5	25,04%	13,04%	2,70%	1,82%	31,32%	25,04%	1,55%
u_6	1,39%	1,02%	2,70%	1,82%	1,36%	1,39%	1,55%
u_7	1,39%	1,30%	32,61%	10,49%	1,36%	1,39%	1,55%
u_8	1,39%	0,99%	2,70%	1,82%	1,36%	1,39%	1,55%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
	Emilia Romagna	**Toscana**	**Umbria**	**Marche**	**Lazio**	**Abruzzo**	**Molise**
u_1	1,69%	1,58%	1,23%	1,36%	1,95%	1,15%	6,21%
u_2	68,82%	84,36%	44,40%	60,55%	1,95%	1,15%	14,31%
u_3	16,01%	4,14%	3,45%	1,36%	86,38%	8,40%	8,13%
u_4	1,69%	1,58%	11,88%	1,36%	1,95%	1,15%	17,97%
u_5	1,69%	1,58%	35,36%	31,32%	1,95%	84,71%	28,88%
u_6	1,69%	1,58%	1,23%	1,36%	1,95%	1,15%	8,24%
u_7	6,74%	1,58%	1,23%	1,36%	1,95%	1,15%	8,99%
u_8	1,69%	3,62%	1,23%	1,36%	1,95%	1,15%	7,27%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

	Campania	Puglia	Basilicata	Calabria	Sicilia	Sardegna
u_1	2,16%	1,15%	4,87%	3,09%	1,15%	3,86%
u_2	1,16%	1,15%	13,45%	3,09%	1,15%	12,20%
u_3	8,14%	8,25%	9,51%	70,17%	8,40%	10,91%
u_4	1,16%	1,15%	29,92%	3,09%	1,15%	33,50%
u_5	83,90%	84,66%	27,32%	3,09%	84,71%	25,11%
u_6	1,16%	1,15%	8,91%	11,30%	1,15%	1,93%
u_7	1,16%	1,33%	3,27%	3,09%	1,15%	2,43%
u_8	1,16%	1,15%	2,76%	3,09%	1,15%	10,05%
v_1	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

The next question is: what happens if we compute average weights and thus a (possibly) unique best performing unit?

Computing the averages of the constrained weights we obtain:

Table 3a - Mean Weights for DEA with no input and non negative constraints

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	v_1
1,95%	32,35%	13,40%	9,98%	34,92%	2,08%	3,21%	2,12%	100,00%

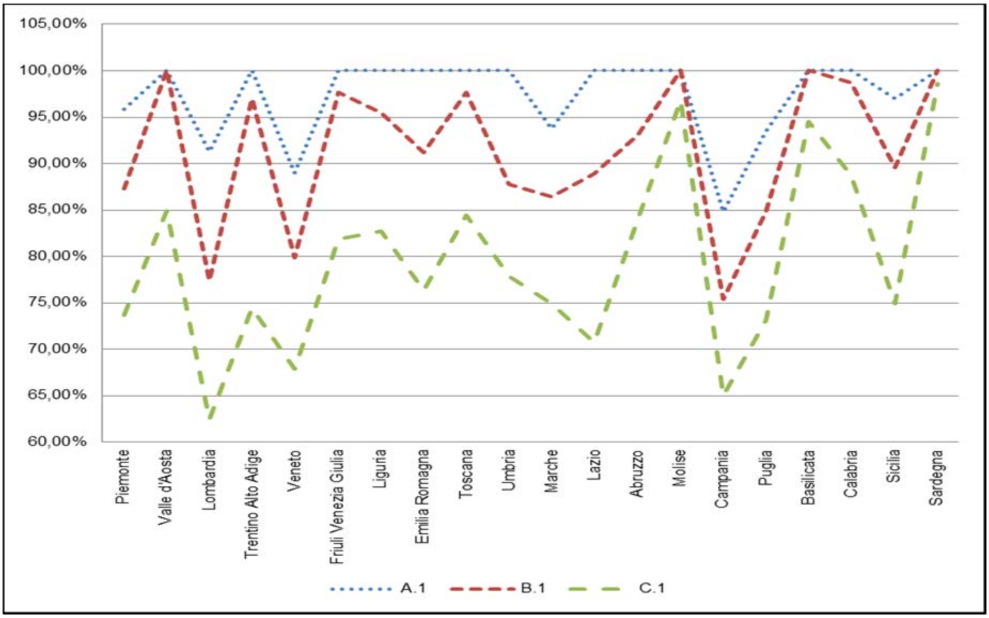
Considering that some of the units of measure of the indicators are per 1.000, others per 10.000, the weights obtained for the different indicators seem not too far from what we could expect.As for efficiency none of the regions appear to be efficient (see table 3b).

Table 3b - Efficiency for DEA with no input and non negative constrained mean weights (scenario C.1)

REGIONS	Efficiency	REGIONS	Efficiency
Piemonte	73,66%	Marche	74,97%
Valle d'Aosta	84,93%	Lazio	70,72%
Lombardia	62,49%	Abruzzo	84,04%
Trentino Alto Adige	74,43%	Molise	96,85%
Veneto	67,92%	Campania	65,03%
Friuli Venezia Giulia	81,85%	Puglia	73,10%
Liguria	82,76%	Basilicata	94,49%
Emilia Romagna	76,39%	Calabria	88,77%
Toscana	84,44%	Sicilia	74,91%
Umbria	77,91%	Sardegna	98,66%

To compare the efficiencies of three scenarios we have figure 1:

Figure 1 - DEA with no input (scenario 1) Efficiency comparison A.1, B.1. C.1



The efficiency rankings appear to be a lot different between the three models and different from the health care infrastructure endowment rankings we would have if we defined the composite indicator as the average of the standardized indicators (thus resorting to so called equal weighting) . This result stands against the use of DEA to construct BoD composite indicators; in fact DEA is an efficiency measure, whereas composite indicators relate to the endowment. This consideration leads us to our original suggestion: the use of DEA to correct a composite endowment indicator taking into account the efficiency of the sub-indicators combination. If we were to choose among the three different scenarios for DEA without input, we would certainly choose to compute our measure of performance by means of average constrained to be positive weights (i.e.Scenario C.1). In fact, as Despotis (2005) comments on the use of DEA without input to compute a Human Development Indicator: "*The DEA approach is meaningful in identifying the 'inefficient' countries. The DEA scores, however, cannot be used to rank the countries in terms of human development, given that the scores are not based on common weights*".

Now, given we need common weights in order to compare the performance of the different regions, the next choice to be made is whether to base our efficiency measure on a production function i.e. DEA with input.

3.2 DEA with different combinations of inputs and outputs

Our next step is to define costs as input indicators, the others as outputs.

First of all we define scenario 2: DEA with one input and choose 1. public health care expenditure per 10.000 residents as an input indicator and all the others as output sub indicators.

Then we add a second input variable (Primary Care Trusts Number x 1.000.000 residents) to define scenario 3.

For both scenarios the other indicators are outputs and the weights are average constrained to be strictly positive weights. First of all let us compare – for these new scenarios – the average constrained to be positive weights. In table 5 we have the different sets of weights for the three scenarios defined so far:

Table 5 - Average weights for the three scenarios C.1,2,3.

	C.1		C.2		C.3
u_1	1,95%	u_1	28,63%	u_1	36,17%
u_2	32,35%	u_2	14,01%	u_2	23,08%
u_3	13,40%	u_3	21,96%	u_3	18,09%
u_4	9,98%	u_4	22,32%	u_4	3,51%
u_5	34,92%	u_5	2,03%	u_5	9,19%
u_6	2,08%	u_6	8,37%	u_6	9,96%
u_7	3,21%	u_7	2,68%	v_2	21,17%
u_8	2,12%	v_1	100,00%	v_1	78,83%
v_1	100,00%				

While in table 6 we compare efficiencies by means of ranks.

Table 6 - DEA Efficiency comparison by means of ranks of all scenarios

REGIONS	Ranks A.1	Ranks B.1	Ranks C.1	Ranks C.2	Ranks C.3
Piemonte	15	15	15	13	18
Valle d'Aosta	1	1	5	4	10
Lombardia	18	19	20	19	19
Trentino Alto Adige	1	8	14	12	14
Veneto	19	18	18	18	20
Friuli Venezia Giulia	1	6	9	9	12
Liguria	1	9	8	8	7
Emilia Romagna	1	11	11	10	13
Toscana	1	7	6	7	6
Umbria	1	14	10	11	15
Marche	16	16	12	14	11
Lazio	1	13	17	17	16
Abruzzo	1	10	7	6	4
Molise	1	1	2	2	2
Campania	20	20	19	20	17
Puglia	17	17	16	16	8
Basilicata	1	1	3	3	3
Calabria	1	5	4	5	5
Sicilia	14	12	13	15	9
Sardegna	1	1	1	1	1

For some regions the efficiency evaluation is stable, for others it changes sensibly.

Unfortunately the weights change significantly and so does the efficiency of the input combination.

As already underlined it is difficult to accept the suggestion (see Cherchye et al., 2007, Despotis 2005, Mahlberg and Obersteiner , 2001) to resort to DEA or to benefit-of-doubt indicators as an alternative to traditional composite indicators. The main reason is that we would use an efficiency measure to compute endowment. A second reason is that even when computing common weights for all regions there is no clear indication on which scenario to prefer.

Our suggestion is to use efficiency to correct an equally weighted composite indicator. Which scenario? What general indication? To use as output variables for an efficiency measure , the same variables the composite indicator is based on. In fact we believe the efficiency measure used to correct the endowment composite indicator must be based on exactly the same sub-indicators, so that in view of non-substitutability their inefficient combination can be accounted for. We will thus take into account DEA without input.

4. Comparison with the Method of Penalties by coefficient of variation

Other suggestions have been introduced in literature to account for non-substitutability of the sub-indicators of infrastructure endowment. For example (Brunini Paradisi Terzi, 2004, and similarly Mazziotta Pareto, 2007) suggest a correction of a composite indicator of infrastructure endowment based on an inverse function of the horizontal variability of the sub-indicators. Under the hypothesis of non-substitutability, equilibrium condition between the different dimensions of the phenomenon implies equality between the sub-indicators. Thus a penalty is attached to the variability among sub-indicators referring to the same unit. The underlying assumption is the availability of a benchmark unit (or region) whose normalized sub-indicators all assume the same value. (In other words the infrastructure endowment for the benchmark region is assumed to be the objective equilibrium endowment; all sub-indicators are normalized with respect to these values).

In particular, according to Mazziotta-Pareto's Method of Penalties by Coefficient of Variation the composite indicator of infrastructure endowment of each region i (Mz_i) can be corrected by taking into account disequilibrium as measured by the square of the coefficient of variation $cv_i = S_{Z_i}/M_{Z_i}$. The corrected indicator (denominated MPI) is defined as

$$MPI = Mz_i(1 - cv_i^2)$$

where Mz_i, is the average of standardized [3] sub-indicators. This means that if for some regions all the sub-indicators have the same value, there is no correction.

Our suggestion is similar in spirit. In fact, to account for non-substitutability we suggest a correction of the composite indicator based on the distance from the efficient frontier, in other words the DEA_{cl} :

$$DEA_{C.l} = Mz_i(\text{efficiency}_i)$$

Table 7 shows our weights:

Table 7 - Penalty case: Mean Weights for DEA with no input and non negative constraints

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	v_1
10,98%	17,69%	11,11%	10,94%	14,07%	12,09%	11,80%	11,31%	100,00%

They are obtained exactly as in scenario 1, however now the variables are standardized to have mean 100 and std 10.

What are the differences between the two suggestions? For MPI the frontier is defined as equally weighted standardized sub-indicators; this is equivalent to saying that for the best performing unit all sub-indicators must have the same weight equal to 12,5% . For DEA instead the weights for the best performing unit are the values for u1, u2,...,u8 shown in table 7.

For an overall comparison we have reported in table 8 the values of the composite

[3] standardized to have mean 100 and std 10.

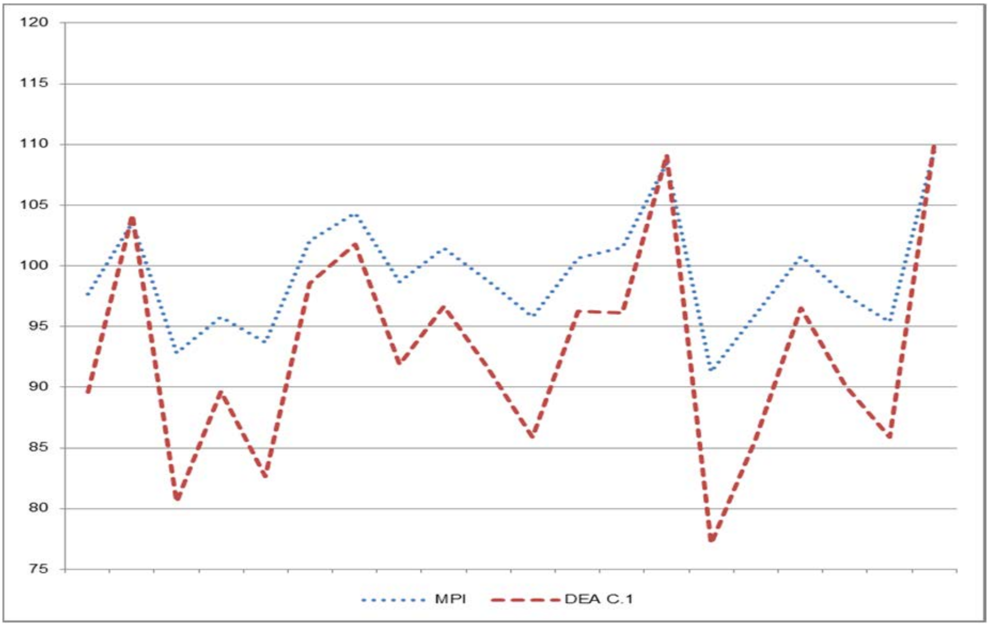
equally weighted indicator Mz_i; its penalties for the different regions, the efficiencies and the corrected indicators MPI and DEA_{C1} as well as their ranks.:

Table 8 - Penalty case: Composite indicator, penalties, efficiencies, corrected indicators

REGIONS	Mzi	(1-cv^2)	EFF	MPI	DEAcl	RankMzi	Rank MPI	RankDEAcl
Piemonte	97,9944	0,9968	0,9144	97,6808	89,6061	13	12	14
Valle	105,526	0,9812	0,9876	103,5421	104,2175	3	4	3
Lombardia	93,3124	0,9949	0,863	92,8365	80,5286	19	19	19
Bolzano	97,3887	0,9833	0,9209	95,7623	89,6853	14	16	13
Veneto	94,1838	0,9948	0,8777	93,694	82,6651	18	18	18
Friuli V. Giulia	102,5557	0,995	0,9611	102,0429	98,5663	5	5	5
Liguria	104,7887	0,996	0,9716	104,3695	101,8127	4	3	4
Emilia Romagna	99,2701	0,9943	0,9255	98,7043	91,8745	11	11	10
Toscana	101,8757	0,9957	0,9488	101,4376	96,6597	9	7	6
Umbria	99,3891	0,9937	0,921	98,7629	91,5374	10	10	11
Marche	96,1131	0,9969	0,8934	95,8151	85,8674	17	15	16
Lazio	101,9784	0,9871	0,9437	100,6629	96,237	6	9	8
Abruzzo	101,9456	0,9957	0,9432	101,5072	96,1551	7	6	9
Molise	109,1713	0,9933	1	108,4399	109,1713	2	2	2
Campania	91,7584	0,9951	0,8401	91,3088	77,0862	20	20	20
Puglia	96,3457	0,9964	0,8895	95,9989	85,6995	15	14	17
Basilicata	101,9352	0,9884	0,947	100,7528	96,5326	8	8	7
Calabria	98,4235	0,9921	0,9156	97,646	90,1166	12	13	12
Sicilia	96,1521	0,9918	0,8932	95,3637	85,8831	16	17	15
Sardegna	109,8921	0,9955	1	109,3976	109,8921	1	1	1

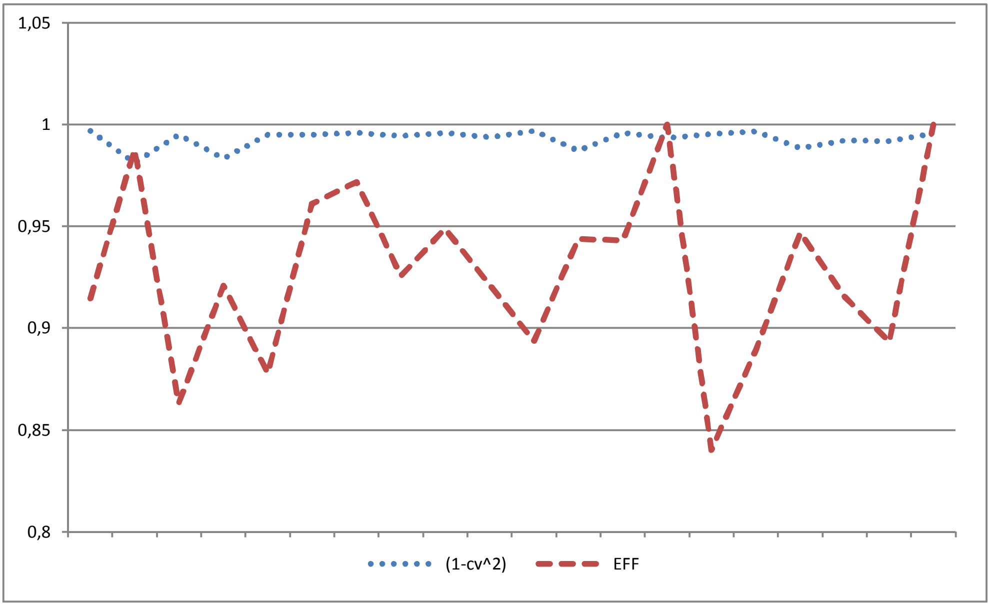
For a comparison between the two corrected indicators we have figure 2

Figure 2 - A comparison between the two indexes DEA_{CI} and MPI



Where as to compare the two different corrections we have figure 3

Figure 3 - Comparison between the two corrections DEA and coefficient of variation



As expected the two corrections give rise to different values of the composite corrected indicator. In particular efficiency shows greater variability among regions and is lower than the MPI correction; thus DEA_{c1} indicator is almost uniformly lower than MPI, whereas comparison among the rankings is not so straightforward.

5. Concluding remarks

The suggestion to correct composite indicators to account for non-substitutability is not new in literature; in particular it is not new within infrastructure endowment indicators. An interesting suggestion that we have taken as a reference guideline is provided by Mazziotta-Pareto's Method of Penalties by Coefficient of Variation. A possible interpretation of their method is that sub-indicators are considered in equilibrium (or optimally combined) whenever for one unit they assume identical values. Moving along this same direction we argue that the tool to measure the efficiency of a combination of indicators is provided by Data Envelopment Analysis. Different DEA models provide different results. In fact DEA is not as objective as it claims to be since it depends on the variable/sub-indicators choice. We believe that further research along this path will lead to more definite suggestions and hope that Istat or individual readers will take up the challenge. The conclusions that can be drawn from our study is that comparing the results of the different scenarios there are differences in the values and in the rankings of the indicators, but these differences are not too strong. With respect to the uncorrected indicator Mz_i for both alternative rankings we have a maximum difference between ranks of corresponding units (maxdiff) equal to 3. Considering its range between 0 and 19 the differences are not huge.

References

Brunini C., Paradisi F., Terzi S. , (2004) Banca dati delle infrastrutture, una proposta di metodo per la sintesi di indicatori, ISTAT *Progetto interdipartimentale "Informazione statistica territoriale e settoriale per le politiche strutturali 2001-2008"* Roma: mimeo

Charnes A., Cooper W. W., Rhodes E., (1978), Measuring the efficiency of decision making units, *European Journal of Operational Research* n.2, 429-444

Cherchye L., Moesen W., Rogge N., Puyenbroeck T. V., (2007), An Introduction to Benefit of Doubt Composite Indicators, *Social Indicators Research* n. 82,111–145

Cherchye L., Moesen W., Rogge N., Puyenbroeck T. V., (2008), Benefit of the doubt composite indicators, KEI-project, *European Commission's Sixth Framework Programme*

Coelli T., (1996), A Guide to DEAP, CEPA Working Paper

Cooper W., Seiford L. M., Zhu J., (2011), Handbook on Data Envelopment

Despotis DK, (2005) , A reassessment of the human development index via data envelopment analysis, Journal of the Operational Research Society , n.56, 969–980

Farrell M.J. , (1957) “The measurement of productive efficiency” *Journal of the Royal Statistical Society*

ISTAT La dotazione di infrastrutture e servizi nella sanità - Anni 2004-2005.

Mahlberg B., Obersteiner M., (2001), Remeasuring the HDI by Data Envelopement Analysis, *International Institute for Applied Systems Analysis Interim Report* 01-069, Laxenburg, Austria

Mazziotta C., Mazziotta M., Pareto A., Vidoli F., (2010) La sintesi di indicatori territoriali di dotazione infrastrutturale: metodi di costruzione e procedure di ponderazione a confronto *Rivista di Economia e Statistica del territorio* v.1, pp 7-33.

Mazziotta M., Pareto A,. (2007) “Un indicatore sintetico di dotazione infrastrutturale: il metodo delle penalità per coefficiente di variazione”, in: *Lo sviluppo regionale nell’Unione Europea – Obiettivi. Strategie, politiche. Atti della XXVIII Conferenza Italiana di Sciene Regionali,* AISRe, Bolzano.

Sharpe A., Andrews B., (2010), An Assessment of Weighting Methodologies for Composite Indicators: The Case of The Index of Economic Well-Being, *CSLS Research Report* No. 10.

Somarriba N., Pena B., (2009) , Synthetic Indicators of Quality of Life in Europe, *Soc Indic Res* n.94,115–133.