Optimization of sampling strata with the SamplingStrata package

Package version 1.1

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Abstract

In stratified random sampling the problem of determining the optimal size and allocation of units in strata is solved by considering the stratification of the population as given. Conversely, the definition of the optimal stratification of a sampling frame for a given survey is investigated without choosing, as objective function, the sampling size required to satisfy given precision constraints on the parameters of interest of a given survey. This package allows the determination of the best stratification of a target population, the one that ensures the minimum sample size (or the minimum fieldwork and interviewing costs) so to satisfy precision constraints in a multivariate and multidomain case. The underlying algorithm is based on a non deterministic evolutionary approach, making use of the genetic algorithm paradigm. The specific functions for the execution of the genetic algorithm are a modified version of those contained in the genalg package.

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1 Introduction

Let us suppose we need to design a sample survey, having a complete frame containing information on the target population (identifiers plus auxiliary information). If our sample design is a stratified one, we need to choose how to form strata in the population, in order to get the maximum advantage by the available auxiliary information. In other words, we have to decide in which way to combine the values of the auxiliary variables (from now on, the 'X' variables) in order to determine a new variable, called 'stratum'. To do so, we have to take into consideration the target variables of our sample survey (from now on, the 'Y' variables): if, to form strata, we choose the X variables most correlated to the Ys, the efficiency of the samples drawn by the resulting stratified frame may be greatly increased. In order to handle the whole auxiliary information in a homogenous way, we have to reduce continuous data to categorical (by mean of a k-means clustering technique, for example). Then, for every set of candidate auxiliary variables Xs, we have to decide (i) what variables to consider as active variables in strata determination, and (ii) for each active variable, what set of values (in general, what aggregation of atomic values) have to be considered. Every combination of values of each active variable determine a particular stratification of the target population, i.e. a possible solution to the problem of 'best' stratification. Here, by best stratification, we mean the stratification that ensures the minimum sample cost, sufficient to satisfy a set of precision constraints, set on the accuracy of the estimates of the survey target variables Ys (constraints expressed as maximum allowable sampling variance on estimates in different domains of interest). When the cost of data collection is uniform over the strata, then the total cost is directly proportional to the overall sample size, and the convenience of a particular stratification can be measured by the associated size of the sample, whose estimates are expected to satisfy given accuracy levels. This minimum size can be determined by applying the Bethel algorithm, with its Chromy variant. In general, the number of possible alternative stratifications for a given population may be very high, depending on the number of variables and on the number of their values, and in these cases it is not possible to enumerate them in order to assess the best one. A very convenient solution to this, is the adoption of the evolutionary approach, consisting in applying a genetic algorithm that may converge towards a near-optimal solution after a finite number of iterations. The methodology is fully described in Ballin and Barcaroli (2013), and a complete illustration of the package, together with a comparison with the stratification package, is in Barcaroli (2014). The implementation of the genetic algorithm is based on a modification of the functions in the genalg package (see Willighagen (2005)).

2 Procedural steps

The optimization of the sampling design starts by making the sampling frame available, defining the target estimates of the survey and establishing the preci-

sion constraints on them. It is then possible to determine the best stratification and the optimal allocation. Finally, we proceed with the selection of the sample. Formalizing, these are the required steps:

- 1. analysis of the frame data: identification of available auxiliary information;
- 2. manipulation of auxiliary information: in case auxiliary variables are of the continuous type, they must be transformed into a categorical form;
- construction of atomic strata: on the basis of the categorical auxiliary variables available in the sampling frame, a set of strata can be constructed by calculating the Cartesian product of the values of all the auxiliary variables;
- 4. characterization of each atomic stratum with the information related to the target variables: in order to optimise both strata and allocation of sampling units in strata, we need information on the distributions of the target variables (means and standard deviations);
- 5. choice of the precision constraints for each target estimate, possibly differentiated by domain;
- 6. optimization of stratification and determination of required sample size and allocation in order to satisfy precision constraints on target estimates;
- 7. analysis of the resulting optimized strata;
- 8. association of new labels to sampling frame units, each of them indicating the new strata resulting by the optimal aggregation of the atomic strata;
- 9. selection of units from the sampling frame with a *stratified random sample* selection scheme;
- evaluation of the found optimal solution in terms of expected precision and bias.

In the following, we will illustrate each step starting from a real sampling frame, the one that comes with the R package sampling (the dataframe swissmunicipalities).

3 Analysis of the frame data and manipulation of auxiliary information

As a first step, we have to define a frame dataframe containing the following information:

- a unique identifier of the unit (no restriction on the name, may be 'cod');
- the (optional) identifier of the stratum to which the unit belongs;

- the values of m auxiliary variables (named from X1 to Xm);
- the (optional) values of p target variables (named from Y1 to Yp);
- the value of the domain of interest for which we want to produce estimates (named 'domainvalue').

By typing the following statements in the R environment:

- > library(SamplingStrata)
- > require(memoise)
- > data(swissmunicipalities)

we get the swissmunicipalities dataframe, that contains 2896 observations (each observation refers to a Swiss municipality). Among the others, there are the following variables (data are referred to 2003):

- REG: Swiss region.
- Nom: municipality name.
- Surfacesbois: wood area.
- Surfacescult: area under cultivation.
- Alp: mountain pasture area.
- Airbat: area with buildings.
- Airind: industrial area.
- Pop020: number of men and women aged between 0 and 19.
- Pop2040: number of men and women aged between 20 and 39.
- Pop4065: number of men and women aged between 40 and 64.
- Pop65P: number of men and women aged between 65 and over.
- POPTOT: total population.

First, we define the identifier of the frame:

- > swissframe <- NULL
- > swissframe\$id <- swissmunicipalities\$Nom

Let us suppose we want to plan a survey whose target estimates are the totals of population by age class in each Swiss region. In this case, our Y variables will be:

- Y1: number of men and women aged between 0 and 19.
- Y2: number of men and women aged between 20 and 39.

- Y3: number of men and women aged between 40 and 64.
- Y4: number of men and women aged between 65 and over.

So we execute the following statements:

```
> swissframe$Y1 <- swissmunicipalities$Pop020
> swissframe$Y2 <- swissmunicipalities$Pop2040
> swissframe$Y3 <- swissmunicipalities$Pop4065
> swissframe$Y4 <- swissmunicipalities$Pop65P</pre>
```

As for the auxiliary variables (Xs), we can use all of those characterising the area use (wood, mountain or pasture, cultivated, industrial, with buildings). As these variables are of the continuous type, first we have to reduce them in a categorical (ordinal) form. A suitable way to do so, is to apply a k-means clustering method (see Hartigan and Wong (1979)) by using the function var.bin:

```
> library(SamplingStrata)
> swissframe$X1 <- var.bin(swissmunicipalities$POPTOT, bins=18)
> swissframe$X2 <- var.bin(swissmunicipalities$Surfacesbois, bins=3)
> swissframe$X3 <- var.bin(swissmunicipalities$Surfacescult, bins=3)
> swissframe$X4 <- var.bin(swissmunicipalities$Alp, bins=3)
> swissframe$X5 <- var.bin(swissmunicipalities$Airbat, bins=3)
> swissframe$X6 <- var.bin(swissmunicipalities$Airind, bins=3)</pre>
```

Now, we have six different auxiliary variables of the categorical type, the first with 18 different modalities, the others with 3 modalities. Finally, we have to set the values of the 'domainvalue' variable, which is mandatory. As we want to obtain estimates for each region, we set:

```
> swissframe$domainvalue <- swissmunicipalities$REG
> swissframe <- data.frame(swissframe)</pre>
```

Now, the swissframe dataframe looks like this way:

> head(swissframe)

```
id
                Y1
                       Y2
                              Y3
                                    Y4 X1 X2 X3 X4 X5 X6
1
      Zurich 57324 131422 108178 66349 18
                                              2
2
      Geneve 32429 60074
                          57063 28398 17
3
       Basel 28161 50349
                           53734 34314 17
                                           1
4
        Bern 19399 44263
                           39397 25575 17
    Lausanne 24291 44202
                                              2
                                                    3
                                                       2
5
                           35421 21000 17
                                           2
                                                1
6 Winterthur 18942 28958 27696 14887 16
                                           3
  domainvalue
1
            4
2
            1
3
            3
4
            2
5
            1
6
            4
```

that is the format required by the package. We write the dataframe to a tab delimited file:

> write.table (swissframe, "swissframe.txt", row.names=FALSE,col.names=TRUE, sep="\t", quote

In any case, this dataframe comes with the package SamplingStrata: it can be made available by executing:

- > library(SamplingStrata)
- > data(swissframe)
- > head(swissframe)

	progr 1	REG	X1	Х2	ХЗ	Х4	Х5	Х6	id	Y1	Y2
1	1	4	18	3	2	1	3	3	Zurich	57324	131422
2	2	1	17	1	1	1	3	2	Geneve	32429	60074
3	3	3	17	1	1	1	3	3	Basel	28161	50349
4	4	2	17	2	3	1	3	3	Bern	19399	44263
5	5	1	17	2	2	1	3	2	Lausanne	24291	44202
6	6	4	16	3	3	1	3	3	${\tt Winterthur}$	18942	28958
	Y3 Y4			domainvalue							
1	108178	663	349				4				
2	57063	283	398	1			1				
3	53734	343	314	3			3				
4	39397	255	575	2			2				
5	35421	210	000	1							
6	27696	148	387				4				

4 Construction of atomic strata and association of the information related to target variables

The strata dataframe reports information regarding each stratum in the population. There is one row for each stratum. The total number of strata is given by the number of different combinations of Xs values in the frame. For each stratum, the following information is required:

- 1. the identifier of the stratum (named 'stratum' or 'strato'), concatenation of the values of the X variables;
- 2. the values of the m auxiliary variables (named from X1 to Xm) corresponding to those in the frame;
- 3. the total number of units in the population (named 'N');
- 4. a flag (named 'cens') indicating if the stratum is to be censused (=1) or sampled (=0);
- 5. a variable indicating the cost of interviewing per unit in the stratum (named 'cost');

- 6. for each target variable y, its mean and standard deviation, named respectively 'Mi' and 'Si');
- 7. the value of the domain of interest to which the stratum belongs ('DOM1').

For example:

- > data(strata)
- > head(strata)

```
stratum
             N X1 X2
                        ХЗ
                                  M1
                                            M2
                                                     S1
        1 2246 x11 x21 x31 148.1598 443.0137 95.41435
1
        2 2972 x11 x21 x32 184.2041 513.8995 81.26956
2
3
        3 1905 x11 x22 x31 193.8927 488.8046 79.66667
        4 3125 x11 x22 x32 181.3437 597.1925 82.77032
5
        5 1733 x12 x21 x31 109.9850 418.2234 88.20289
6
        6 1060 x12 x21 x32 114.7943 489.8292 52.71574
        S2 cens cost DOM1
1 202.4569
              0
                    1
                       tot
2 214.9999
              0
                    1
                       tot
3 261.1876
              0
                    1
                       tot
4 226.5086
              0
                       tot
5 179.1571
              0
                    1
                       tot
6 166.0292
              0
                       tot
```

If in the frame dataframe are also present the values of the target Y variables (from a census, or from administrative data), it is possible to automatically generate the strata dataframe by invoking the buildStrataDF function. Let us consider again the swissframe dataframe that we have in built in previous steps. On this frame we can apply the function buildStrataDF:

> swissstrata <- buildStrataDF(swissframe)

Computations have been done on population data

The function takes as unique argument the name of the frame, and also writes out in the working directory the strata file, always named 'strata.txt'. This is the structure of the created dataframe:

> head(swissstrata)

```
STRATO
                         M1
                                   M2
1 1*1*1*1*1*1 184 48.31522
                             49.40217
                                       61.44022 28.40761
                1 98.00000 106.00000 116.00000 43.00000
2 1*1*1*1*1*2
3 1*1*1*2*1*1
                2 57.00000
                             64.00000
                                       70.00000 50.00000
               11 77.72727
                             81.18182
                                       92.36364 47.00000
4 1*1*2*1*1*1
5 1*2*1*1*1*1
                9 58.22222
                             61.55556
                                       66.77778 36.22222
                8 61.00000
6 1*2*1*2*1*1
                             68.00000
                                       84.62500 58.37500
        S1
                 S2
                           S3
                                    S4 COST CENS DOM1 X1 X2
```

```
1 26.81536 28.49831 32.63062 14.63922
                                                0
                                                        1
   0.00000 0.00000 0.00000 0.00000
                                          1
                                                     1
                                                           1
  4.00000 0.00000 1.00000 15.00000
4 15.24998 18.69768 17.03084 11.12736
                                          1
                                                0
                                                     1
                                                        1
                                                           1
5 25.46360 20.27100 24.89881 15.49751
                                          1
                                                0
                                                        1
                                                           2
6 24.56624 19.48076 26.35307 26.55625
                                                        1
                                                           2
  X3 X4 X5 X6
   1
      1
         1
            1
      1
         1
            2
3
   1
      2
         1
            1
      1
         1
            1
5
   1
      1
         1
            1
      2
         1
```

It is worth while noting that the total number of different atomic strata is 641, lower than the dimension of the Cartesian product of the Xs (which is 4374): this is due to the fact that not all combinations of the value of the auxiliary variables are present in the sampling frame. Variables 'cost' and 'cens' are initialised respectively to 1 and 0 for all strata. It is possible to give them different values:

- 1. for variable 'cost', it is possible to differentiate the cost of interviewing per unit by assigning real values;
- 2. for variable 'cens', it is possible to set it equal to 1 for all strata that are of the 'take-all' type (i.e. all units in that strata must be selected).

The swissstrata dataframe comes together with SamplingStrata package, it can be made available by typing:

> data(swissstrata)

On the contrary, if there is no information in the frame regarding the target variables, it is necessary to build the strata dataframe starting from other sources, for instance a previous round of the same survey, or from other surveys. In this case, we need to read sample data by executing:

> samp <- read.delim("samplePrev.txt")</pre>

The only difference is that computed mean and variances of the Ys are sampling estimates, whose reliability should be evaluated by carefully considering their sampling variances. In addition to the naming constraints previously introduced, this case requires that a variable named 'WEIGHT' is present in the samp dataframe. Then we can execute this function in this way:

> strata <- buildStrataDF(samp)</pre>

The result is much the same than in the previous case: the function creates a new dataframe, strata, and writes out in the working directory the strata file, named 'strata.txt'.

Note that in all cases, for each target variable Y, mean and standard deviation are calculated excluding NAs.

5 Choice of the precision constraints for each target estimate

The errors dataframe contains the accuracy constraints that are set on target estimates. This means to define a maximum coefficient of variation for each variable and for each domain value. Each row of this frame is related to accuracy constraints in a particular subdomain of interest, identified by the DOM1 value. In the case of the Swiss municipalities, we have chosen to define the following constraints:

> data(swisserrors)

> swisserrors

```
DOM CV1 CV2 CV3 CV4 domainvalue
1 DOM1 0.08 0.12 0.08 0.12 1
2 DOM1 0.08 0.12 0.08 0.12 2
3 DOM1 0.08 0.12 0.08 0.12 3
4 DOM1 0.08 0.12 0.08 0.12 4
5 DOM1 0.08 0.12 0.08 0.12 5
6 DOM1 0.08 0.12 0.08 0.12 6
7 DOM1 0.08 0.12 0.08 0.12 7
```

This example reports accuracy constraints on variables Y1, Y2, Y3 and Y4 that are the same for all the 7 different subdomains (Swiss regions) of domain level DOM1. Of course we can differentiate the precision constraints region by region. It is important to underline that the values of 'domainvalue' are the same than those in the frame dataframe, and correspond to the values of variable 'DOM1' in the strata dataframe. Once having defined dataframes containing frame data, strata information and precision constraints, it is worth while to check their internal and reciprocal coherence. It is possible to do that by using the function checkInput:

> checkInput(swisserrors,swissstrata,swissframe)

Input data have been checked and are compliant with requirements

For instance, this function controls that the number of auxiliary variables is the same in the frame and in the strata dataframes; that the number of target variables indicated in the frame dataframe is the same than the number of means and standard deviations in the strata dataframe, and the same than the number of coefficient of variations indicated in the errors dataframe.

If we try to determine the total size of the sample required to satisfy these precision constraints, considering the current stratification of the frame (the 641 atomic strata), we can do it by simply using the function **bethel**. This function requires a slightly different specification of the constraints dataframe:

```
> cv <- swisserrors[1,]
> cv
```

```
DOM CV1 CV2 CV3 CV4 domainvalue
1 DOM1 0.08 0.12 0.08 0.12 1
```

because the bethel function does not permit to differentiate precision constraints by subdomain. In any case, the result of the application of the Bethel algorithm (see Bethel (1989)) is:

> sum(bethel(swissstrata,cv))

[1] 893

That is, the required amount of units to be selected, with no optimization of sampling strata. In general, after the optimization, this number is sensibly reduced.

6 Optimization of frame stratification

Once the strata and the constraints dataframes have been prepared, it is possible to apply the function that optimises the stratification of the frame, that is optimizeStrata. This function operates on all subdomains, identifying the best solution for each one of them. The fundamental parameters to be passed to optimizeStrata are:

- 1. errors: the (mandatory) dataframe containing the precision levels expressed in terms of maximum allowable coefficients of variation that regard the estimates on target variables of the survey
- 2. strata: the (mandatory) dataframe containing the information related to 'atomic' strata, i.e. the strata obtained by the Cartesian product of all auxiliary variables Xs. Information concerns the identifiability of strata (values of Xs) and variability of Ys (for each Y, mean and standard deviation in strata)
- 3. cens: the (optional) dataframe containing the 'take-all' strata, those strata whose units must be selected in whatever sample. It has same structure than strata dataframe
- 4. strcens: flag (TRUE/FALSE) to indicate if 'take-all' strata do exist or not. Default is FALSE
- 5. initialStrata: the initial limit on the number of strata for each solution.

 Default is 3000
- 6. addStrataFactor: this parameter indicates the probability that at each mutation the number of strata may increase with respect to the current value. Default is 0.01
- 7. minnumstr: indicates the minimum number of units that must be allocated in each stratum. Default is 2

- 8. iter Indicated the maximum number of iterations (= generations) of the genetic algorithm. Default is 20
- 9. pops The dimension of each generations in terms of individuals. Default is 50
- 10. mut_chance (mutation chance): for each new individual, the probability to change each single chromosome, i.e. one bit of the solution vector. High values of this parameter allow a deeper exploration of the solution space, but a slower convergence, while low values permit a faster convergence, but the final solution can be distant from the optimal one. Default is 0.05
- 11. elitism_rate: this parameter indicates the rate of better solutions that must be preserved from one generation to another. Default is 0.2.
- 12. highvalue: parameter for genetic algorithm. Its default value should not be changed
- 13. suggestions: optional parameter for genetic algorithm that indicates one possible solution (maybe from previous runs) that will be introduced in the initial population. Default is NULL.
- 14. realAllocation: if FALSE, the allocation is based on INTEGER values; if TRUE, the allocation is based on REAL values. Default is FALSE.
- 15. writeFile: indicates if at the end of the processing the resulting strata will be outputted in a delimited file. Default is "YES".

In the case of the Swiss municipalities, this is a possible choice of the value of the parameters:

```
> solution <- optimizeStrata(
          errors = swisserrors,
          strata = swissstrata,
          cens = NULL,
          strcens = FALSE,
          initialStrata = nrow(strata),
          addStrataFactor = 0.00,
          minnumstr = 2,
          iter = 40,
          pops = 10,
          mut\_chance = 0.05,
          elitism_rate = 0.2,
          highvalue = 1e+08,
          suggestions = NULL,
          realAllocation = TRUE,
          writeFiles = TRUE)
```

```
*** Domain: 1 1
Number of strata: 119
GA Settings
Population size = 10
Number of Generations = 40
Elitism = 2
Mutation Chance = 0.05
> sum(ceiling(solution$aggr_strata$SOLUZ))
```

[1] 464

The execution of optimizeStrata produces the solution of 7 different optimization problems, one for each domain. We have reported in Figure 1 the convergence plot regarding the third domain. The results of the execution are contained in the list 'solution', composed by two elements:

- 1. solution\$indices: the vector of the indices that indicates to what aggregated stratum each atomic stratum belongs;
- 2. solution\$aggr_strata: the dataframe containing information on the optimal aggregated strata.

7 Analysis of results

We want to analyse what kind of aggregation of the atomic strata the genetic algorithm did produce. To do so, we apply the function updateStrata, that assigns the labels of the new strata to the initial one in the dataframe strata, and produces:

- 1. a new file named 'newstrata.txt' containing all the information in the strata dataframe, plus the labels of the new strata;
- 2. a table, contained in the dataset 'strata_aggregation.txt', showing in which way the auxiliary variables Xs determine the new strata.

The function is invoked in this way:

```
> newstrata <- updateStrata(swissstrata, solution, writeFiles = TRUE)
```

Now, the atomic strata are associated to the aggregate strata defined in the optimal solution, by means of the variable LABEL. If we want to analyse in detail the new structure of the stratification, we can look at the 'strata_aggregation.txt' file:

```
> strata_aggregation <- read.delim("strata_aggregation.txt")
> head(strata_aggregation)
```

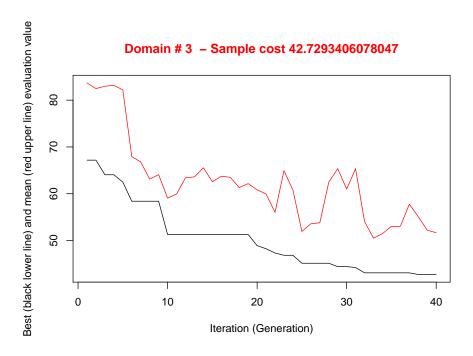


Figure 1: This graph illustrates the convergence of the solution to the final one starting from the initial one (i.e. the one related to the atomic strata). Along the x-axis are reported the executed iterations, from 1 to the maximum, while on the y-axis are reported the size of the sample required to satisfy precision constraints. The upper (red) line represent the average sample size for each iteration, while the lower (black) line represents the best solution found until the i-th iteration.

```
DOM1 AGGR_STRATUM X1 X2 X3 X4 X5 X6
     1
                     1
                        1
                            1
                               1
1
2
3
     1
                     1
                        6
4
     1
                     2
5
                     2
                        2
                            2
     1
                                   2
                                      1
                                          1
                     2 10
                            2
```

In this structure, for each aggregate stratum the values of the X's variables in each contributing atomic stratum are reported. It is then possible to understand the meaning of each aggregate stratum produced by the optimization.

8 Updating the frame and selecting the sample

Once the optimal stratification has been obtained, to be operational we need to accomplish the following two steps:

- 1. to update the frame units with new stratum labels (combination of the new values of the auxiliary variables Xs);
- 2. to select the sample from the frame.

As for the first, we execute the following command:

```
> framenew <- updateFrame(swissframe, newstrata, writeFiles=TRUE)</pre>
```

The function updateFrame receives as arguments the indication of the dataframe in which the frame information is memorised, and of the dataframe produced by the execution of the updateStrata function. The execution of this function produces a dataframe framenew, and also a file (named 'framenew.txt') with the labels of the new strata produced by the optimisation step. The allocation of units is contained in the 'soluz' column of the dataset 'outstrata.txt'. At this point it is possible to select the sample from the new version of the frame:

> sample <- selectSample(framenew, solution\$aggr_strata, writeFiles=TRUE)

```
*** Sample has been drawn successfully ***
464 units have been selected from 216 strata
```

==> There have been 51 take-all strata from which have been selected 81 units

that produces two .csv files:

- 1. 'sample.csv' containing the units of the frame that have been selected, together with the weight that has been calculated for each one of them;
- 2. 'sample.chk.csv' containing information on the selection: for each stratum, the number of units in the population, the planned sample, the number of selected units, the sum of their weights that must equalise the number of units in the population.

9 Evaluation of the found solution

In order to be confident about the quality of the found solution, the function evalSolution allows to run a simulation, based on the selection of a desired number of samples from the frame to which the stratification, identified as the best, has been applied. The user can invoke this function also indicating the number of samples to be drawn:

```
> evalSolution(framenew, solution$aggr_strata, nsampl=50, writeFiles=TRUE)
```

For each drawn sample, the estimates related to the Y's are calculated. Their mean and standard deviation are also computed, in order to produce the CV related to each variable in every domain. These CV's are written to an external csv file:

```
> expected_cv <- read.csv("expected_cv.csv")
> expected_cv
```

```
CV1 CV2 CV3 CV4 dom
1 0.05562670 0.05084577 0.05416888 0.05268873 DOM1
2 0.07399172 0.07860820 0.07645720 0.08082185 DOM2
3 0.07459086 0.07558794 0.07917990 0.08857938 DOM3
4 0.07277576 0.06944485 0.07131466 0.06657854 DOM4
5 0.06083712 0.06264083 0.06399747 0.05989139 DOM5
6 0.07445364 0.07197639 0.07435478 0.07575349 DOM6
7 0.07143245 0.07149233 0.06882229 0.07488334 DOM7
```

These values are on average compliant with the precision constraints set (see also Figure 2).

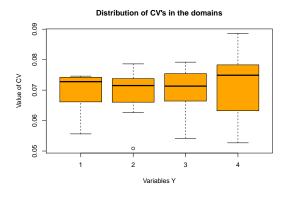


Figure 2: Distribution of the CV's in the different domains for each target variable

Moreover, the estimates of each drawn sample are compared to the known values in the population. The distribution of the differences are reported in the

boxplots of Figure 3. It can be seen that the average of the estimates are on average close to the value zero for all the Y's in all domains.

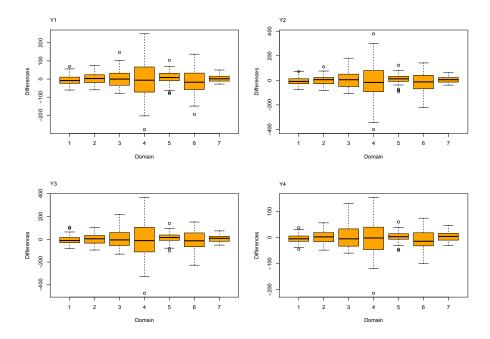


Figure 3: Distribution of the differences between sample estimates and true values of the parameters in the different domains

10 Handling 'take-all' strata in the optimization step

As input to the optimization step, together with proper sampling strata, it is also possible to provide 'take-all' strata. These strata will not be subject to optimisation as the proper strata, but they will contribute to the determination of the best stratification, as their presence in a given domain will permit to satisfy precision constraint with a lower number of units belonging to sampling strata.

In order to correctly execute the optimization and further steps, it is necessary to perform a pre-processing of the overall input. The first step to be executed consists in the bi-partition of units to be censused and of units to be sampled, in order to build two different frames:

- > data(swisserrors)
- > data(swissstrata)
- > data(swissframe)

```
> #----Selection of units to be censused from the frame
> framecens <- swissframe[ (swissframe$domainvalue == 1 |</pre>
                            swissframe$domainvalue == 4) &
                            (swissframe$X2 == 1 &
                             swissframe$X3 == 1 &
                             swissframe$X4 == 1 &
                             swissframe$X5 == 1 &
                             swissframe$X6 == 1) , ]
> #----Selection of units to be sampled from the frame
> # (complement to the previous)
> framesamp <- swissframe[!((swissframe$domainvalue == 1 |</pre>
                              swissframe$domainvalue == 4) &
                             (swissframe$X2 == 1 &
                              swissframe$X3 == 1 &
                              swissframe$X4 == 1 &
                              swissframe$X5 == 1 &
                              swissframe$X6 == 1)) , ]
```

In this way, we have included all units belonging to 'take-all' strata in 'frame-cens', and the remaining in 'framesamp'. At the end of the process, the sample will be selected from 'framesamp', while the units in 'framecens' will be simply added to the sample.

We can obtain census strata and sampling strata by applying 'buildStrataDF' respectively to to 'framecens' and framesamp':

```
> # Build strata to be censused and sampled
> cens <- buildStrataDF(framecens)

Computations have been done on population data
> sum(cens$N)
[1] 405
> strata <- buildStrataDF(framesamp)

Computations have been done on population data
> sum(strata$N)
```

Now we have all required inputs to run 'optimizeStrata' in presence of 'take-all' stata:

```
> solution <- optimizeStrata(
+ errors = swisserrors,
+ strata = strata,
+ cens = cens,</pre>
```

[1] 2491

```
strcens = TRUE,
         initialStrata = nrow(strata),
         addStrataFactor = 0.00,
         minnumstr = 2,
         iter = 40,
         pops = 10,
         mut_chance = 0.05,
         elitism_rate = 0.2,
         highvalue = 1e+08,
         suggestions = NULL,
         realAllocation = TRUE,
         writeFiles = TRUE)
*** Domain : 1 1
Number of strata: 110
GA Settings
 Population size = 10
 Number of Generations = 40
 Elitism
 Mutation Chance
                       = 0.05
```

Once the optimized solution has been produced, the next steps are executed by considering only the sampling part of the frame:

```
> newstrata <- updateStrata(strata, solution)
> # updating sampling frame with new strata labels
> framenew <- updateFrame(frame=framesamp,newstrata=newstrata)
> # selection of sample from sampling strata
> sample <- selectSample(frame=framenew,outstrata=solution$aggr_strata)
*** Sample has been drawn successfully ***
432 units have been selected from 185 strata
==> There have been 39 take-all strata
from which have been selected 78 units
```

Finally, the units in the 'take-all' strata can be added to sampled ones. First, the census frame needs to be made homogeneous to the sample frame in order to permit the 'rbind' step:

```
> # addition of necessary variables to
> colnames(framesamp) <- toupper(colnames(framesamp))
> colnames(framecens) <- toupper(colnames(framecens))
> framecens$WEIGHTS <- rep(1,nrow(framecens))
> framecens$FPC <- rep(1,nrow(framecens))
> framecens$LABEL <- rep("999999",nrow(framecens))
> framecens$STRATUM <- rep("999999",nrow(framecens))
> framecens$STRATO <- rep("999999",nrow(framecens))</pre>
```

```
The overall set of units to be surveyed is obtainable in this way:
```

References

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