The decomposition of the chained price index rate of change: generalization and interpretative effectiveness

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Abstract

The paper deals with the method of decomposition of the rate of change of a chained price index into the sum of the effects of the item groups covered by the index, as suggested by M. Ribe (1999). The aim is twofold: firstly, to provide a generalization of the envisaged method to the case of the decomposition of the average rates of change of the aggregate index over different time intervals (such as, for example, annual or quarterly average rates of change); secondly, to investigate the formal properties of the decomposition in order to evaluate its interpretative effectiveness as a measure of the impact of the evolution of the prices of different components on the overall inflation.

Keywords: Chained price index, decomposition of the price index rate of change, contribution to inflation.

1. Introduction

In analyzing inflation, it is usually important to evaluate to what extent the development of the overall index is influenced by the price changes of one commodity or a group of commodities. However, the estimation of the single sub-indices contribution to inflation is not straightforward when the overall index is calculated as a chained index. To this aim, a method of decomposition of the monthly and annual rates of change of a chained price index into the sum of the effects of its sub-indices has been suggested by M. Ribe [1999]. In the present paper, we argue that the envisaged method can be usefully generalized in order to cover the case of the decomposition of the average rates of change of the overall index over different time intervals (such as, for example, annual or quarterly average rates of change). Moreover, some formal properties of the decomposition are discussed in order to evaluate its interpretative effectiveness as a measure of the impact of distinct sub-indices on the overall inflation.

The paper is structured as follows: after a brief reference to the methodological underpinnings of the chained price index (section 2), Ribe's method is presented (section 3), where the decomposition of the rates of change is calculated both between two different months of the same year and between the same month of two consecutive years. In section 4, we show how Ribe's formulas can be generalized to the case of average rates of change over time periods of different length. The formal properties of this generalised method are considered in section 5, where we investigate the relationship between the price development of a component on a given time period and its effect on the rate of change of

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the overall index. As a result we argue that, generally, a zero yearly rate of change of a sub-index (or even a zero yearly average rate of change) is neither a necessary nor a sufficient condition for its effect on the rate of change of the overall index to be null in the same time interval. In the concluding section, an application of the decomposition method to the Italian Harmonized Index of Consumer Price is presented.

2. Chain system for the construction of the price index

A fixed based index provides a measure of the average changes in the price levels by means of the series of binary comparisons between a base reference period and other reporting periods in a specified time interval. On the contrary, in the chained index approach, binary indices are used only to assess the change of price going from one period to the following. The evolution of the price levels over time is then obtained by linking the rates of change of the single-periods in a sequence[2].

The main advantage of the chain system for the construction of a price index consists in the possibility of updating its base at each step of the sequence. Therefore, from the interpretative point of view, the chained price index (of the Laspeyres type) represents the changing cost of a basket of products which is fixed at the beginning of each period and renewed from period to period. More precisely, with reference to chained price indices computed monthly by most of the National Statistical Institute, the basket and its weighting structure are fixed at December of every year and kept fixed in the following twelve months[3].

Formally, let $I_{0,y}^{n,y}$ be the price index of month *n* of year *y* expressed in base December *y-1* = 1 (conventionally indicated hereafter as month 0 of year *y*). The index is computed as:

$$I_{0,y}^{n,y} = \sum_{k=1}^{K} \pi^{0,y}(k) \cdot I_{0,y}^{n,y}(k) \qquad n = 1, 2, \ldots, 12$$

where the weights $\pi^{0,y}(k)$ and the price indices $I_{0,y}^{n,y}(k)$ of the K components of the overall index are given by:

$$\pi^{0,y}(k) = \frac{q^{0,y}(k) \cdot p^{0,y}(k)}{\sum_{k=1}^{K} q^{0,y}(k) \cdot p^{0,y}(k)}; \qquad I_{0,a}^{n,y}(k) = \frac{p^{n,y}(k)}{p^{0,y}(k)} \qquad k = 1, 2, \ldots, K$$

and $q^{0,y}(k)$ and $p^{0,y}(k)$ are respectively the quantity and the price of the component *k* in month 0 of year *y*.

[2] The chained price index can be considered as an approximation in discrete time of the continuous time Divisia index (see. among others R.D.G. Allen (1975), B.M. Balk (2008), F.G. Forsyth and R.F. Fowler (1981), ILO (2004)).

[3] For more information about the adoption of the chained price index by the Italian Statistical Institute, see Quaranta V., Di Iorio F. (1997).

Assuming that year $(y-j)$ is arbitrarily chosen as the base year (henceforth BY), the series of chained price indices are calculated as follows:

$$I_{BY}^{n,y-j} = \frac{I_{0,y-j}^{n,y-j}}{\bar{I}_{0,y-j}^{y-j}} \quad \text{where} \quad \bar{I}_{0,y-j}^{y-j} = \frac{1}{12}\sum_{n=1}^{12} I_{0,y-j}^{n,y-j}$$

and

$$I_{BY}^{n,y-j+1} = I_{BY}^{12,y-j} \cdot I_{0,y-j+1}^{n,y-j+1} \qquad n = 1,2,\ldots,12$$

$$I_{BY}^{n,y-j+2} = \left(I_{BY}^{12,y-j} \cdot I_{0,y-j+1}^{12,y-j+1}\right) \cdot I_{0,y-j+2}^{n,y-j+2} = I_{BY}^{12,y-j+1} \cdot I_{0,y-j+2}^{n,y-j+2} \qquad n = 1,2,\ldots,12$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$I_{BY}^{n,y} = \left(I_{BY}^{12,y-j} \cdot I_{0,y-j+1}^{12,y-j+1} \cdot I_{0,y-j+2}^{12,y-j+2} \cdot \ldots \cdot I_{0,y-1}^{12,y-1}\right) \cdot I_{0,y}^{n,y} = I_{BY}^{12,y-1} \cdot I_{0,y}^{n,y} \qquad n = 1,2,\ldots,12$$

It is useful to draw attention to the well-known fact that indices expressed in their reference base do not satisfy the additive property. That is[4]:

$$I_{BY}^{n,y} = I_{BY}^{12,y-1} \cdot I_{0,y}^{n,y} \neq \sum_{k=1}^{K} \pi^{y}(k) \cdot I_{BY}^{n,y}(k) = \sum_{k=1}^{K} I_{BY}^{12,y-1}(k) \cdot \pi^{y}(k) \cdot I_{0,y}^{n,y}(k) \cdot$$

3. Ribe's decomposition of the monthly and annual rates of change of a chained price index

We start by considering the decomposition of the rates of change calculated between two months of the same year, and then the attention will be drawn to the case of price changes between the same month of two adjacent years.

Let ${}_{m,y}\Delta_{n,y}$ be the rate of change of the overall index between months *m* and *n* of the same year *y*, or alternatively in the period [(*m,y*); (*n,y*)]:

$${}_{m,y}\Delta_{n,y} = \frac{I_{BY}^{n,y}}{I_{BY}^{m,y}} - 1 = \frac{\sum_{k=1}^{K} \pi^{y}(k) \cdot \left[I_{0,y}^{n,y}(k) - I_{0,y}^{m,y}(k)\right]}{I_{0,y}^{m,y}}$$

where $0 \le m < n \le 12$.

[4] The equality holds only in the special case in which the links of the *K* components of the aggregate index, as well as the link of the aggregate index, are all at the same level. Notably, in order to minimize the notation, concerning the weights, the reference to the base 0 will be omitted from now on.

It is then clearly apparent that:

$$_{m,y}\Delta_{n,y} = \sum_{k=1}^{K} \varepsilon^{m,y}(k) \cdot {}_{m,y}\Delta_{n,y}(k) \qquad (1)$$

in which $\varepsilon^{m,y}(k) \equiv \pi^{y}(k) \cdot \frac{I_{0,y}^{m,y}(k)}{I_{0,y}^{m,y}}$.

The interpretation of (1) is straightforward: the rate of change of the overall index measured between months *m* and *n* can be expressed as the weighted arithmetic mean of the rates of change of its sub-indices, with weights given by $\varepsilon^{m,y}(k)$. Accordingly, it is possible to define a measure of the effect of the change of the price of the component *k* on the overall index[5]:

$$_{m,y}C_{n,y}(k) = \pi^{y}(k) \cdot \frac{I_{0,y}^{m,y}(k)}{I_{0,y}^{m,y}} \cdot {}_{m,y}\Delta_{n,y}(k) \qquad (2)$$

In other words, $_{m,y}C_{n,y}(k)$ provides a measure of the contribution of the sub-index *k* to the dynamics of the overall index, in the period considered.

It is worth noting that if, and only if, $I_{0,y}^{m,y}(k) = I_{0,y}^{m,y}$, the effect of the component *k* can be calculated by multiplying its corresponding rate of change and weight. Specifically, this case occurs when $m = 0$: when the lower bound of the time interval is placed on the month representing the base for the computation of the index of a given year, the equality $I_{0,y}^{m,y}(k) = I_{0,y}^{m,y} = 1$ holds, by definition, for every *k*.

Consider now the yearly rate of change of the overall index:

$$_{n,y-1}\Delta_{n,y} = \frac{I_{BY}^{n,y}}{I_{BY}^{n,y-1}} - 1 = \frac{I_{0,y}^{n,y} \cdot I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} - 1$$

It is, then, possible to write:

[5] By putting m = n-1, expression (2) is equivalent to the monthly effect as defined in Ribe (1999) (see page 5 and 6).

$$ {}_{n,y-1}\Delta_{n,y} = \frac{I_{0,y}^{n,y} \cdot I_{0,y-1}^{12,y-1} - I_{0,y-1}^{12,y-1} + I_{0,y-1}^{12,y-1} - I_{0,y-1}^{n,y-1}}{I_{0,y-1}^{n,y-1}} = $$

$$ = \frac{I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} \cdot {}_{0,y}\Delta_{n,y} + {}_{n,y-1}\Delta_{12,y-1} $$

The decomposition (1) and definition (2) can now be applied to the expression of ${}_{n,y-1}\Delta_{n,y}$:

$$ {}_{n,y-1}\Delta_{n,y} = \frac{I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} \cdot \sum_{k=1}^{K} \pi^{y}(k) \cdot {}_{0,y}\Delta_{n,y}(k) + \sum_{k=1}^{K} \varepsilon^{n,y-1}(k) \cdot {}_{n,y-1}\Delta_{12,y-1}(k) \quad \Rightarrow $$

$$ {}_{n,y-1}\Delta_{n,y} = \sum_{k=1}^{K} \left(\frac{I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} \cdot {}_{0,y}C_{n,y}(k) + {}_{n,y-1}C_{12,y-1}(k) \right) \qquad (3) $$

Finally, the effect of the sub-index k on the yearly rate of change of the overall index is defined according to[6]:

$$ {}_{n,y-1}C_{n,y}(k) = \frac{I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} \cdot {}_{0,y}C_{n,y}(k) + {}_{n,y-1}C_{12,y-1}(k) \qquad (4) $$

4. The generalized decomposition method

In this section, we show how the envisaged decomposition formulas can be generalized to the case of the average rates of change of the overall index over different time intervals. To this end, it can be useful to look firstly at the decomposition of the annual average rate of change, and then consider the more general case.

The rate of change of the aggregate index between year *y-1* and year *y* is given by:

[6] Expression (4) corresponds to the "twelve-month effect" defined in Ribe (1999) (see pages 6-8).

$$
{}_{y-1}\Delta_y = \frac{\overline{I}_{BY}^{y}}{\overline{I}_{BY}^{y-1}} - 1 = \sum_{n=1}^{12}\left[\frac{I_{BY}^{n,y-1}}{\sum_{n=1}^{12} I_{BY}^{n,y-1}} \cdot \left(\frac{I_{BY}^{n,y} - I_{BY}^{n,y-1}}{I_{BY}^{n,y-1}}\right)\right] =
$$

$$
= \sum_{n=1}^{12}\left[\frac{I_{0,y-1}^{n,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}\Delta_{n,y}\right] \qquad (5)
$$

By substituting (3) and (4) in (5), we have:

$$
{}_{y-1}\Delta_y = \sum_{n=1}^{12}\left[\frac{I_{0,y-1}^{n,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot \sum_{k=1}^{K} {}_{n,y-1}C_{n,y}(k)\right] = \sum_{k=1}^{K}\left\{\sum_{n=1}^{12}\left[\frac{I_{0,y-1}^{n,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}C_{n,y}(k)\right]\right\}
$$

The effect of component k on ${}_{y-1}\Delta_y$ can thus be defined in the following way:

$$
{}_{y-1}C_y(k) = \sum_{n=1}^{12}\left[\frac{I_{0,y-1}^{n,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}C_{n,y}(k)\right] \qquad (6)
$$

According to (6), ${}_{y-1}C_y(k)$ corresponds to the weighted arithmetic mean of the contributions of the sub-index k to the yearly rates of change of the overall index, in the twelve months of the current year.

In the more general case, let ${}_{y-1}\Delta_y^{T,\tau}$ be the average rate of change of the overall index calculated in the time interval $(T;\, T+\tau)$, that is:

$$
{}_{y-1}\Delta_y^{T,\tau} = \frac{\sum_{n=T}^{T+\tau} I_{BY}^{n,y}}{\sum_{n=T}^{T+\tau} I_{BY}^{n,y-1}} - 1
$$

where $T = 1, 2, \ldots, 12$ and $0 \leq \tau \leq 12 - T$.

It is easy to show that the following formula holds:

$$_{y-1}\Delta_y^{T,\tau} = \sum_{k=1}^{K} {}_{y-1}C_y^{T,\tau}(k)$$

in which

$$_{y-1}C_y^{T,\tau}(k) = \sum_{n=T}^{T+\tau} \left[\frac{I_{0,y-1}^{n,y-1}}{\sum_{n=T}^{T+\tau} I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}C_{n,y}(k) \right] \qquad (7)$$

As a concluding remark, it is worth noting that expression (4) can be obtained, as a special case, by setting $\tau = 0$ in (7).

5. The effect of sub-indices as a measure of their contribute to overall inflation

The rest of the paper focuses on the formal properties of the decomposition formulas hitherto presented. The principal aim is to investigate the relationship between the rate of change of the index of a component *k* and the measure of its effect on the overall index. Specifically, it will be shown that the two magnitudes not necessarily have the same sign. More precisely, under well defined conditions, the rate of change of a sub-index and its contribution to the overall rate of change have opposite signs. However, this possibility never occurs when the effect on the monthly rate of change of the all-items index is considered. This case will be discussed at first.

5.1 The monthly rate of change of component *k* and its effect on the aggregate index

According to expression (2), the effect of the sub-index *k,* measured by $_{m,y}C_{n,y}(k)$, is a number representing how much of the monthly rate of change of the overall index can be imputed to the development of the price of the component *k,* in the same period.

As the definition of $_{m,y}C_{n,y}(k)$ makes clear, the size of the effect depends on three different factors:

- the rate of change of the sub-index *k* between month *m* and *n* of the current year;
- the relative weight of the sub-index *k*;
- the ratio between the level of the sub-index and the level of the overall index, in the lower bound of the time interval considered.

Regarding the first two factors, the direct relationship tying them to the effect on the overall index is intuitive enough: the higher is ${}_{m,y}\Delta_{n,y}(k)$ and the higher is $\pi^{y}(k)$, the higher is ${}_{m,y}C_{n,y}(k)$. As for the third element, it represents a "scale factor" in the computation of the contribution of the sub-index *k*. That is, assuming that two different sub-indices k_1 and k_2 have the same weight and exhibit the same rate of change in [(*m,y*);(*n,y*)], if $I_{0,y}^{m,y}(k_1)$ is twice as much as $I_{0,y}^{m,y}(k_2)$, their respective contributions to the rate of change of the overall index, in the considered period, will be in the same proportion.

Formally, with reference to expression (2), the following propositions can be immediately verified:

a.1 ${}_{m,y}C_{n,y}(k)$ *is a linear (strictly) increasing function of the monthly rate of change of the index of component k in* [(*m,y*);(*n,y*)].

b.1 *The contribution* ${}_{m,y}C_{n,y}(k)$ *is null if, and only if,* ${}_{m,y}\Delta_{n,y}(k)=0$.

Notably, proposition ***a.1*** and ***b.1*** imply that ${}_{m,y}C_{n,y}(k)$ is a sign conservative function of ${}_{m,y}\Delta_{n,y}(k)$. Moreover, as a corollary of ***b.1***, the rate of change of the aggregate index, that would be measured under the hypothesis of constancy of the sub-index *k* in [(*m,y*);(*n,y*)], is given by the sum of the contributions of the other sub-indices, different from *k*.

Finally, let $K_H=\{k_1, k_2,\ldots,k_H\}$ be a subset of *H* components of the overall index and $I_{0,y}^{n,y}(K_H)$ the corresponding synthetic index. The following proposition can also be easily derived from expression (2)[7]:

c.1 *The contribution of* K_H *to the rate of change of the overall index in [(m,y);(n,y)] is given by the sum of the contributions of the* k_H *components. That is:* ${}_{m,y}C_{n,y}(K_H)=\sum_{k\in K_H}{}_{m,y}C_{n,y}(k)$.

In the next subsection, we will turn the attention to the decomposition of the rates of change of the overall index calculated on two adjacent years. We will argue that, in such a case, the failure to satisfy the additive property by the chained price index has relevant implications on the formal properties of the contribution function.

[7] In practical terms, the proposition ***c.1*** states that the effect of an index corresponding, for example, to a COICOP division on the monthly inflation rate is equal to the sum of the effects of the elementary indices belonging to the same COICOP division.

5.2 The "year on year" rate of change of component *k* and its effect on the aggregate index

Similarly to the previous case, the decomposition (3) allows a measure of the effect of the single sub-index *k* on the yearly rate of change of the all-items index. In other words, it provides the basis for the measurement of the contribution of *k* to the dynamics of the overall index in a given year. However, as shown by expression (4), the effect of the sub-index *k* on the overall index in $[(n, y-1);(n, y)]$ is not generally reducible to the sum of the contributions of *k* on the two subintervals of the year, $\theta_1 = [(n, y-1);(12, y-1)]$ and $\theta_2 = [(0, y);(n, y)]$. Moreover, the effect of the sub-index *k* on the yearly overall rate of change is not independent from the dynamics of the other components in θ_1.

More in detail, taking into account the expressions of ${}_{0,y}C_{n,y}(k)$ and ${}_{n,y-1}C_{12,y-1}(k)$, the contribution function (4) can be written as:

$$ {}_{n,y-1}C_{n,y}(k) = \frac{I_{0,y-1}^{12,y-1}}{I_{0,y-1}^{n,y-1}} \cdot \pi^{y}(k) \cdot {}_{0,y}\Delta_{n,y}(k) + \pi^{y-1}(k) \cdot \frac{I_{0,y-1}^{n,y-1}(k)}{I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}\Delta_{12,y-1}(k) \quad (8) $$

The following propositions can then be stated:

a.2 *${}_{n,y-1}C_{n,y}(k)$ is an increasing function of the rate of change of the index of the component k in both θ_1 and θ_2;*

b.2 *${}_{n,y-1}C_{n,y}(k) = 0$ if, and only if:*

$$ I_{0,y-1}^{12,y-1} \cdot {}_{0,y}C_{n,y}(k) = - I_{0,y-1}^{n,y-1} \cdot {}_{n,y-1}C_{12,y-1}(k) \quad (9) $$

or, in terms of (8):

$$ {}_{0,y}\Delta_{n,y}(k) = - \frac{\pi^{y-1}(k)}{\pi^{y}(k)} \cdot \frac{I_{0,y-1}^{n,y-1}(k)}{I_{0,y-1}^{12,y-1}} \cdot {}_{n,y-1}\Delta_{12,y-1}(k) \quad (10) $$

The interpretation of (9) is straightforward: the expression on the left hand side of the equality, $I_{0,y-1}^{12,y-1} \cdot {}_{0,y}C_{n,y}(k)$, is the amount of the change of the level of the overall index, measured in θ_2 (i.e. $I_{0,y-1}^{n,y} - I_{0,y-1}^{12,y-1}$), that can be imputed to the sub-index *k*. Similarly, the expression at the right hand side, $I_{0,y-1}^{n,y-1} \cdot {}_{n,y-1}C_{12,y-1}(k)$ represents how much of the difference $I_{0,y-1}^{12,y-1} - I_{0,y-1}^{n,y-1}$ can be attributed to the same component. Accordingly, ${}_{n,y-1}C_{n,y}(k)=0$ if the first magnitude exactly offsets the second one.

It should be noted - see (10) - that a trivial case, in which the contribution ${}_{n,y-1}C_{n,y}(k)$ vanishes, occurs when ${}_{0,y}\Delta_{n,y}(k)$ and ${}_{n,y-1}\Delta_{12,y-1}(k)$ are both equal to zero. In this case, the yearly rate of change of the index of the sub-index *k*, ${}_{n,y-1}\Delta_{n,y}(k)$, is zero as well. However, it is important to stress that, generally, the condition ${}_{n,y-1}\Delta_{n,y}(k)=0$ is neither necessary nor sufficient for ${}_{n,y-1}C_{n,y}(k)=0$.

It order to investigate this issue more in depth, it is useful to write the condition (10) as follows:

$$ {}_{0,y}\Delta_{n,y}(k) = -\frac{\pi^{y-1}(k)}{\pi^{y}(k)} \cdot \frac{I_{0,y-1}^{12,y-1}(k)}{I_{0,y-1}^{12,y-1}} \cdot \frac{{}_{n,y-1}\Delta_{12,y-1}(k)}{1+{}_{n,y-1}\Delta_{12,y-1}(k)} \qquad (11) $$

Let ${}_{0,y}\widetilde{\Delta}_{n,y}(k)$ indicate the rate of change of the sub-index *k,* in the subinterval θ_2 that satisfies (11).

Now, the annual rate of change of the sub-index *k,* ${}_{n,y-1}\Delta_{n,y}(k)$ is null if, and only if :

$$ {}_{0,y}\Delta_{n,y}(k) = -\frac{{}_{n,y-1}\Delta_{12,y-1}(k)}{1+{}_{n,y-1}\Delta_{12,y-1}(k)} \equiv -\,{}_{12,y-1}\Delta_{n,y-1}(k) \qquad (12) $$

In a similar way, let ${}_{0,y}\hat{\Delta}_{n,y}(k)$ be the rate of change of the sub-index *k,* in θ_2 that satisfies (12).

Finally, the necessary and sufficient condition for ${}_{n,y-1}C_{n,y}(k)=0$ can be written as:

$$ {}_{0,y}\Delta_{n,y}(k) = {}_{0,y}\widetilde{\Delta}_{n,y}(k) = \frac{\pi^{y-1}(k)}{\pi^{y}(k)} \cdot \frac{I_{0,y-1}^{12,y-1}(k)}{I_{0,y-1}^{12,y-1}} \cdot {}_{0,y}\hat{\Delta}_{n,y}(k) \qquad (13) $$

Equation (13) states that the difference between ${}_{0,y}\widetilde{\Delta}_{n,y}(k)$ and ${}_{0,y}\hat{\Delta}_{n,y}(k)$ depends on the change in the weight of component *k* between the two years and on the *links ratio*, that is on the size of the link of the sub-index *k* with respect to the (weighted arithmetic) mean of the links of all the components. Formally:

$$\left|{}_{0,y}\tilde{\Delta}_{n,y}(k)\right| \underset{<}{\overset{>}{=}} \left|{}_{0,y}\hat{\Delta}_{n,y}(k)\right| \quad \Leftrightarrow \quad \frac{\pi^{y-1}(k)}{\pi^{y}(k)} \cdot \frac{I_{0,y-1}^{12,y-1}(k)}{I_{0,y-1}^{12,y-1}} \underset{<}{\overset{>}{=}} 1 \qquad (14)$$

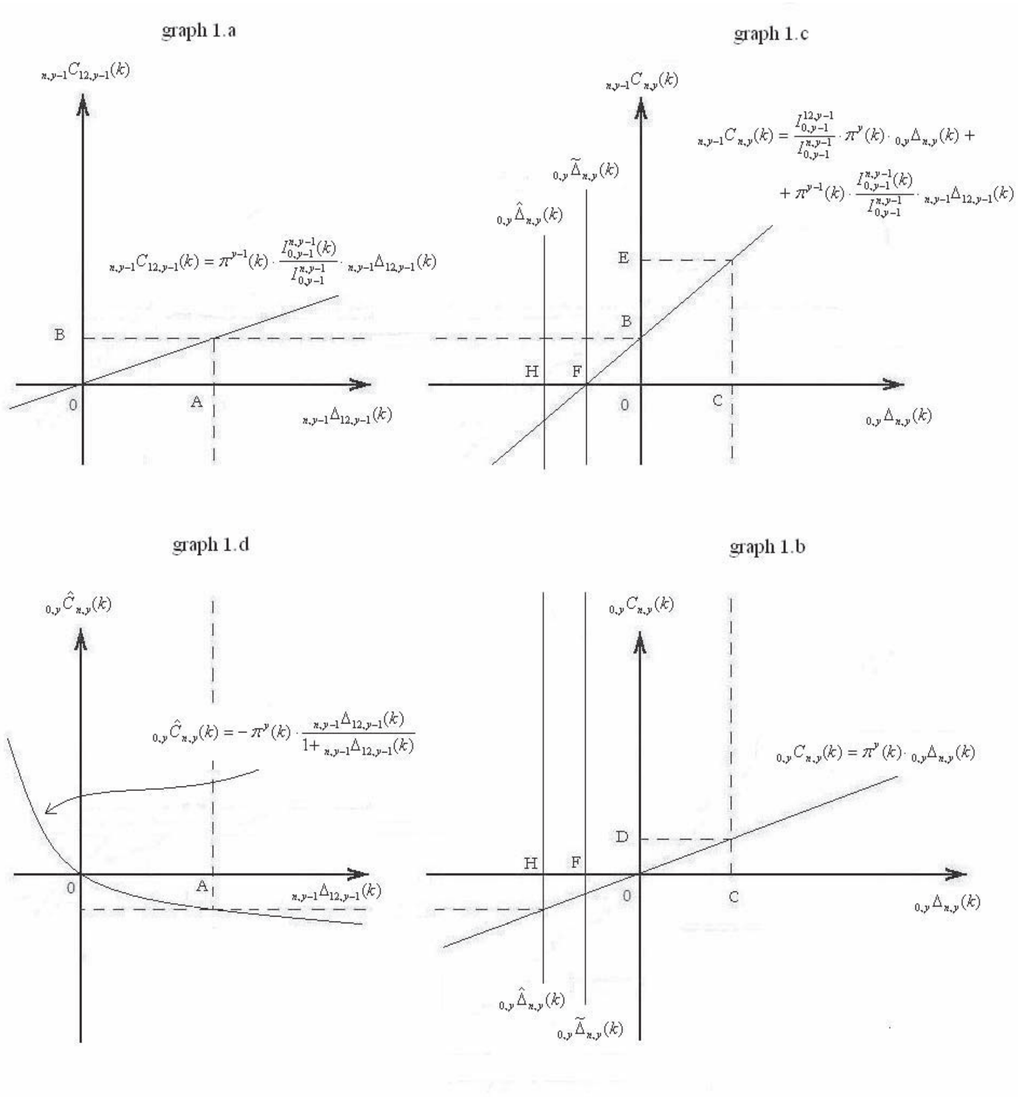
As a crucial implication of the condition (13), the sign of the yearly rate of change of the sub-index *k* and the sign of its contribution to the overall yearly rate of change can differ. In order to clarify this point, it could be useful to show graphically the relation between ${}_{0,y}\tilde{\Delta}_{n,y}(k)$ and ${}_{0,y}\hat{\Delta}_{n,y}(k)$. To this aim, in Figure 1, the contribution function ${}_{n,y-1}C_{12,y-1}(k)$ has been plotted (see graph 1.a). If we assume, for example, that the rate of growth of the sub-index *k*, between month *n* of year *y*-1 and December of the same year, is given by the segment OA, the corresponding effect on the aggregate index, in θ_1, would be equal to OB. Similarly, in graph 1.b, the contribution function ${}_{0,y}C_{n,y}(k)$ is drawn: should the rate of change of the sub-index, in the first *n* months of the following year, be OC, the contribution of *k* to the rate of change of the overall index in θ_2 would be OD, while the effect on the inflation rate would be $OE > OB + OD$ (see graph 1.c).

The point of intersection F between the function ${}_{n,y-1}C_{n,y}(k)$ and the horizontal axis in graph 1.c measures the rate of change of the sub-index *k* in θ_2 that satisfies (10). Moreover, having assumed ${}_{n,y-1}\Delta_{12,y-1}(k) = OA > 0$, the yearly rate of change of the sub-index *k* is zero if (and only if) its rate of decline, in θ_2, is equal to OH (graph 1.b). The position of the point *H* in graph 1.b can be easily set through the curve drawn in graph 1.d. For any given ${}_{n,y-1}\Delta_{12,y-1}(k)$, this curve defines the contribution ${}_{0,y}\hat{C}_{n,y}(k)$ which is associated with the rate of change of *k*, in θ_2, that satisfies condition (12).

In the example shown in the picture, the point H lies at the left of F: in this case, there is a set of different values of ${}_{0,y}\Delta_{n,y}(k)$, in the interval $(H;F)$, for which the yearly rate of change of the sub-index *k* is positive, even though its contribution to the all-items index is negative. However, it is also possible to consider the opposite case, in which the point H is at the right of F. As it has already been noticed, the occurrence of the first or of the second eventuality strictly depends on the condition (14).

It is also important to stress that, as an implication of proposition ***b.2***, the sum of the contributions of the other sub-indices different from *k* cannot be considered as the measure of the yearly rate of change that the overall index would have if it is assumed that ${}_{n,y-1}\Delta_{n,y}(k) = 0$.

Figure 1 - The change of the index of the component k and its effect on the aggregate index.



Lastly, with reference to the subset of H components $K_H = \{k_1, k_2, \ldots, k_H\}$ of the overall index, the following proposition can be proved[8]:

[8] The proof can be obtained by decomposition (3), taking into account that the proposition ***c.1*** applies to ${}_{0,y}C_{n,y}(k)$ and to ${}_{n,y-1}C_{12,y-1}(k)$.

c.2 *The contribution of* K_H *to the rate of change of the overall index in [(n,y-1);(n,y)] is given by the sum of the contributions of the* k_H *components. That is:* ${}_{n,y-1}C_{n,y}(K_H) = \sum_{k \in K_H} {}_{n,y-1}C_{n,y}(k)$.

5.3 The annual average rate of change of *k* and its effect on the overall index

In this subsection we will limit the analysis to the decomposition of the overall yearly average rate of change, given by (6)[9]. Putting expression (4) in (6) and rearranging, we have:

$$ {}_{y-1}C_y(k) = \frac{I_{0,y-1}^{12,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot \sum_{n=1}^{12} {}_{0,y}C_{n,y}(k) + \sum_{n=1}^{12} \frac{I_{0,y-1}^{n,y-1}}{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}} \cdot {}_{n,y-1}C_{12,y-1}(k) $$

If we take into account the expressions of ${}_{0,y}C_{n,y}(k)$ and ${}_{n,y-1}C_{12,y-1}(k)$, with some algebra, it is possible to write the contribution function ${}_{y-1}C_y(k)$ as follows:

$$ {}_{y-1}C_y(k) = \frac{I_{0,y-1}^{12,y-1}}{\bar{I}_{0,y-1}^{y-1}} \cdot \pi^y(k) \cdot {}_{0,y}\Delta_y(k) + \pi^{y-1}(k) \cdot \frac{\bar{I}_{0,y-1}^{y-1}(k)}{\bar{I}_{0,y-1}^{y-1}} \cdot {}_{y-1}\Delta_{12,y-1}(k) \quad (15) $$

where :

- $\bar{I}_{0,y-1}^{y-1} = \frac{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}}{12}$ and $\bar{I}_{0,y-1}^{y-1}(k) = \frac{\sum_{n=1}^{12} I_{0,y-1}^{n,y-1}(k)}{12}$;
- ${}_{0,y}\Delta_y(k) \equiv \left\{\bar{I}_{0,y}^{y}(k) - 1\right\}$ is the measure of the rate of change of the yearly average sub-index *k* with respect to its base, $I_{0,y}^{0,y}(k)$, which is by definition equal to one (conventionally, ${}_{0,y}\Delta_y(k)$ will be referred to as the rate of change on the period $[(0,y);y]$);

[9] However, the results can be easily extended to the more general case.

- ${}_{y-1}\Delta_{12,y-1}(k) \equiv \left\{ \dfrac{I_{0,y-1}^{12,y-1}(k) - \bar{I}_{0,y-1}^{y-1}(k)}{\bar{I}_{0,y-1}^{y-1}(k)} \right\}$ is the measure of the relative change of the December (*y-1*) index of the sub-index *k* with respect to the yearly average index of the same component (similarly, ${}_{y-1}\Delta_{12,y-1}(k)$ will be referred to as the rate of change on period $[y-1;(12,y-1)]$)[10].

Regarding (15), the following propositions hold:

a.3 ${}_{y-1}C_y(k)$ *is an increasing function of* ${}_{0,y}\Delta_y(k)$ *and* ${}_{y-1}\Delta_{12,y-1}(k)$;

b.3 ${}_{y-1}C_y(k)=0$ *if, and only if:*

$$ {}_{0,y}\Delta_y(k) = -\frac{\pi^{y-1}(k)}{\pi^y(k)} \cdot \frac{\bar{I}_{0,y-1}^{y-1}(k)}{I_{0,y-1}^{12,y-1}} \cdot {}_{y-1}\Delta_{12,y-1}(k) $$

or alternatively:

$$ {}_{0,y}\Delta_y(k) = -\frac{\pi^{y-1}(k)}{\pi^y(k)} \cdot \frac{I_{0,y-1}^{12,y-1}(k)}{I_{0,y-1}^{12,y-1}} \cdot \frac{{}_{y-1}\Delta_{12,y-1}(k)}{1+{}_{y-1}\Delta_{12,y-1}(k)} \qquad (16) $$

Let ${}_{0,y}\tilde{\Delta}_y(k)$ be the rate of change of the sub-index *k,* in $[(0,y);y]$, satisfying (16). Moreover, let ${}_{0,y}\hat{\Delta}_y(k)$ be the rate of change of the sub-index *k,* in $[(0,y);y]$, such that ${}_{y-1}\Delta_y(k)=0$. It is easy to show that:

$$ {}_{0,y}\hat{\Delta}_y(k) = -\frac{{}_{y-1}\Delta_{12,y-1}(k)}{1+{}_{y-1}\Delta_{12,y-1}(k)} \qquad (17) $$

[10] It is worth noting that, generally speaking, ${}_{y-1}\Delta_{12,y-1}$ and ${}_{0,y}\Delta_y$ correspond (approximately) to a decomposition of the annual average rate of change ${}_{y-1}\Delta_y$ in two parts: the first one measures the amount of ${}_{y-1}\Delta_y$ which is due to the development of prices in the final months of the year *y-1*, while the second one corresponds to that part of ${}_{y-1}\Delta_y$ which depends on the price changes in the current year. For this reason, sometimes they have been respectively considered the "*inflation rate inherited*" by the year *y* from the year *y-1* and the "*proper inflation rate*" of the year *y*. See Predetti A. (1994), page. 111 and the followings, for more details.

Since generally ${}_{0,y}\tilde{\Delta}_y(k) \neq {}_{0,y}\hat{\Delta}_y(k)$, concerning the decomposition of the overall yearly average rate of change, the condition ${}_{y-1}\Delta_y(k) = 0$ is neither necessary nor sufficient in order to have ${}_{y-1}C_y(k) = 0$.

As in the previous case, proposition ***b.3*** implies that the sum of the contributions of the other components different by *k* cannot be considered as the measure of the annual average rate of change that the overall index would exhibit under the hypothesis of ${}_{y-1}\Delta_y(k) = 0$.

Considering the subset of *H* components $K_H = \{k_1, k_2, \ldots, k_H\}$ of the overall index, it is possible to prove that[11]:

c.3 *The contribution of* K_H *to the rate of change of the overall index in [y-1;y] is given by the sum of the contributions of the* k_H *components. That is:* ${}_{y-1}C_y(K_H) = \sum_{k \in K_H} {}_{y-1}C_y(k)$.

6. Conclusion

In the present paper the method of decomposition of the monthly and yearly rates of change of a chained price index into the sum of the effects deriving from price changes of its sub-indices has been generalized to the case of the average rates of change over different time intervals (such as yearly or quarterly average rates of change). Moreover, the formal properties of the decomposition have been discussed in order to allow an evaluation of its interpretative effectiveness as a measure of the impact of the evolution of the prices of different components on overall inflation. We showed that, in the more general case, the rate of change of a sub-index and its effect on the overall rate of change do not necessarily have the same algebraic sign, that is: the contribution to inflation of the component, in a given time interval, may be positive even though the rate of change of the price index of the same component is negative (and vice versa). As a complement to the previous analysis, it might be of some utility to provide some evidence derived from the Italian Harmonized Index of Consumer Prices (HICP). Specifically, we consider the yearly average rates of change of the HICP sub-indices, corresponding to the three digits of the COICOP-HICP classification and their contribution to the rate of change of the all-items index for year 2009.

[11] Proposition ***c.3*** can be proved from definition (6) and proposition ***c.2***.

Table. 1 - Distribution of the HICP three-digit COICP sub-indices according to the sign of their annual average rate of change and to the sign of their contribution to the yearly average rate of change of the all-items index - Year 2009

		sign of the contribution to the annual average rate of change of the all-items index		total
		+	-	
sign of the annual average rates of change of the HICP components	+	31	1	32
	-	0	7	7
total		31	8	39

Table 1 represents the distribution of the sub-indices according to the sign of their rate of change and to the sign of contribution to rate of change of the overall index (see Table 2 in the Appendix for more details).

One sub-index (C.03.1 – "*clothing*"), out of 39, proves to have a (slight) deflationary effect in the concerned year, even though the corresponding price sub-index exhibits a positive rate of growth.

This result can be easily explained by verifying that: ${}_{0,2009}\hat{\Delta}_{2009}(C.03.1) < {}_{0,2009}\Delta_{2009}(C.03.1) < {}_{0,2009}\tilde{\Delta}_{2009}(C.03.1)$. It is in fact possible to show that, in the case at hand:

$$\begin{cases} {}_{0,2009}\hat{\Delta}_{2009}(C.03.1) = -\,0.0528; \\ {}_{0,2009}\Delta_{2009}(C.03.1) = -\,0.0509; \\ {}_{0,2009}\tilde{\Delta}_{2009}(C.03.1) = -\,0.0505 \end{cases}$$

Counterintuitive as it may appear, this event depends on specific conditions that have been formally investigated in the previous sections of the paper.

Appendix

Table. 2 - Yearly average rates of change and contribution to the yearly inflation of the sub-indices of the Italian HICP - Year 2009

Coicop-hicp	annual average rates of change	contribution to annual inflation
C.00 - All-items HICP	**0.8**	**-**
C.01.1 - Food	1.6	0.271
C.01.2 - Non-alcoholic beverages	1.4	0.018
C.02.1 - Alcoholic beverages	2.8	0.023
C.02.2 - Tobacco	4.1	0.092
C.03.1 - Clothing	0.2	-0.003
C.03.2 - Footwear including repair	1.5	0.032
C.04.1 - Actual rentals for housing	3.3	0.077
C.04.3 - Maintenance and repair of the dwelling	2.9	0.038
C.04.4 - Water supply and miscellaneous services relating to the dwelling	4.7	0.107
C.04.5 - Electricity, gas and other fuels	-5.0	-0.221
C.05.1 - Furniture and furnishings, carpets and other floor coverings	1.6	0.056
C.05.2 - Household textiles	1.3	0.006
C.05.3 - Household appliances	0.4	0.005
C.05.4 - Glassware, tableware and household utensils	2.8	0.023
C.05.5 - Tools and equipment for house and garden	1.6	0.005
C.05.6 - Goods and services for routine household maintenance	1.9	0.056
C.06.1 - Medical products, appliances and equipment	4.5	0.077
C.06.2 - Out-patient services	2.2	0.029
C.06.3 - Hospital services	1.7	0.010
C.07.1 - Purchase of vehicles	1.2	0.057
C.07.2 - Operation of personal transport equipment	-3.6	-0.346
C.07.3 - Transport services	-3.0	-0.069
C.08.1 - Postal services	5.6	0.009
C.08.2 - Telephone and telefax equipment and services	-0.6	-0.015
C.09.1 - Audio-visual, photographic and information processing equipment	-5.9	-0.055
C.09.2 - Other major durables for recreation and culture	1.2	0.004
C.09.3 - Other recreational items and equipment, gardens and pets	0.9	0.012
C.09.4 - Recreational and cultural services	2.6	0.047
C.09.5 - Newspapers, books and stationery	2.4	0.043
C.09.6 - Package holidays	-0.5	-0.002
C.10 - Education	2.7	0.029
C.11.01 - Catering services	2.3	0.205
C.11.02 - Accommodation services	-2.4	-0.072
C.12.01 - Personal care	1.7	0.049
C.12.03 - Personal effects n.e.c.	4.4	0.053
C.12.04 - Social protection	2.4	0.020
C.12.05 - Insurance	2.7	0.038
C.12.06 - Financial services n.e.c.	3.1	0.028
C.12.07 - Other services n.e.c.	2.0	0.025

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