Recent advances in time series analysis

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The analysis of current economic conditions is done by assessing the real-time trend-cycle of major economic indicators (*leading, coincident and lagging*) using percentage changes, based on seasonally adjusted data calculated for months or quarters in chronological sequence. This is known as recession and recovery analysis.

Major economic and financial changes of global nature have introduced more variability in the data ⇒ statistical agencies have shown an interest in providing trend-cycle or smoothed seasonally adjusted graphs to facilitate recession and recovery analysis.

The linear filter developed by Henderson (1916) is the most frequently applied to estimate the trend-cycle component of seasonally adjusted economic indicators.

It is available in nonparametric seasonal adjustment software, such as the U.S. Bureau of the Census X11 method (Shiskin et al., 1967) and its variants, X11ARIMA, X12ARIMA, and X13.

The Henderson smoother has the property that fitted to exact cubic functions will reproduce their values, and fitted to stochastic cubic polynomials it will give smoother results than those obtained by ordinary least squares.

The study of the properties of the Henderson filters have been extensively discussed by many authors, among them, Cholette (1981); Kenny and Durbin (1982); Dagum and Laniel (1987); Dagum (1996); Gray and Thomson (1996); Loader (1999); Ladiray and Quenneville (2001); Findley and Martin (2006); Dagum and Luati (2009a, 2012).

Dagum and Bianconcini (2008) are the first to derive the symmetric Henderson smoother using the Reproducing Kernel Hilbert Space (RKHS) methodology.

A RKHS is a Hilbert space characterized by a kernel that reproduces, via an inner product, every function of the space or, equivalently, a Hilbert space of real valued functions with the property that every point evaluation functional is bounded and linear.

The RKHS approach followed in our study is strictly nonparametric.

Berlinet (1993) $\rightarrow$ A kernel estimator of order p can always be decomposed into the product of a reproducing kernel R_{p-1}, belonging to the space of polynomials of degree at most $p-1$, times a probability density function f_0 with finite moments up to order $2p$.

Asymmetric filters associated with the Henderson filter of length $2m+1$ are developed by Musgrave (1964), and applied to the m first and last observations.

They are derived to minimize the mean squared revision between final and preliminary estimates subject to the constraint that the sum of the weights is equal to one. The assumption made is that at the end of the series, the seasonally adjusted values follow a linear trend-cycle plus a purely random irregular.

Several authors have studied the statistical properties of the Musgrave filters, among others, Laniel (1985); Doherty (2001); Gray and Thomson (2002); Quenneville et al. (2003); Dagum and Luati (2009b, 2012).

Dagum and Bianconcini (2008, 2013) $\rightarrow$ first to introduce a RKHS approach to derive asymmetric filters that were close to those of Musgrave (1964).

Trend estimates for the first and last m data points are subject to revisions due to new observations entering in the estimation and to filter changes.

Reduction of revisions due to filter changes $\rightarrow$ important property that the asymmetric filters should possess together with a fast detection of turning points.

In the RKHS framework, the bandwidth parameter strongly affects the statistical properties of the asymmetric filters.

Aim of the study

We propose time-varying bandwidth parameters in agreement with the time-varying asymmetric filters.

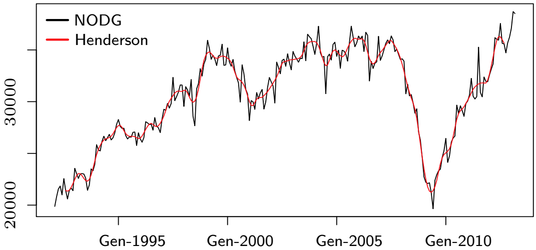
Criteria of bandwidth selection based on the minimization of

1. the distance between the gain functions of asymmetric and symmetric filters $\rightarrow$ *reliability*
2. the phase shift function over the domain of the signal $\rightarrow$ *timeliness*
3. the distance between the transfer functions of asymmetric and symmetric filters $\rightarrow$ optimal compromise between reducing revisions (*increasing reliability*) and reducing phase shift (*increasing timeliness*).

Another important property is the time path of the last point predicted trend as new observations are added to the series. An optimal asymmetric filter should have a *time path that converges fast and monotonically to the final estimates*.

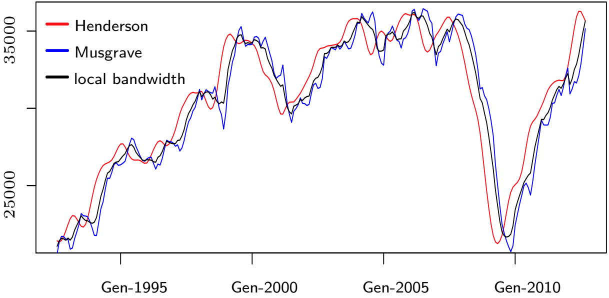
US New Orders for Durable Goods

Important indicator of the state of the economy, often allowing to detect shifts in the US economy up to six months in advance.



New Orders for consumer Durable Goods, US [February 1992 - March 2013]: original series and two-sided nonparametric trend estimates obtained by the Henderson filter. Source: U.S. Census Bureau.

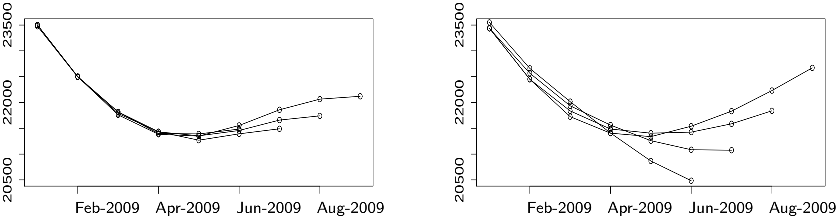
Reducing the size of real time trend-cycle revisions



New Orders for consumer Durable Goods, US: trend-cycle estimates based on symmetric Henderson filter, last point Musgrave and RKHS filters based on local bandwidth parameters.

Detection of true turning points

The reduction in the revisions can be achieved at the expenses of an increase in the time lag of detecting a true turning point.



*US NODG series: revision path before June 2009 turning point of the optimal asymmetric kernel (*left*) and Musgrave (*right*) filters.*

Linear filters in RKHS

Basic assumptions

$$y_t = g_t + u_t \qquad t = 1, \cdots, N$$

- $\{y_t, t = 1, \cdots, N\}$ input series: seasonally adjusted or without seasonality.
- u_t noise: either a white noise, $WN(0, \sigma_u^2)$, or an ARMA process.
- $g_t, t = 1, \cdots, T$, signal: smooth function of time, *locally* represented by a polynomial of degree p in a variable j, which measures the distance between y_t and its neighboring observations $y_{t+j}, j = -m, ..., m$.

This is equivalent to estimate the trend-cycle $\hat{g}_t$ as a weighted moving average as follows

$$\hat{g}_t = \sum_{j=-m}^{m} w_j y_{t+j} = \mathbf{w}'\mathbf{y} \qquad t = m+1, \cdots, N-m,$$

- $\mathbf{w}' = \begin{bmatrix} w_{-m} & \cdots & w_0 & \cdots & w_m \end{bmatrix}$: weights
- $\mathbf{y}' = \begin{bmatrix} y_{t-m} & \cdots & y_t & \cdots & y_{t+m} \end{bmatrix}$: input series.

Several nonparametric estimators, based on different sets of weights $\mathbf{w}$, have been developed in the literature.

Henderson kernel representations

[Henderson, 1916;Kenny and Durbin, 1982; Ladiray and Quenneville, 2001]

The Henderson filter consists of fitting a cubic polynomial by means of weighted least squares to the input $\mathbf{y}$, where the weights are given by $W_j \propto \{(m+1)^2 - j^2\}\{(m+2)^2 - j^2\}\{(m+3)^2 - j^2\}$.

[Loader, 1999]

- $\hat{g}_t = \sum_{j=-m}^{m} \phi(j) W_j y_{t+j}$.
  - $\phi(j)$: cubic polynomial on j.
- For large $m \rightarrow$ equivalent kernel representation, witht W_j approximated by the tri-weight function $m^6(1-(j/m)^2)^3$, such that the weight diagram is approximately $\frac{315}{512}(3 - 11(j/m)^2)(1-(j/m)^2)^3$.

RKHS representation

[Dagum and Bianconcini 2008 and 2013]

$$K_4(t) = \sum_{i=0}^{3} P_i(t)P_i(0)f_0(t) \qquad t \in [-1, 1]$$

- Dagum and Bianconcini (2008) proposed the biweight function $f_{0B}(t) = \frac{15}{16}(1 - t^2)^2, t \in [-1, 1]$: better approximation than the triweight function when the Henderson filters are of short length, such as 5, 7, 9, 13 and 23 terms.
- $P_i, i = 0, 1, 2, 3,$: corresponding orthonormal polynomials.

Equivalently,

$$K_4(t) = \frac{\det(\mathbf{H}_4^0[1, \mathbf{t}])}{\det(\mathbf{H}_4^0)} f_{0B}(t) \qquad t \in [-1, 1]$$

- $\mathbf{H}_4^0$ is the Hankel matrix whose elements are the moments of f_0, that is $\mu_r = \int_{-1}^{1} t^r f_0(t) dt$.

- $\mathbf{H}_4^0[1, \mathbf{t}]$ is the matrix obtained by replacing the first column of $\mathbf{H}_4^0$ by the vector $\mathbf{t} = \left[\begin{array}{cccc} 1 & t & t^2 & t^3 \end{array}\right]'$.

Applied to real data,

$$w_j = \left[\frac{\mu_4 - \mu_2 \left(\frac{j}{b}\right)^2}{S_0\mu_4 - S_2\mu_2} \right] \frac{1}{b} f_{0B}\left(\frac{j}{b}\right) \qquad j = -m, \cdots, m.$$

- $S_r = \sum_{j=-m}^{m} \frac{1}{b}\left(\frac{j}{b}\right)^r f_0\left(\frac{j}{b}\right)$: discrete approximation of μ_r (function of m and b).

Fundamental choice of the bandwidth parameter b

- to ensure that only $2m+1$ data values surrounding the target point will receive nonzero weight;
- approximate, as close as possible, the continuous moments with the discrete ones, as well as the biweight density function;
- global time-invariant bandwidth b equal to $m+1$ [Dagum and Bianconcini 2008 and 2013].

Asymmetric filters in RKHS

At both ends of the sample period, only $2m, \cdots, m+1$ data values are available $\Rightarrow$ the effective domain of the kernel function K_4 is not $[-1,1]$ as for any interior point, but $[-1, q^*]$, where $q^* = q/b, q = 0, ..., m-1$.

- Symmetry of the kernel is lost $\rightarrow \int_{-1}^{q^*} K_4(t)dt \neq 1$.

- Moment conditions are not longer satisfied, that is $\int_{-1}^{q*} t^i K_4(t)dt \neq 0$ for $i = 1, 2, 3$.

"Cut-and-normalize" boundary kernels [Gasser and Muller, 1979; Kyung-Joon and Schucany, 1998]

$$K_4^{q*}(t) = \frac{K_4(t)}{\int_{-1}^{q^*} K_4(t)dt} \quad t \in [-1, q^*].$$

Equivalently,

$$K_4^{q*}(t) = \frac{\det(\mathbf{H}_4^0[1, \mathbf{t}])}{\det(\mathbf{H}_4^0[1, \boldsymbol{\mu}^{q*}])} f_{0B}(t) \quad t \in [-1, q^*].$$

$\mu_r^{q*} = \int_{-1}^{q*} t^r f_{0B}(t) dt$ being proportional to the moments of the truncated biweight density f_{0B} on the support $[-1, q^*]$.

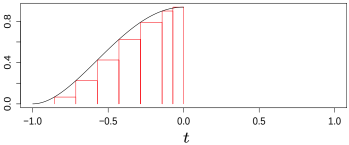
Applied to real data, $w_j = \left[\frac{\mu_4 - \mu_2 \left(\frac{j}{b_q} \right)^2}{S_0^q \mu_4 - S_2^q \mu_2} \right] \frac{1}{b_q} f_{0B} \left(\frac{j}{b_q} \right)$

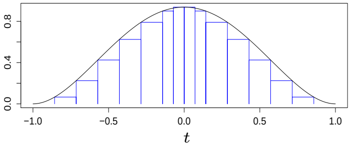
$j = -m, \cdots, q; q = 0, \cdots, m-1$

- $S_r^q = \sum_{j=-m}^{q} \frac{1}{b_q} \left(\frac{j}{b_q} \right)^r f_{0B} \left(\frac{j}{b_q} \right)$ is the discrete approximation of truncated continuous moment μ_r^{q*} (function of m, q, and b_q).
- $b_q, q = 0, \cdots, m-1$: time-varying local bandwidths, specific for each asymmetric filter.

In this study, we select optimal time-varying bandwidth b_q in order to improve the statistical properties of the filters for real time trend-cycle prediction.

- Reduction of the revisions due to time-varying filters for the last m data points.
  - function of the relationship between the truncated S_r^q and untruncated S_r discrete moments, and their respective density functions.





Left: Truncated biweight density (black) and discrete approximation (red). Right: Biweight density (black) and discrete approximation (blue).

- Fast detection of turning points.

Frequency domain analysis

The main effects induced by a linear filter on a given input are fully described by the Fourier transform of the filter weights, $w_j, j = -m, \cdots, m$,

$$\begin{aligned} \Gamma(\omega) &= \sum_{j=-m}^{m} w_j \exp(-i2\pi\omega j) \\ &= G(\omega)\exp(-i2\pi\phi(\omega)) \qquad \omega \in [-1/2, 1/2] \end{aligned}$$

- $G(\omega) = |\Gamma(\omega)|$: gain function $\rightarrow$ it measures the amplitude of the output for a sinusoidal input of unit amplitude.
- $\phi(\omega)$: phase function $\rightarrow$ it shows the shift in phase of the output compared with the input.

$\Gamma(\omega)$ plays a fundamental role to measure that part of revisions due to filter changes.

Measure of total revisions [Musgrave, 1964]

$$E\left[\sum_{j=-m}^{q} w_{q,j} y_{t-j} - \sum_{j=-m}^{m} w_j y_{t-j}\right]^2 \qquad q = 0, \cdots, m-1$$

This criterion can be also reexpressed in the frequency domain as follows

$$\int_{-1/2}^{1/2} |\Gamma_q(\omega) - \Gamma(\omega)|^2 e^{i4\pi\omega t} h_y(\omega) d\omega$$

- $h_y(\omega)$: unknown spectral density of y_t
- $\Gamma_q(\omega)$ and $\Gamma(\omega)$: transfer functions corresponding to the asymmetric and symmetric filters, respectively.

The total revisions are function of filter changes and new innovations entering in the input series. The quantity $|\Gamma_q(\omega) - \Gamma(\omega)|^2$ accounts for the revisions due to filter changes.

Using the law of cosines, [Wildi, 2008]

$$\int_0^{1/2} |\Gamma_q(\omega) - \Gamma(\omega)|^2 d\omega = \int_0^{1/2} |G_q(\omega) - G(\omega)|^2 d\omega + 4\int_0^{1/2} G_q(\omega)G(\omega) \sin\left(\phi\left(\frac{\omega}{2}\right)\right)^2 d\omega$$

- $\int_0^{1/2} |G_q(\omega) - G(\omega)|^2 d\omega$: part of the total mean square filter error which is attributed to the amplitude function of the asymmetric filter.
- $\int_0^{1/2} G_q(\omega)G(\omega) \sin\left(\phi\left(\frac{\omega}{2}\right)\right)^2 d\omega$ measures the distinctive contribution of the phase shift.

Optimal bandwidth criteria

$$b_{q,\Gamma} = \min_{b_q} \sqrt{2\int_0^{1/2} |\Gamma_q(\omega) - \Gamma(\omega)|^2 d\omega}$$

$$b_{q,G} = \min_{b_q} \sqrt{2\int_0^{1/2} |G_q(\omega) - G(\omega)|^2 d\omega}$$

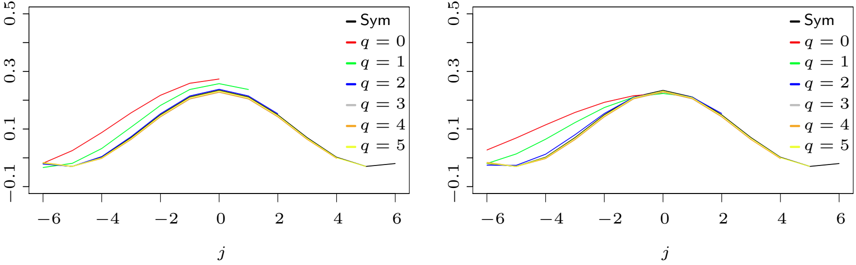
$$b_{q,\phi} = \min_{b_q} \sqrt{2\int_{\Omega_S} G_q(\omega)G(\omega)\left[1-\cos(\phi_q(\omega))\right]} \approx \min_{b_q}\left[\frac{1}{0.06}\int_{\Omega_S}\frac{\phi(\omega)}{2\pi\omega}d\omega\right]$$

Optimal bandwidth values (13-term symmetric filter)

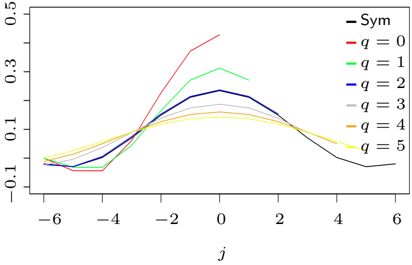
q	0	1	2	3	4	5
$b_{q,\Gamma}$	9.54	7.88	7.07	6.88	6.87	6.94
$b_{q,G}$	11.78	9.24	7.34	6.85	6.84	6.95
$b_{q,\phi}$	6.01	6.01	7.12	8.44	9.46	10.39

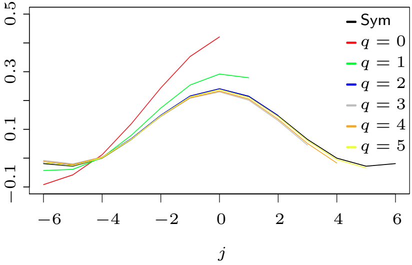
The main impact of these bandwidth parameters on the weight given to the last input point is that the larger is the bandwidth, the closer is the weight to the corresponding final one. This means that a larger b implies smaller variance and larger bias.

Time path of the asymmetric filters based on $b_{q,\Gamma}$ (left) and $b_{q,G}$ (right)

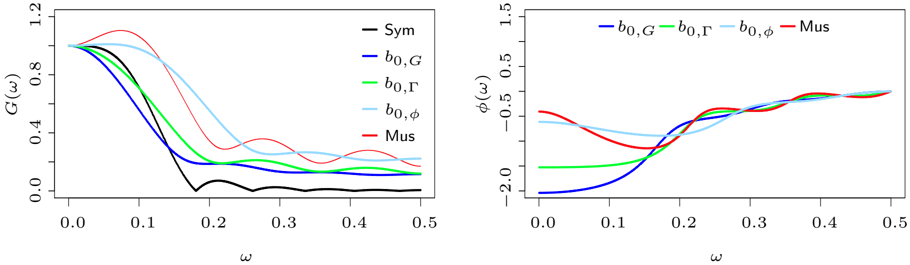


Time path of the asymmetric filters based on $b_{q,\phi}$ (left) and of the Musgrave asymmetric filters (right)

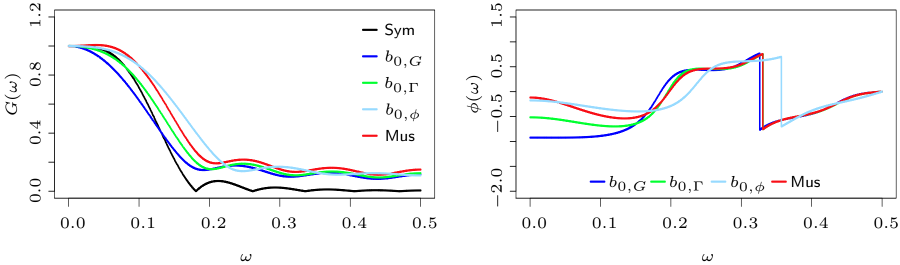




Gain (left) and phase shift (right) functions for the last point asymmetric filters based on $b_{0,\Gamma}$, $b_{0,G}$, and $b_{0,\phi}$ compared with the last point Musgrave filter.



Gain (left) and phase shift (right) functions for the previous to the last point asymmetric filters based on $b_{1,\Gamma}$, $b_{1,G}$, and $b_{1,\phi}$ compared with the previous to the last point Musgrave filter.



Empirical illustration

We evaluate the properties of the asymmetric filters derived following the RKHS methodology versus the Musgrave ones in terms of both total revision reduction and detection of turning points.

- Set of series consisting of leading, coincident, and lagging indicators of the US economy.
- The series considered are seasonally adjusted and cover different periods.
- Symmetric filter lengths selected according to the I/C (noise to signal) ratio. In the sample, the ratio ranges from 0.20 to 1.98, hence filters of length 9 and 13 terms are applied.

Reduction of total revisions of real time trend-cycle estimates

The comparisons are based on the relative trend-cycle revisions between the final F and last point L estimates, that is,

$$R_t = \frac{F_t - L_t}{F_t}, \quad t = m+1, \cdots, N-m$$

For each series, we calculate the ratio between the Mean Square Percentage Error (MSPE) of the revisions derived following the RKHS methodology and those corresponding to the last point Musgrave filter.

Macro-area	Series	$\frac{b_{0,G}}{Mus}$	$\frac{b_{0,\Gamma}}{Mus}$	$\frac{b_{0,\phi}}{Mus}$
Leading	Average weekly overtime hours: manufacturing	0.492	0.630	0.922
	New orders for durable goods	0.493	0.633	0.931
	New orders for nondefense capital goods	0.493	0.633	0.931
	New private housing units authorized by building permits	0.475	0.651	0.927
	S&P 500 stock price index	0.454	0.591	0.856
	M2 money stock	0.508	0.655	0.932
	10-year treasury constant maturity rate	0.446	0.582	0.849
	University of Michigan: consumer sentiment	0.480	0.621	0.912
Coincident	All employees: total nonfarm	0.517	0.666	0.951
	Real personal income excluding current transfer receipts	0.484	0.627	0.903
	Industrial production index	0.477	0.616	0.884
	Manufacturing and trade sales	0.471	0.606	0.869
Lagging	Average (mean) duration of unemployment	0.509	0.649	0.937
	Inventory to sales ratio	0.483	0.618	0.894
	Index of total labor cost per unit of output	0.515	0.663	0.983
	Commercial and industrial loans at all commercial banks	0.473	0.610	0.871

Timeliness of the real time trend-cycle estimates based on $b_{0,G}$, $b_{0,\Gamma}$, $b_{0,\phi}$, and Musgrave filters (in number of months)

Macro-area	Series	$b_{0,G}$	$b_{0,\Gamma}$	$b_{0,\phi}$	$Musgrave$
Leading	Average weekly overtime hours: manufacturing	2	2	1	1
	New orders for durable goods	2	2	1	1
	New orders for nondefense capital goods	2	2	1	2
	New private housing units authorized by building permits	1	1	1	1
	S&P 500 stock price index	1	1	1	1
	10-year treasury constant maturity rate	1	1	1	1
	University of Michigan: consumer sentiment	2	2	1	1
Coincident	All employees: total nonfarm	1	1	1	1
	Real personal income excluding current transfer receipts	1	1	1	1
	Industrial production index	2	2	1	1
	Manufacturing and trade sales	1	1	1	1
Lagging	Average (mean) duration of unemployment	1	1	1	1
	Inventory to sales ratio	2	2	1	1
	Index of total labor cost per unit of output	1	1	1	1
	Commercial and industrial loans at all commercial banks	1	1	1	1

Turning point detection

A turning point is generally defined to occur at time t if (downturn):

$$y_{t-k} \leq \cdots \leq y_{t-1} > y_t \geq y_{t+1} \geq \cdots \geq y_{t+m}$$

or (upturn)

$$y_{t-k} \geq \cdots \geq y_{t-1} < y_t \leq y_{t+1} \leq \cdots \leq y_{t+m}$$

Here, $k = 3$ and $m = 1$ given the smoothness of the trend cycle data [Zellner et al., 1991].

Time lag in detecting true turning points based on $b_{q,G}$, $b_{q,\Gamma}$, $b_{q,\phi}$, and Musgrave filters

Macro-area	Series	$b_{q,G}$	$b_{q,\Gamma}$	$b_{q,\phi}$	$Musgrave$
Leading	Average weekly overtime hours: manufacturing	1	1	1	1
	New orders for durable goods	1	2	3	2
	New orders for nondefense capital goods	1	2	2	3
	New private housing units authorized by building permits	2	2	3	3
	S&P 500 stock price index	1	2	2	2
	10-year treasury constant maturity rate	1	1	1	2
	University of Michigan: consumer sentiment	1	1	1	1
Coincident	All employees: total nonfarm	1	1	1	2
	Real personal income excluding current transfer receipts	1	1	1	1
	Industrial production index	1	1	1	1
	Manufacturing and trade sales	1	2	3	3
Lagging	Average (mean) duration of unemployment	3	3	4	3
	Inventory to sales ratio	1	1	1	2
	Index of total labor cost per unit of output	2	2	3	2
	Commercial and industrial loans at all commercial banks	1	1	1	1
	Average time lag in months	1.27	1.67	1.93	2.00

Conclusions

We made use of the RKHS methodology, according to which hierarchies of kernels are generated via the multiplication of the biweight density function with corresponding orthonormal polynomials.

Optimal bandwidth parameters have been selected to ensure optimal boundary kernels in terms of reducing total revisions and fast detection of turning points as new observations are added to the series.

We applied the new set of asymmetric filters to leading, coincident and lagging indicators of the US economy.

The empirical results show that the bandwidth selected to minimize the distance between the gain functions of the asymmetric and symmetry filters should be preferred since they gave 50% reduction of total revisions relative to the Musgrave filter and smaller time lag in detecting true turning points.

Thank you for your time

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